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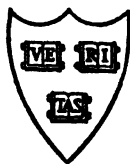
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KEY
TO THE
NATIONAL ARITHMETIC,

EXHIBITING THE OPERATION OF
THE MORE DIFFICULT QUESTIONS

IN THAT WORK;
FOR THE USE OF TEACHERS ONLY.

By BENJAMIN GREENLEAF, A. M.,
AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," ETC.

NEW ELECTROTYPE EDITION.

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P R E F A C E .

THE object of the author, in this publication, is to aid the teacher in communicating instruction to his pupils, and in detecting any error which they may have made in the operation of the examples.

Every instructor who has a large number of scholars under his care is aware that it is a great tax on his time, especially when in school, to examine the operation of many arithmetical questions; whereas, by the aid of a Key, he may readily detect any mistake in the operation. Besides, amid the labors of the school-room, it is often very difficult for the most able arithmetician to recollect, at the moment, all the principles involved in the solution of difficult questions; but, by recurring to a Key, this difficulty will be obviated.

The author would recommend to teachers never to point out *directly* to the pupil the method of solving a problem, nor perform the labor for him; but suggest and explain such principles as will enable him to perform the question himself.

The answers to all the examples in the Arithmetic are inserted in the Key, for the convenience of those teachers who may prefer to use the edition of the Arithmetic which does not contain the answers.

B. GREENLEAF.

BRADFORD, MASS., *September*, 1857.

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KEY

TO

GREENLEAF'S ARITHMETIC.

NUMERATION.

1. (Art. 40, p. 22.)	29
2.	407
3.	23,007
4.	5,000,027
5.	7,205,005
6.	2,207,604,009
7.	105,909,308,201
8.	9,000,000,008,000,000,046
9.	15,000,000,000,031,000,017
10.	507,000,000,000,203,000,057,000,018
11.	9,000,000,000,000,000,047,007,002,000,392
12.	15,000,000,000,000,000,000,000,010,127,026,320,426

NOTE. — The above is the French method.

ADDITION.

(ART. 47, p. 26.)

17.	30,530	24.	300,000	31.	3,837,156
18.	31,643	25.	264,088	32.	150
19.	26,798	26.	357,477	33.	26,199
20.	28,578	27.	276,605	34.	264
21.	34,383	28.	3,980,839	35.	4,801,393
22.	29,340	29.	4,183,478	36.	5,067,696
23.	283,649	30.	31,881,050	37.	5,840,428

1*

(5)

38.	4,344,737	42.	62,075	46.	119
39.	4,935,497	43.	10,601	47.	\$ 228
40.	1,937,678	44.	11,087	48.	\$ 37,443
41.	118,106	45.	\$ 82,871		

(ART. 48, p. 28.)

2.	296	4.	25,976	6.	936,318
3.	1,832	5.	643,322	7.	23,191,876

SUBTRACTION.

4. (ART. 52, p. 31.)	5,676	24.	408,881,883,715
5.	5,119	25.	61,475,423
6.	4,409	26.	999,999
7.	2,589	27.	1
8.	48,447	28.	6,686,136
9.	46,698	29.	760,702,380
10.	17,672	30.	31,309,891
11.	53,859	31.	16,680,605
12.	411,001	32.	10,014,098,379
13.	426,944	33.	85
14.	6,202,102	34.	110
15.	799,081	35.	190
16.	1,439	36.	993,044
17.	92,690	37.	\$ 11,819
18.	243,334	38.	
19.	617,441	39.	173
20.	900,981	40.	1,026
21.	98,999,080	41.	4,004
22.	788,889	42.	45 and 38
23.	9,393,239,896,461	43.	519,853,026
1. (ART. 53, p. 33.)	138	5.	1,237,311
2.	525 dollars.	6.	2,500,000
3.	389 dollars.	7.	49,632 dollars.
4.	9,187 dollars.	8.	572,206 dollars.

MULTIPLICATION.

5. (ART. 63, p. 40.)	3,156,492	26.	59,784
6.	6,172,835	27.	3,545,304
7.	1,979,796	28.	584,720,181,340
8.	30,316,704	29.	594,731,545
9.	16,294,896	30.	119,109,094,835
10.	22,204,188	31.	406,781,410,014
11.	45,005,091	32.	318,697,622,634
12.	77,377,566	33.	230,896,467,247
13.	293,468,329	34.	137,260,338,494
14.	161,539,842	35.	213,255,462,816
15.	274,135,320	36.	395,018,272
16.	17,247,986,832	37.	70,136,114,040
17.	\$26,645	38.	475,065,601,536
18.	\$5,529	39.	20,406,081,008,060,402
19.	\$2,779	40.	915,527,086,788,307
20.	\$21,053	41.	454,115,186,861,492
21.	13,505	42.	12,032,109,124,168,023
22.	24,386	43.	81,000,108,000,036
23.	\$4,886	44.	52,370,625
24.	4,888	45.	114,972
25.	9,021	46.	29,657,416,470,704

(ART. 64, p. 42.)

2.	252,801	5.	2,639,559,272
3.	\$11,025	6.	897,264
4.	2,784		

(ART. 64, p. 43.)

3.	18,190	8.	63,126,063,000
4.	410,600	9.	3,720
5.	70,000,000	10.	\$888,000
6.	9,594,000,000	11.	\$2,050,000
7.	700,000,000	12.	2,850,000,000

DIVISION.

(ART. 77, p. 49.)				Quotients.	Rem.
	Quotients.	Rem.			
	25,569	2	30.	17,327	
8.			31.	69,255	1
9.	151,617	2	32.	71,451	
10.	66,930	2	33.	8,650	111
11.	12,090,447	2	34.	90,365	28
12.	20,747	8	35.	4,598	297
13.	39,936	4	36.	226,447	174
14.	260,171	28	37.	5,091	5091
15.	68,241	80	38.	7,060,504	4267
16.	111,946,492	1	39.	88,888	2341
17.	23,762,387	2	40.	800,008	4567
18.	20,166,474	1	41.	908,007,004	8765
19.	17,964,186	4	42.	2,069	
20.	33,081,425	3	43.	2,700 pounds.	
21.	13,698,246	4	44.	134	
22.	26,316,692	1	45.	987	
23.	169,739,167	3	46.	17	
24.	133,557,795	1	47.	85 $\frac{1}{2}$	
25.	129,629,629	3	48.	384 $\frac{1}{3}$ hours.	
26.	126,984,126	6	49.	\$12,402	
27.	17,166	0	50.	35	
28.	153,227	44	51.	A's 76; B's 68; C's 48.	
29.	275,175	0			
3.	(ART. 78, p. 51.)	321	7.	138	45
4.		308 1	8.	278	18
5.		38 38	9.	121 $\frac{1}{2}$	
6.		507 40			
3.	(ART. 79, p. 52.)		8.	37	411,111
	12,345,678	9	9.	89,765	432,156
4.	9,876,543	0	10.	164,000	
5.	5	2,100	11.	\$21.42 $\frac{1}{2}$	
6.	11	91,853	12.	494 $\frac{1}{2}$	
7.	3	137,851			

CANCELLATION.

(ART. 85, p. 55.)

$$3. \frac{\overset{2}{24} \times 16}{\underset{1}{12}} = 32.$$

$$4. \frac{\overset{3}{48} \times 8}{\underset{1}{16}} = 24.$$

$$5. \frac{\overset{1}{7} \times \overset{5}{10} \times \overset{2}{12} \times 5}{\underset{2}{14} \times \underset{9}{18} \times 6} = \underset{3}{2} = 2\frac{2}{3}.$$

$$6. \frac{\overset{1}{15} \times \overset{1}{7} \times \overset{4}{27} \times \overset{4}{40}}{\underset{2}{54} \times \underset{2}{14} \times \underset{1}{10} \times 2} = \underset{2}{1} = 7\frac{1}{2}.$$

$$7. \frac{\overset{1}{13} \times \overset{5}{15} \times \overset{2}{20} \times 5}{\underset{2}{26} \times \underset{2}{10} \times 2 \times 3} = \underset{2}{2} = 12\frac{1}{2}.$$

$$8. \frac{\overset{4}{28} \times \overset{1}{27} \times \overset{3}{21} \times 15 \times \overset{2}{18}}{\underset{1}{7} \times \underset{2}{54} \times \underset{1}{7} \times 3 \times 9} = 60$$

$$9. \frac{\overset{2}{56} \times 11}{\underset{28}{28}} = 22.$$

$$10. \frac{\overset{2}{14} \times \overset{8}{24} \times \overset{1}{9}}{\underset{7}{63} \times \underset{1}{3}} = 16 \text{ cents.}$$

$$11. \frac{\overset{3}{5} \times \overset{3}{12} \times 7}{\underset{3}{3} \times \underset{4}{4}} = 35.$$

$$12. \frac{\overset{2}{8} \times \overset{5}{60} \times \overset{15}{75}}{\underset{3}{30} \times \underset{5}{40}} = 10.$$

(ART. 87, p. 57.)

$$\begin{array}{r} (2.) \\ 63013 \times 17 \\ \hline 441091 \\ 1071221 \end{array}$$

$$\begin{array}{r} (3.) \\ 79245 \times 19 \\ \hline 713205 \\ 1505655 \end{array}$$

$$\begin{array}{r} (4.) \\ 82067812 \times 16 \\ \hline 192406872 \\ 518084992 \end{array}$$

(ART. 88, p. 57.)

$$\begin{array}{r} (2.) \\ 6651 \times 108 \\ \hline 53208 \\ 718308 \end{array}$$

$$\begin{array}{r} (3.) \\ 111223 \times 104 \\ \hline 444892 \\ 11567192 \end{array}$$

$$\begin{array}{r} (4.) \\ 2042 \times 1009 \\ \hline 18378 \\ 2060378 \end{array}$$

(ART. 89, p. 58.)

$$\begin{array}{r} (2.) \\ 18317 \times 51 \\ \hline 66585 \\ 679167 \end{array}$$

$$\begin{array}{r} (3.) \\ 71389 \times 21 \\ \hline 142778 \\ 1499169 \end{array}$$

$$\begin{array}{r} (4.) \\ 12062 \times 91 \\ \hline 108558 \\ 1097642 \end{array}$$

(ART. 90, p. 58.)

$$\begin{array}{r} (2.) \\ 8360 \times 7001 \\ \hline 58520 \\ 58528360 \end{array}$$

$$\begin{array}{r} (3.) \\ 10613 \times 801 \\ \hline 84904 \\ 8501013 \end{array}$$

$$\begin{array}{r} (4.) \\ 91603 \times 2001 \\ \hline 183206 \\ 183297603 \end{array}$$

(ART. 91, p. 59.)

$$\begin{array}{r} (2.) \\ 915 \\ \hline 22\frac{2}{5} \\ 1830 \\ \hline 1830 \end{array}$$

$$\begin{array}{r} (3.) \\ 1224\frac{1}{2} \\ \hline 18 \\ \hline 9792 \\ \hline 1224 \end{array}$$

$$\begin{array}{r} (4.) \\ 180 \\ \hline 69\frac{1}{6} \\ \hline 1620 \\ \hline 1080 \end{array}$$

$$20130 = \text{product by } 22.$$

$$22032$$

$$30 = \frac{1}{6} \text{ of } 180.$$

$$366 = \text{product by } \frac{2}{3}.$$

$$2\frac{2}{3} = \frac{1}{7} \text{ of } 18.$$

$$12450$$

$$20496 = \text{product by } 22\frac{2}{5}. \quad 22034\frac{4}{5}$$

(ART. 92, p. 59.)

$$\begin{array}{r} (2.) \\ 8)6805600 \\ \hline 850700 \end{array}$$

$$\begin{array}{r} (3.) \\ 4)179240 \\ \hline 44810 \end{array}$$

$$\begin{array}{r} (4.) \\ 6)19237800 \\ \hline 3206300 \end{array}$$

$$\begin{array}{r} (5.) \\ 8)12345678000 \\ \hline 1548209750 \end{array}$$

$$\begin{array}{r} (6.) \\ 3)3130 \\ \hline 1043\frac{1}{3} \end{array}$$

$$\begin{array}{r} (7.) \\ 3)53400 \\ \hline 17800 \end{array}$$

$$\begin{array}{r} (8.) \\ 4)771000 \\ \hline 192750 \end{array}$$

$$\begin{array}{r} (9.) \\ 6)9168000 \\ \hline 1528000 \end{array}$$

$$\begin{array}{r} (10.) \\ 8)1993000 \\ \hline 249125 \end{array}$$

$$\begin{array}{r} (11.) \\ 3)28044000 \\ \hline 9348000 \end{array}$$

(ART. 93, p. 61.)

$$\begin{array}{r}
 (3.) \quad 61370913 \\
 \quad \quad 96488 \\
 \hline
 \quad \quad 490967304 = \text{the product by 8.} \\
 \quad \quad 2945803824 = \text{the last product} \times 6. \\
 \quad \quad 5891607648 = \text{the last product} \times 2. \\
 \hline
 \quad \quad 5921556653544
 \end{array}$$

$$\begin{array}{r}
 (4.) \quad 8649347864 \text{ multiplicand.} \\
 \quad \quad 1325769612 \text{ multiplier.} \\
 \hline
 \quad \quad 103792174368 = \text{the product by 12.} \\
 \quad \quad 830337394944 = \text{the foregoing product} \times 8 \text{ for 96.} \\
 \quad \quad 4982024369664 = \text{the last product} \times 6 \text{ for 576.} \\
 \quad \quad 1141713918048 = \text{the first product} \times 11 \text{ for 132.} \\
 \hline
 11467042561708308768 \text{ product, Ans.}
 \end{array}$$

(ART. 94, p. 61.)

$ \begin{array}{r} (2.) \\ 77777770000 \\ \quad 7777777 \\ \hline 77769992223 \end{array} $	$ \begin{array}{r} (3.) \\ 41623100000 \\ \quad 416231 \\ \hline 41622683769 \end{array} $
$ \begin{array}{r} (4.) \\ 987654000000 \\ \quad 987654 \\ \hline 3)987653012346 \\ \hline 329217670782 \end{array} $	$ \begin{array}{r} (5.) \\ 87654300000 \\ \quad 876543 \\ \hline 3)87653423457 \\ \hline 29217807819 \\ \quad \quad 2 \\ \hline 58435615638 \end{array} $
$ \begin{array}{r} (6.) \\ 9999990000 \\ \quad 999999 \\ \hline 99989900001 \end{array} $	$ \begin{array}{r} (7.) \\ 325678950000 \\ \quad 32567895 \\ \hline 3)325646382105 \\ \hline 108548794035 \end{array} $

(8.)
 6666600000
 66666
3)6666533334
 2222177778
 2
4444855556

(9.)
 91234567800
 912345678
90322222122

(10.)
 12345670000
 1234567
12344435438

(11.)
 98123452000000
 98123452
98123353876548

CONTRACTIONS IN DIVISION.

(ART. 95, p. 62.)

(2.)
 89630
 3
26889|0

(3.)
 123450
 6
7407|00

(4.)
 18621
 8
1489|68

(5.)
 317121
 4
126848|4

(6.)
 876735
 3
26302|05

(7.)
 123456
 8
987|648

(8.)
 61678500
 4
246714|000

(9.)
 9500
 6
57|000

(10.)
 12000
 3
36|000

(11.)
 150000
 8
1200|000

(12.)
 333½
 3
10|00

(13.)
 120
 4
48|0

(14.)
 616350
 4
24654|00

(ART. 96, p. 64.)

$$\begin{array}{r|l} (4.) & \\ 12332 & 655 \\ 12 & 987 \\ 1 & 999 \\ \hline 12345 & \text{Ans.} \end{array}$$

$$\begin{array}{r|l} (5.) & \\ 98755 & 1235 \\ 9 & 9990 \\ \hline 98765 & \text{Ans.} \end{array}$$

$$\begin{array}{r|l} (5.) & \\ 98755 & 1235 \\ 9 & 9990 \\ 1 & 9999 \\ \hline 98765 & \text{Ans.} \end{array}$$

$$\begin{array}{r|l} (6.) & \\ 9123456779 & 876543211 \\ 9 & 999999990 \\ 1 & 999999999 \\ \hline 9123456789 & \text{Ans.} \end{array}$$

(ART. 97, p. 64.)

$$\begin{array}{r} (2.) \\ 44)39006(886\frac{2}{3} \\ \underline{380} \\ 286 \\ \underline{22} \end{array}$$

$$\begin{array}{r} (3.) \\ 34)1088(32 \\ \underline{68} \\ 0 \end{array}$$

$$\begin{array}{r} (4.) \\ 191)5157(27 \\ \underline{1337} \\ 0 \end{array}$$

$$\begin{array}{r} (5.) \\ 7750)1133123(146\frac{1}{3} \\ \underline{35812} \\ 48123 \\ \underline{1623} \end{array}$$

(ART. 98, p. 65.)

$$\begin{array}{r} (2.) \\ 9\frac{1}{2})13120 \\ 4 \quad 4 \\ \hline 37)52480(1418\frac{1}{3} \\ \underline{37} \\ 154 \\ \underline{148} \\ 68 \\ \underline{37} \\ 310 \\ \underline{296} \\ 14 \end{array}$$

$$\begin{array}{r} (3.) \\ 36) 76672\frac{2}{3} \\ 7 \quad 7 \\ \hline 252)536710(2129\frac{2}{3} \\ \underline{504} \\ 327 \\ \underline{252} \\ 751 \\ \underline{504} \\ 2470 \\ \underline{2268} \\ 202 \end{array}$$

$$\begin{array}{r}
 \text{(4.)} \\
 205\overline{)2090\frac{1}{2}} \\
 \underline{3 \quad 3} \\
 615\overline{)6271(10\frac{1}{2}\frac{1}{2}} \\
 \underline{615} \\
 121
 \end{array}$$

$$\begin{array}{r}
 \text{(5.)} \\
 16\frac{1}{2}\overline{)10626} \\
 \underline{2 \quad 2} \\
 33\overline{)21252(644} \\
 \underline{198} \\
 145 \\
 \underline{132} \\
 132 \\
 \underline{132}
 \end{array}$$

$$\begin{array}{r}
 \text{(6.)} \\
 69\frac{1}{2}\overline{)12450} \\
 \underline{6 \quad 6} \\
 415\overline{)74700(180} \\
 \underline{415} \\
 3320 \\
 \underline{3320} \\
 0
 \end{array}$$

$$\begin{array}{r}
 \text{(7.)} \\
 17\frac{1}{2}\overline{)5591\frac{1}{2}} \\
 \underline{4 \quad 4} \\
 71\overline{)22365(315} \\
 \underline{213} \\
 106 \\
 \underline{71} \\
 355 \\
 \underline{355}
 \end{array}$$

$$\begin{array}{r}
 \text{(8.)} \\
 10\frac{1}{2}\overline{)2667} \\
 \underline{2 \quad 2} \\
 21\overline{)5334(254} \\
 \underline{42} \\
 113 \\
 \underline{105} \\
 84 \\
 \underline{84}
 \end{array}$$

$$\begin{array}{r}
 \text{(9.)} \\
 272\frac{1}{2}\overline{)136125} \\
 \underline{4 \quad 4} \\
 1089\overline{)544500(500} \\
 \underline{5445} \\
 00
 \end{array}$$

$$\begin{array}{r}
 \text{(10.)} \\
 2\frac{1}{2}\overline{)119} \\
 \underline{7 \quad 7} \\
 17\overline{)833(49} \\
 \underline{68} \\
 153 \\
 \underline{153}
 \end{array}$$

$$\begin{array}{r}
 \text{(11.)} \\
 31\frac{1}{2}\overline{)12968\frac{1}{2}} \\
 \underline{4 \quad 4} \\
 125\overline{)51875(415} \\
 \underline{500} \\
 187 \\
 \underline{125} \\
 625 \\
 \underline{625}
 \end{array}$$

PROBLEMS.

1. (ART. 99, p. 67.) $\$2763 + \$4650 + \$8950 = \16363 .
2. $\$929 - \$279 = \$650$.
3. $6476 - 242 = 6234$ feet.

4. $1519 + 328 = 1847$.
5. $1963 - 199 = 1764$; $1764 \div 2 = 882$, miles B travelled;
 $882 + 199 = 1081$, miles A travelled.
6. $\$250 + 410 = \660 ; $\$4698 - 660 = \4038 ; $\$4038 \div 3 = \1346 , George received; $\$1346 + \$250 = \$1596$, James received; $\$1346 + \$410 = \$1756$, Edwin received.
7. $\$8463 \div 217 = \39 .
8. $19 \times 3 = 57$; $684 \div 57 = 12$ weeks.
9. $3808 \div 224 = 17$ men.
10. $\$575 \times 99 = \56925 .
11. $96 \times 22 = 2112$; $63360 \div 2112 = 30$.
12. $1101 \times 13 = 14313$.

MISCELLANEOUS EXAMPLES.

(PAGE 68.)

1. $200 + 305 + 230 + 282 + 171 = 1188$, Ans.
2. $\$175 + \$87 + \$31 = \293 ; $\$38 + \$12 = \$50$; $\$293 - \$50 = \$243$, Ans.
3. $97 \times 5 = \$485$; $97 - 17 = 80$; $80 \times 8 = \$640$;
 $\$640 - \$485 = \$155$, gain, Ans.
4. $3787 \times 1728 = 6543936$ cubic inches, Ans.
5. $175686\text{lb.} \div 987 = 178\text{lb.}$, Ans.
6. $120 + 80 + 160 = 360$ acres, Ans.
7. $\$8395 \div 365 = \23 , Ans.
8. $12 \times 6 = 72$; $12 \times 12 \times 6 = 864$; $864 - 72 = 792$, Ans.
9. $\$7 \times 8 = \56 ; $\$8 \times 3 = \24 ; $\$56 + \$24 = \$80$, Ans.
10. $\$31 + \$45 = \$76$; $3952 \div 76 = 52$, Ans.
11. $13 \times 4 = 52$, Ans.
12. $\$250,000 \div 500 = \500 , Ans.
13. $127 + 212 = 339$; $500 - 339 = 161$; $\$47 \times 127 = \5969 ; $\$96 \times 212 = \20352 ; $\$37 \times 161 = \5957 ; $\$5969 + \$20352 + \$5957 = \$32,278$; $\$32,278 - \$17,876 = \$14,402$, Ans.

14. $17 - 7 = 10$; $18 - 10 = 8$ miles, Ans.
 15. $15 \times 5 = 75$ days, Ans.
 16. $2 + 8 = 10$; $10 \times 4 = 40$; $40 + 32 = 72$, $72 \div 2 = 36$; $36 \times 10 = 360$; $360 \div 24 = 15$, Ans.
 17. $16 + 4 = 20$; $20 \times 2 = 40$; $126 + 40 = 166$; $48 \div 2 = 24$; $34 \times 6 = 204$; $17 - 5 = 12$; $204 \div 12 = 17$; $17 + 24 = 41$; $166 - 41 = 125$, Ans.
 18. $688 - 16 = 667$; $667 \div 23 = 29$ pupils, Ans.
 19. $12 + 40 = 52$; $40 \times 5 = 200$; $12 \times 6 = 72$; $52 + 200 + 72 + 7 = 331$ miles, Ans.
 20. $9891 - 1211 = 8680$; $8680 \div 2 = 4340$, A received; $9891 - 4340 = 5551$, B received, Ans.
 21. $15 \times 16 = 240$; $4080 \div 240 = 17$, third number, Ans.
 22. $\$4 \times 17 = \68 ; $\$2 \times 32 = \64 ; $\$68 + \$64 = \$132$; $132 \div 6 = 22$ tons, Ans.
 23. $100 + 200 + 300 = 600$; $5608 - 600 = 5008$; $5008 \div 4 = 1252$, first year; $1252 + 100 = 1352$, second year; $1352 + 100 = 1452$, third year; $1452 + 100 = 1552$, fourth year, Ans.
 24. $\$45 - \$35 = \$10$; $\$10 \times 12 = \120 ; $\$1100 - \$620 = \$480$; $480 \div 120 = 4$ years, Ans.
 25. $19782 + 31 = 19813$, Ans.
 26. $1885 \div 65 = 29$; $\$4473 + \$812 = \$5285$; $\$65 + \$6 = \$71$; $4473 \div 71 = 63$ shares; $63 - 29 = 34$ shares remaining; $\$5285 - \$1885 = \$3400$; $\$3400 \div 34 = \100 , Ans.

 UNITED STATES MONEY.

(ART. 107, p. 73.)		5.	\$ 12.345
2.	76500 cents.	6.	123560 mills.
3.	72 $\frac{2}{10}$ cents.	7.	2220 cents.
4.	\$ 3.29		

ADDITION.

(ART. 108, p. 73.)		
3.	\$ 328.722	6. \$ 1717.062
4.	\$ 1805.847	7. \$ 154.355
5.	\$ 3320.675	8. \$ 7.105

SUBTRACTION.

(ART. 109, p. 74.)		
3.	\$ 278.486	7. \$ 13.875
4.	\$ 69.066	8. \$ 909.75
5.	\$ 154.069	9. \$ 3.43
6.	\$ 2058.08	10. \$ 2.36

MULTIPLICATION OF UNITED STATES MONEY.

(ART. 110, p. 75.)		
2.	\$ 16.38	10. \$ 44.748
3.	\$ 58.59	11. \$ 109.25
4.	\$ 591.25	12. \$ 790.92
5.	\$ 3.358	13. \$ 65.10
6.	\$ 249.28	14. \$ 142.02
7.	\$ 1120.32	15. \$ 48.222
8.	\$ 358.28	16. \$ 48910.95
9.	\$ 102.69	17. \$ 67.16
		18. \$ 70.664

DIVISION OF UNITED STATES MONEY.

(ART. 111, p. 77.)		
4.	\$ 1.625	13. \$ 3.75
5.	425	14. \$ 2.28
6.	\$ 3.89	15. 47
7.	\$ 1.75	16. 17
8.	329	17. 691
9.	\$ 0.75	18. \$ 4.68
10.	\$ 4.00	19. \$ 0.18
11.	144	20. \$ 132.55
12.	\$ 0.06	21. \$ 1.12

PRACTICAL QUESTIONS BY ANALYSIS.

(ART. 116, p. 79.)		27.	\$60.17 $\frac{1}{2}$
2.	\$118.82	28.	\$12467.25
3.	\$7.24 $\frac{1}{2}$	30.	\$28.20
4.	\$310.20	31.	\$42.75
5.	\$1126.93 $\frac{1}{2}$	32.	\$9.035 $\frac{1}{4}$
6.	\$4668595.00	33.	\$771.65
8.	\$120.66 $\frac{2}{3}$	34.	\$283.12 $\frac{1}{2}$
9.	\$105.00	35.	\$35.526 $\frac{1}{4}$
10.	\$72.68 $\frac{3}{4}$	37.	\$0.13
12.	\$3800.00	38.	\$4.16 $\frac{2}{3}$
13.	\$337.50	39.	\$72.25
14.	\$502.25	40.	\$29.70
15.	\$22.50	41.	\$2.50
16.	\$6996.00	42.	\$0.22
18.	\$190.00	43.	\$0.20
19.	\$3255.75	45.	152
20.	\$103.35	46.	84
21.	\$52.95	47.	362
22.	\$129.525	48.	27
24.	\$876.375	49.	50
25.	\$472.50	50.	216
26.	\$7.98 $\frac{3}{4}$		

BILLS.

(ART. 122, p. 83.)		(1.)	JAMES DOW.	(2.)	SAMUEL SMITH.
		\$0.45	$\times 17 =$	\$7.65	\$0.98 $\times 13 =$ \$12.74
		.37	$\times 19 =$	7.03	.15 $\times 16 =$ 2.40
		.46	$\times 16 =$	7.36	.13 $\times 36 =$ 4.68
		.87	$\times 13 =$	11.31	.9 $\times 47 =$ 4.23
		.63	$\times 9 =$	5.67	.19 $\times 12 =$ 2.28
		.56	$\times 25 =$	14.00	.17 $\times 7 =$ 1.19
		.31	$\times 17 =$	5.27	.61 $\times 13 =$ 7.93
		.16	$\times 19 =$	3.04	
					<hr/>
					\$35.45
					<hr/>
			\$61.33		

(3.) WILSON, NILES & Co.

\$ 6.00	×	2	=	\$ 12.00
5.00	×	3	=	15.00
1.80	×	5	=	9.00
.25	×	17	=	4.25
.60	×	13	=	7.80
.50	×	19	=	9.50
.60	×	3	=	1.80
1.12½	×	7	=	7.87½
.22	×	15	=	3.30
15.00	×	5	=	75.00
4.50	×	3	=	13.50
1.50	×	5	=	7.50
4.50	×	1	=	4.50
				<hr/>
				\$ 171.02½

(4.) ALBERT CRAWFORD Dr.

\$ 5.25	×	17	=	\$ 89.25
1.62	×	29	=	46.98
.17	×	60	=	10.20
.27	×	49	=	13.23
3.19	×	18	=	57.42
2.75	×	27	=	74.25
.61	×	75	=	45.75
.75	×	36	=	27.00
.18	×	49	=	8.82
				<hr/>
				\$ 372.90

Cr.

Cash,	\$ 83.00
\$ 30.00 × 3 =	90.00
4.00 × 7 =	28.00
2.00 × 4 =	8.00
1.75 × 5 =	8.75
2.25 × 7 =	15.75
Cash,	= 60.00
Draft,	= 45.00
	<hr/>
	\$ 338.50
	<hr/>
Remains due, \$	34.40

(5.) BENJAMIN TREAT.

\$ 25.50	×	37	=	\$ 943.50
16.17	×	41	=	662.97
97.75	×	40	=	3910.00
169.37	×	13	=	2201.81
				<hr/>
				\$ 7718.28

(6.) J. C. PORTER.

\$ 8.25	×	17	=	\$ 140.25
.50	×	50	=	25.00
.08½	×	140	=	11.90
.63	×	120	=	75.60
				<hr/>
				\$ 252.75

(7.)

JOHN CUMMINGS.

\$6.25 × 97 =	\$606.25	Am't brought up	\$12811.36
5.95 × 167 =	993.65	1.61 × 39 =	62.79
6.07 × 87 =	528.09	.17 × 197 =	33.49
5.75 × 196 =	1127.00	.69 × 86 =	59.34
7.25 × 275 =	1993.75	1.17 × 78 =	91.26
1.16 × 69 =	80.04	.85 × 187 =	158.95
.67 × 136 =	91.12	11.61 × 91 =	1056.51
.76 × 68 =	51.68	17.15 × 88 =	1423.45
1.37 × 169 =	231.53	3.16 × 47 =	148.52
9.67 × 76 =	734.92	18.15 × 35 =	635.25
69.70 × 89 =	6203.30	9.47 × 47 =	445.09
3.47 × 49 =	170.03	6.83 × 57 =	389.31
Am't carried up,	\$12811.36		\$17315.32

LEDGER ACCOUNTS.

(ART. 123, p. 87.)

1. \$ 461.97	2. \$ 3165.60
3. \$ 4130.23	4. \$ 21995.54

REDUCTION OF COMPOUND NUMBERS.

ENGLISH MONEY.

(ART. 128, p. 90.)

(3.)	(4.)
127£. 15s. 8d.	4)122672far.
<u>20</u>	12)30668d.
2555s.	20)2555£. 8d.
<u>12</u>	127£. 15s. 8d.
30668d.	
<u>4</u>	
122672far.	

(5.)

$$\begin{array}{r} 28\text{£. } 19\text{s. } 11\text{d. } 3\text{far.} \\ \underline{20} \\ 579\text{s.} \\ \underline{12} \\ 6959\text{d.} \\ \underline{4} \\ 27839\text{far.} \end{array}$$

(6.)

$$\begin{array}{r} 4)27839\text{far.} \\ \underline{12}6959\text{d. } 3\text{far.} \\ 20)579\text{s. } 11\text{d.} \\ \underline{28\text{£. } 19\text{s. } 11\text{d. } 3\text{far.}} \end{array}$$

(7.)

$$\begin{array}{r} 378\text{£.} \\ \underline{20} \\ 7560\text{s.} \\ \underline{12} \\ 90720\text{d.} \end{array}$$

(8.)

$$\begin{array}{r} 12)90720\text{d.} \\ \underline{20}7560\text{s.} \\ 378\text{£.} \end{array}$$

AVOIRDUPOIS WEIGHT.

(ART. 129, p. 91.)

(1.) 165T. 13cwt. 3qr. 19lb. 14oz.

$$\begin{array}{r} 20 \\ \underline{3313\text{cwt.}} \\ 4 \\ \underline{13255\text{qr.}} \\ 25 \\ \underline{66284} \\ 26511 \\ \underline{331394\text{lb.}} \\ 16 \\ \underline{1988368} \\ 331395 \\ \underline{5302318\text{oz.}} \end{array}$$

(2.) 16)5302318oz.

$$\begin{array}{r} 25)331394\text{lb. } 14\text{oz.} \\ 4)13255\text{qr. } 19\text{lb.} \\ 20)3313\text{cwt. } 3\text{qr.} \\ \underline{165\text{T. } 13\text{cwt. } 3\text{qr. } 19\text{lb. } 14\text{oz.}} \end{array}$$

(3.) 3T. 16cwt. 2qr. 18lb.

$$\begin{array}{r}
 20 \\
 \hline
 76\text{cwt.} \\
 4 \\
 \hline
 306\text{qr.} \\
 25 \\
 \hline
 1538 \\
 613 \\
 \hline
 7668\text{lb.} \\
 16 \\
 \hline
 46008 \\
 7668 \\
 \hline
 122688\text{oz.}
 \end{array}$$

(4.) 16)122688oz.

$$\begin{array}{r}
 25)7668\text{lb.} \\
 4)306\text{qr. 18lb.} \\
 20)76\text{cwt. 2qr.} \\
 3\text{T. 16cwt. 2qr. 18lb.}
 \end{array}$$

(5.) 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.

$$\begin{array}{r}
 20 \\
 \hline
 57\text{cwt.} \\
 4 \\
 \hline
 231\text{qr.} \\
 25 \\
 \hline
 1161 \\
 463 \\
 \hline
 5791\text{lb.} \\
 16 \\
 \hline
 34751
 \end{array}$$

(6.) 16)1482749dr.

$$\begin{array}{r}
 16)92671\text{oz. 13dr.} \\
 25)5791\text{lb. 15oz.} \\
 4)231\text{qr. 16lb.} \\
 20)57\text{cwt. 3qr.}
 \end{array}$$

Ans. 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.

$$\begin{array}{r}
 34751 \\
 5792 \\
 \hline
 92671\text{oz.} \\
 16 \\
 \hline
 556029 \\
 92672 \\
 \hline
 1482749\text{dr., Ans.}
 \end{array}$$

(7.) 7T. 17cwt.

$$\begin{array}{r}
 20 \\
 \hline
 157\text{cwt.} \\
 4 \\
 \hline
 628\text{qr.} \\
 25 \\
 \hline
 3140 \\
 1256 \\
 \hline
 15700\text{lb.} \\
 .07 \\
 \hline
 \$1099.00, \text{Ans.}
 \end{array}$$

(8.) 19cwt. 3qr. 20lb.

$$\begin{array}{r}
 4 \\
 \hline
 79\text{qr.} \\
 25 \\
 \hline
 395 \\
 160 \\
 \hline
 1995\text{lb.} \\
 .09 \\
 \hline
 \$179.55, \text{Ans.}
 \end{array}$$

TROY WEIGHT.

(ART. 130, p. 92.)

(1.) 28lb. 11oz. 12pwt. 15gr.

$$\begin{array}{r}
 12 \\
 \hline
 347\text{oz.} \\
 20 \\
 \hline
 6952\text{pwt.} \\
 24 \\
 \hline
 27813 \\
 13905 \\
 \hline
 166863\text{gr., Ans.}
 \end{array}$$

(2.) 24)166863gr.

$$20)6952\text{pwt. 15gr.}$$

$$12)347\text{oz. 12pwt.}$$

$$\text{Ans. 28lb. 11oz. 12pwt. 15gr.}$$

(3) 3lb. 10oz.

$$\begin{array}{r}
 12 \\
 \hline
 46\text{oz.} \\
 20 \\
 \hline
 920\text{pwt.} \\
 24 \\
 \hline
 22080\text{gr., Ans.}
 \end{array}$$

(4.) 24)22080gr.

$$20)920\text{pwt.}$$

$$12)46\text{oz.}$$

$$3\text{lb. 10oz., Ans.}$$

(5) 73lb. 11oz.

$$\begin{array}{r}
 12 \\
 \hline
 887\text{oz.} \\
 20 \\
 \hline
 17740\text{pwt.} \\
 \$.062 \\
 \hline
 35480 \\
 106440 \\
 \hline
 \$ 1099\ 88, \text{Ans.}
 \end{array}$$

(6.) .062)\$ 1099.88

$$20)17740\text{pwt.}$$

$$12)887\text{oz.}$$

$$\text{Ans. 73lb. 11oz.}$$

KEY TO

(7.) 57lb. 7oz.	(8.) 19lb. 6oz. 16pwt.	(9.) 13lb. 9oz.
<u>12</u>	<u>12</u>	<u>12</u>
691oz.	234oz.	165
<u>20.593½</u>	20	1.3857
2073	4696pwt.	1155
6219	.98	825
3455	<u>\$ 4367.28, Ans.</u>	1320
1382		495
<u>138½</u>		165
\$ 14229.901½, Ans.		\$ 228.6405, Ans.

APOTHECARIES' WEIGHT.

(ART. 131, p. 93.)

(1.) 23℔ 9⅔ 0⅜ 2℥ 13gr.

$$\begin{array}{r}
 12 \\
 285\frac{3}{4} \\
 \underline{8} \\
 2280\frac{3}{4} \\
 \underline{8} \\
 6842\cancel{0} \\
 \underline{20} \\
 136853\text{gr.}, \text{Ans.}
 \end{array}$$

(2.) 20)136853gr.

$$\begin{array}{r}
 3)6842\cancel{0} \\
 8)2280\frac{3}{4} \\
 12)285\frac{3}{4} \\
 \text{Ans. } 23\ell\text{b } 9\frac{3}{4} 0\frac{3}{8} 2\cancel{0} 13\text{gr.}
 \end{array}$$

(3.) 23℔

$$\begin{array}{r}
 12 \\
 276\frac{3}{4} \\
 \underline{8} \\
 2208\frac{3}{4} \\
 \underline{8} \\
 6624\cancel{0}, \text{Ans.}
 \end{array}$$

(4.) 3)6624℥

$$\begin{array}{r}
 8)2208\frac{3}{4} \\
 12)276\frac{3}{4} \\
 23\ell\text{b}, \text{Ans}
 \end{array}$$

(5.) 47lb 03 03 19gr.

$$\begin{array}{r} 12 \\ \cdot 5643 \\ \underline{8} \\ 45123 \\ \underline{3} \\ 135379 \\ \underline{20} \end{array}$$

270759gr., Ans.

(6.) 20)270759gr.

$$\begin{array}{r} 3)135379 \text{ 19gr.} \\ \underline{8}45123 \text{ 19} \\ 12)5643 \end{array}$$

Ans. 47lb 03 03
[19 19gr.

(7.) 1lb

$$\begin{array}{r} 12 \\ \underline{12}3 \\ 8 \\ \underline{96}3 \\ 3 \\ \underline{288}9 \\ 20 \end{array}$$

5)5760gr.

$$\begin{array}{r} 1152 \\ \underline{121} \end{array}$$

\$144.00

1.80

\$142.20, Ans.

AVOIRDUPOIS, TROY, AND APOTHECARIES' WEIGHT
COMPARED.

(ART. 132, p. 94.)

(1.) 13lb. 6oz.

$$\begin{array}{r} 7000 \text{ 437}\frac{1}{2}\text{gr} \\ \underline{91000} \quad \underline{6} \\ 2625 \text{ 2625gr.} \end{array}$$

5760)93625(16lb.

$$\begin{array}{r} 576 \\ \underline{3602} \\ 3456 \\ \underline{1465} \\ 12 \end{array}$$

5760)17580(3oz.

$$\begin{array}{r} 1728 \\ \underline{300} \\ 20 \end{array}$$

6000 (Carried forward to p. 26.)

3

(2.) 16lb. 3oz. 1pwt. 1gr.

$$\begin{array}{r} 12 \\ \underline{195}\text{oz.} \\ 20 \end{array}$$

3901pwt.

24

7000)93625(13lb.

$$\begin{array}{r} 7000 \\ \underline{23625} \\ 21000 \end{array}$$

2625

16

7000)42000(6oz.

$$\begin{array}{r} 42000 \end{array}$$

(Brought forward.)

5760)6000(1pwt.

5760

240

24

5760)5760(1gr.

5760

(3.)

3lb. 8oz. 10pwt.

12

44oz.

20

890pwt.

24

20)21360gr.

3)1068

356dr., Ans.

(4.)

356dr.

3

1068

20

24)21360

20)890

12)44oz. 10pwt.

Ans. 3lb. 8oz. 10pwt.

(5.)

2lb. 8oz.

16

40oz.

2.40

.\$ 96.00

2lb. 8oz.

7000 437½gr.

14000 8

3500 3500gr.

20)17500gr.

8750

.20

\$ 175

96

\$ 79 gain, Ans.

(6.)

12lb.

16

192oz.

37½

\$ 72.00

Ans. \$ 2.00 loss.

12lb.

7000

24)84000gr.

20)3500pwt.

175oz.

.40

\$ 70.00

LINEAR OR LONG MEASURE.

(ART. 133, p. 96.)

(1.) 96deg. 56m. 7fur. 32rd. 12ft. 6in.

$$\begin{array}{r}
 69\frac{1}{8} \\
 \hline
 870 \\
 581 \\
 \hline
 16 \\
 6696m. \\
 8 \\
 \hline
 53575fur. \\
 40 \\
 \hline
 2143032rd. \\
 16\frac{1}{2} \\
 \hline
 12858194 \\
 2143033 \\
 \hline
 1071516 \\
 35360040ft. \\
 12 \\
 \hline
 \end{array}$$

Ans. 424320486in.

(2.) 12)424320486in.

16 $\frac{1}{2}$)35360040ft. 6in.

40)2143032rd. 12ft.

8)53575fur. 32rd.

69 $\frac{1}{8}$)6696m. 7fur.

Ans. 96deg. 56m. 7fur. 32rd.

[12ft. 6in., Ans.

(3.) 79m.

8

632fur.

40

25280rd.

16 $\frac{1}{2}$

417120ft., Ans.

(4.) 16 $\frac{1}{2}$)417120ft.

40)25280rd.

8)632fur.

Ans. 79m.

(5.) 396

40

15840rd.

16 $\frac{1}{2}$

95040

15840

7920

261360ft.

12

8136320in., Ans.

(6.) 12)8136320in.

16 $\frac{1}{2}$)261360ft.

2

2

33)522720

40)15840rd.

396fur., Ans.

(7.) 30ms.

$$\begin{array}{r}
 8 \\
 \hline
 240\text{fur.} \\
 40 \\
 \hline
 9600\text{rd.} \\
 16\frac{1}{2} \\
 \hline
 57600 \\
 9600 \\
 \hline
 4800 \\
 \hline
 158400\text{ft.} \\
 12 \\
 \hline
 1900800\text{in., Ans.}
 \end{array}$$

(8.) 12)1900800in.

$$\begin{array}{r}
 16\frac{1}{2})158400\text{ft.} \\
 2 \quad 2 \\
 \hline
 33 \quad 316800 \\
 40 \quad 9600\text{rd.} \\
 \hline
 8 \quad 240\text{fur.} \\
 \hline
 30\text{ms., Ans.}
 \end{array}$$

CLOTH MEASURE.

(ART. 134, p. 96.)

(1.)

$$\begin{array}{r}
 17\text{yd. 3qr. 2na.} \\
 4 \\
 \hline
 71\text{qr.} \\
 4 \\
 \hline
 286\text{na., Ans.}
 \end{array}$$

(2.)

$$\begin{array}{r}
 4)286\text{na.} \\
 4)71\text{qr. 2na.} \\
 \hline
 17\text{yd. 3qr. 2na., Ans.}
 \end{array}$$

(3.)

$$\begin{array}{r}
 365\text{yd. 1qr. 3na.} \\
 4 \\
 \hline
 1461\text{qr.} \\
 4 \\
 \hline
 5847\text{na., Ans.}
 \end{array}$$

(4.)

$$\begin{array}{r}
 4)5847\text{na.} \\
 4)1461\text{qr. 3na.} \\
 \hline
 365\text{yd. 1qr. 3na., Ans.}
 \end{array}$$

(5.)

$$\begin{array}{r}
 71\text{E.E. 4qr.} \\
 5 \\
 \hline
 359\text{qr.} \\
 4 \\
 \hline
 1436\text{na., Ans.}
 \end{array}$$

(6.)

$$\begin{array}{r}
 4)1436\text{na.} \\
 5)359\text{qr.} \\
 \hline
 \text{Ans. 71E.E. 4qr.}
 \end{array}$$

<p>(7.)</p> $ \begin{array}{r} 47\text{yd. } 3\text{qr.} \\ \underline{4} \\ 191\text{qr.} \\ 1.25 \\ \underline{955} \\ 382 \\ \underline{191} \\ \$238.75, \text{ Ans.} \end{array} $	<p>(8.)</p> $ \begin{array}{r} 31\frac{1}{2}\text{E.E.} \\ \underline{5} \\ 4)156\text{qr.} \\ \underline{39} \\ 3 \\ \$117, \text{ Ans.} \end{array} $
--	--

SURFACE OR SQUARE MEASURE.

(ART. 135, p. 97.)

<p>(1.) 57A. 3R. 27p. 21yd. 8ft. 57in.</p> $ \begin{array}{r} \underline{4} \\ 231\text{R} \\ \underline{40} \\ 9267\text{p.} \\ \underline{30\frac{1}{4}} \\ 278031 \\ \underline{2316\frac{3}{4}} \\ 280347\frac{1}{4}\text{yds.} \\ \underline{9} \\ 2523137\frac{3}{4}\text{ft.} \\ \underline{144} \\ 10092555 \\ 10092553 \\ 2523127 \\ \underline{108} \\ 363331893\text{in., Ans.} \end{array} $	<p>(2.)</p> $ \begin{array}{r} 144)363331893\text{in.} \\ \underline{9)2523138\text{ft. } 21\text{in.}} \\ 30\frac{1}{4})280348\text{yd. } 6\text{ft.} \\ \underline{4 \quad 4} \\ 121)1121392 \\ \underline{40)9267\text{p. } 85 \div 4 = 21\text{yd. } 2\text{ft. } 36\text{in.}} \\ 4)231\text{R. } 27\text{p.} \\ \underline{57\text{A. } 3\text{R. } 27\text{p. } 21\text{yd. } 2\text{ft. } 36\text{in.}} \\ \underline{6 \quad 21} \\ \text{Ans. } 57\text{A. } 3\text{R. } 27\text{p. } 21\text{yd. } 8\text{ft. } 57\text{in.} \end{array} $
---	--

(3.) 25A.	(4.) $272\frac{1}{4}$	1089000ft.	(5.) 365S. M.
<u>4</u>	<u>4</u>	<u>4</u>	<u>640</u>
100R.	1089)4356000	14600
<u>40</u>		40)4000p.	2190
4000p.		4)100R.	233600A.
<u>272$\frac{1}{4}$</u>		25A., Ans.	<u>160</u>
8000			14016000
28000			2336
8000			<u>Ans. 37376000sq. rd.</u>
<u>1000</u>			

1089000ft., Ans.

(6.) 160)37376000 square rods.	(7.) 144)12345678
<u>640)233600 acres.</u>	9)85733ft. 126 in.
365 square miles.	30 $\frac{1}{4}$)9525yd. 8ft.
	40)314p. 26 $\frac{1}{2}$ yd.
	<u>4)7R. 34p.</u>
	1A. 3R.
	1A. 3R. 34p. 26 $\frac{1}{2}$ yd. 8ft. 126in.
	<u>$\frac{1}{2}$yd. = 4ft. 72in.</u>
	Ans. 1A. 3R. 34p. 27yd. 4ft. 54in.

(8.) 39A. 2R. 16p.	6336 poles.
<u>4</u>	<u>272$\frac{1}{4}$</u>
158R.	12672
<u>40</u>	44352
6336p.	12672
<u>3.75</u>	<u>1584</u>
31680	1724976 square feet.
44352	<u>0.25</u>
<u>19008</u>	<u>8224880</u>
\$23760.00	3449952
	431244.00
	<u>23760.00</u>
	\$407,484.00, Ans.

SURVEYOR'S MEASURE.

(1.)	(ART. 136, p. 98.)	(2.)
46m. 3fur. 5ch. 25l.		100)371525l.
<u>8</u>		<u>10</u>)3715ch. 25l.
371fur.		<u>8</u>)371fur. 5ch.
<u>10</u>		<u>46m. 3fur.</u>
3715ch.		Ans. 46m. 3fur. 5ch. 25l.
<u>100</u>		
371525l., Ans.		

(3.)	(5.)	(6.)
97m.	1377sq. ch.	\$ 80 ÷ 160 = \$ 0.50.
<u>8</u>	<u>16</u>	625)2,126,250 square links.
776fur.	<u>8262</u>	<u>3402</u> sq. rd.
<u>10</u>	<u>1377</u>	<u>0.50</u>
7760ch.	40)22032	\$ 1701.00, Ans.
<u>100</u>	4)550R. 32p.	
776000l., Ans.	<u>137</u> A. 2R.	
	137A. 2R. 32p., Ans.	

CUBIC OR SOLID MEASURE.

(1.)	(ART. 137, p. 100.)	(2.)
29 cords.		1728)6414336 cubic inches.
<u>128</u>		<u>128</u>)3712 cubic feet.
232	(3.)	<u>29</u> cords, Ans.
<u>58</u>	<u>19</u> tons.	
29	<u>40</u>	(4)
3712ft.	760ft.	1728)1313280 cubic inches.
<u>1728</u>	<u>1728</u>	<u>40</u>)760 feet.
29696	6080	<u>19</u> tons, Ans.
<u>7424</u>	<u>1520</u>	
25984	5320	
<u>3712</u>	<u>760</u>	
Ans. 6414336in.	1313280 cubic inches, Ans.	

- (5.) $128 \times 128 = 16384$ cubic feet, Ans.
 (6.) $16384 \div 128 = 128$ cords, Ans.
 (7.) $4 \times 40 \times 7 = 1120$ cubic feet, Ans.
 (8.) $8650 \text{ ft.} \div 128 = 67$ cords, 74 cubic feet, Ans.
 (9.) $17 \times 11 \times 9 = 1683$ cubic feet, Ans.

LIQUID OR WINE MEASURE.

(ART. 138, p. 101.)

- (1.) 57T. 3hhd. 50gal. 3qt. (3.) 96hhd. 47gal. 2qt.

$$\begin{array}{r} 4 \\ 231\text{hhd.} \\ 63 \\ \hline 14603\text{gal.} \\ 4 \\ \hline 58415\text{qt.} \\ 2 \\ \hline 116830\text{pt.} \end{array}$$

$$\begin{array}{r} (2.) \\ 2)116830 \text{ pints.} \\ 4)58415\text{qt.} \\ 63)14603\text{gal. 3qt.} \\ 4)231\text{hhd. 50gal.} \\ 57\text{T. 3hhd.} \\ 57\text{T. 3hhd. 50gal. 3qt., Ans.} \end{array}$$

$$\begin{array}{r} 63 \\ 295 \\ 580 \\ \hline 6095\text{gal.} \\ 4 \\ \hline 24382\text{qt.} \\ 2 \\ \hline 48764\text{pt.} \\ 4 \\ \hline 195056 \text{ gills, Ans.} \end{array}$$

$$\begin{array}{r} (4.) \\ 4)195056 \text{ gills.} \\ 2)48764\text{pt.} \\ 4)24382\text{qt.} \\ 63)6095\text{gal. 2qt.} \\ 96\text{hhd. 47gal.} \\ \text{Ans. 96hhd. 47gal. 2qt.} \end{array}$$

$$\begin{array}{r} (5.) \\ 40\text{hhd.} \\ 63 \\ 120 \\ 240 \\ \hline 2520\text{gal.} \\ 4 \\ \hline 10080\text{qt.} \\ 2 \\ \hline 20160\text{pt.} \\ 0.37\frac{1}{2} \\ \hline 141120 \\ 60480 \\ 10080 \end{array}$$

$$\begin{array}{r} (6.) \quad 2 \quad 126 \\ 63 \quad 4 \\ \hline 126 \quad 504 \\ 0.40 \quad 0.12 \\ \hline \$50.40 \quad \$60.48 \\ 50.40 \\ \hline \$10.08, \text{ Ans.} \end{array}$$

$$\begin{array}{r} \$7560.00, \text{ Ans.} \end{array}$$

BEER MEASURE.

(ART. 139, p. 102.)

- (1.) $46 \times 54 = 2484\text{gal.}$; $2484\text{gal.} + 49\text{gal.} = 2533\text{gal.}$;
 $2533 \times 4 = 10132\text{qt.}$; $10132 \times 2 = 20264\text{pt.}$, Ans.
- (2.) $20264 \div 2 = 10132\text{qt.}$; $10132 \div 4 = 2533\text{gal.}$; $2533 \div 54 = 46\text{hhd.}$ 49gal., Ans.
- (3.) $368 \times 54 = 19872\text{gal.}$; $19872 \times 4 = 79488\text{qt.}$; $79488 \times 2 = 158976\text{pt.}$, Ans.
- (4.) $158976 \div 2 = 79488\text{qt.}$; $79488 \div 4 = 19872\text{gal.}$;
 $19872 \div 54 = 368\text{hhd.}$, Ans.
- (5.) $76 \times 54 = 4104\text{gal.}$; $4104 \times 0.29 = \$1190.16$, Ans.
- (6.) $47 \times 54 = 2538\text{gal.}$; $2538\text{gal.} + 36\text{gal.} = 2574\text{gal.}$;
 $2574 \times 4 = 10296\text{qt.}$; $10296 \times 0.05 = \$514.80$,
 Ans.

DRY MEASURE.

(ART. 140, p. 103.)

- (1.) $35 \times 4 = 140\text{pk.}$; $140\text{pk.} + 3\text{pk.} = 143\text{pk.}$; $143 \times 8 = 1144\text{qt.}$; $1144 \times 2 = 2288\text{pt.}$, Ans.
- (2.) $2288 \div 2 = 1144\text{qt.}$; $1144 \div 8 = 143\text{pk.}$; $143 \div 4 = 35\text{bu.}$ 3pk., Ans.
- (3.) $676 \times 36 = 24336\text{bu.}$; $24336 \times 4 = 97344\text{pk.}$, Ans.
- (4.) $97344 \div 4 = 24336\text{bu.}$; $24336 \div 36 = 676\text{ch.}$, Ans.
- (5.) $50 \times 0.60 = \$30.00$; $50 \times 5 = 250\text{pk.}$; $250 \times 0.15 = \$37.50$; $\$37.50 - \$30.00 = \$7.50$ gain, Ans.
- (6.) $96 \times 0.42 = \$40.32$; $2304 \div 32 = 72\text{bu.}$; $72 \times 0.45 = \$32.40$; $\$40.32 - \$32.40 = \$7.92$ loss, Ans.

DRY, LIQUID, AND BEER MEASURE COMPARED.

(ART. 141, p. 104.)

- (1.) 4hhd. 15gal. beer = 231gal.; $231 \times 282 = 65142$;
 $65142 \div 231 = 282$ gal. wine = 4hhd. 30gal., Ans.
- (2.) 4hhd. 30gal. = 282 wine gal.; $282 \times 231 = 65142$;
 $65142 \div 282 = 231$ beer gal. = 4hhd. 15gal., Ans.
- (3.) $2820 \times 4 = 11280$ qt.; $11280 \times 0.04 = \$451.20$; $2820 \times 282 = 795240$; $795240 \div 231 = 3442\frac{4}{7}$ gal. wine measure; $3442\frac{4}{7} \times 4 = 13770\frac{4}{7}$ qt.; $13770\frac{4}{7} \times 0.06 = \$826.22\frac{2}{7}$; $\$826.22\frac{2}{7} - \$451.20 = \$375.02\frac{2}{7}$,
 Ans.
- (4.) $2538 \times 4 \times 0.04 = \406.08 ; $2538 \times 231 = 586278$;
 $586278 \div 282 = 2079$; $2079 \times 4 \times 0.06 = \498.96 ;
 $\$498.96 - \$406.08 = \$92.88$ gain, Ans.
- (5.) $385 \times 4.00 = \$1540.00$; 385 bu. $\times 268\frac{4}{5} = 103488$;
 $103488 \div 231 = 448$ bu.; $448 \times 32 \times 0.20 = \2867.20 ;
 $\$2867.20 - \$1540.00 = \$1327.20$ gain, Ans.
- (6.) $1000 \times 1.05 = \$1050.00$; 1000 bu. $\times 268\frac{4}{5} = 268800$;
 $268800 \div 231 = 1163\frac{7}{11}$ bu.; $1163\frac{7}{11} \times 1.12 = \1303.27 ;
 $\$1303.27 - \$1050.00 = \$253.27$ gain,
 Ans.
- (7.) $30' \times 1728 = 51840$ cubic inches; $51840 \div 282 = 183\frac{4}{9}$ gal.;
 $51840 \div 268\frac{4}{5} = 192\frac{1}{2}$ gal.; $192\frac{1}{2} - 183\frac{4}{9} = 9\frac{5}{18}$ gal., Ans.
- (8.) $365 \times 4 \times 0.05 = \73.00 ; $365 \times 231 = 84315$; $84315 \div 282 = 299$ gal.;
 $299 \times 4 \times 0.05 = \59.80 ; $\$73.00 - \$59.80 = \$13.20$, Ans.

TIME.

(ART. 142, p. 106.)

(1.) $365\text{da. } 5\text{h. } 48\text{min. } 49\text{sec.}$

$$\begin{array}{r} 24 \\ 1465 \\ 730 \\ \hline 8765\text{h.} \\ 60 \\ \hline 525948\text{m.} \\ 60 \\ \hline \end{array}$$

31556929sec., Ans.

(2.)

$$\begin{array}{r} 60)31556929\text{sec.} \\ 60)525948\text{m. } 49\text{sec.} \\ 24)8765\text{h. } 48\text{m.} \\ 365\text{da. } 5\text{h.} \end{array}$$

Ans. 365da. 5h. 48m. 49sec.

- (3.) $2348\text{yrs.} + 1855\text{yrs.} = 4203\text{yrs.}$; $365\text{da. } 6\text{h.} = 31557600\text{sec.}$; $31557600\text{sec.} \times 4203 = 132636592800\text{sec.}$, Ans.
 (4.) 31556929 seconds in a solar year; therefore $74726807872 \div 31556929 = 2368$ years, Ans.
 (5.) June 13da. + July 31 + August 31 + September 30 + October 31 + November 30 + December 31 + January 31 + February 16 + 365 + 365 = 974da. = 2 yrs. 244da., Ans.

CIRCULAR MEASURE.

(ART. 143, p. 107.)

(1.) $27\text{S. } 27^\circ 43'$

$$\begin{array}{r} 30 \\ 837^\circ \\ 60 \\ \hline 50263', \text{ Ans.} \end{array}$$

(2.)

$$\begin{array}{r} 60)50263' \\ 30)837^\circ 43' \\ 27\text{S. } 27^\circ \\ \hline \text{Ans. } 27\text{S. } 27^\circ 43'. \end{array}$$

- (3.) $44 \times 30 = 1320^\circ$; $1320^\circ + 18^\circ = 1338^\circ$; $1338 \times 60 = 80280'$; $80280' + 57' = 80337'$; $80337' \times 60 = 4820220''$; $4820220'' + 23'' = 4820243''$, Ans.
 (4.) $4820243 \div 60 = 80337' 23''$; $80337 \div 60 = 1338^\circ 57'$; $1338 \div 30 = 44\text{S. } 18^\circ$; $44\text{S. } 18^\circ 57' 23''$, Ans.
 (5.) $360 \times 60 = 21600'$; $21600 \times 60 = 1296000$, Ans.
 (6.) $1296000 \div 60 = 21600'$; $21600 \div 60 = 360^\circ$, Ans.

MISCELLANEOUS.

(ART. 144, p. 108.)

- (1.) $4 \times 5 + 4 = 24$; $24 \times 2 + 1 = 49$; $49 \times 20 + 10 = 990$; $990 \times 24 = 23760$ sheets, Ans.
- (2.) $23760 \div 24 = 990$ qr.; $990 \div 20 = 49$ rm. 10qr.; $49 \div 2 = 24$ bun. 1rm.; $24 \div 5 = 4$ ba. 4bun.; 4ba. 4bun. 1rm. 10qr., Ans.
- (3.) $10 \times 8 + 6 = 86$; $86 \times 21\frac{1}{2} + 8 = 1857$; $1857 \times 14 = 25998$ lbs.
- (4.) $25998 \div 14 = 1857$; $1857 \div 21\frac{1}{2} = 86$ pigs 8 stone; $86 \div 8 = 10$ fother 6p.; 10 fother 6 pigs 8 stone, Ans.
- (5.) $56 \times 12 \times 0.23 = \154.56 , Ans.
- (6.) $\$22.00 \div 200 = \0.11 , Ans.
- (7.) $302 \times 8 \times 0.30 = \724.80 , Ans.
- (8.) $\$4.00 \div 100 = \0.04 ; $50.24 \div .04 = 1256$ pounds.
- (9.) $12 \times 12 \times 12 \times \$0.045 = \$77.76$.
- (10.) $56 \times 2 = 112$ lb.; $112 \times 0.20 = \$22.40$; 196 lb. $\div 2 = 98$ lb.; $98 \times 0.05 = \$4.90$; $\$22.40 - \$4.90 = \$17.50$, Ans.

MISCELLANEOUS EXAMPLES IN REDUCTION.

(PAGE 109.)

- (1.) $57 \times 20 + 15 = 1155$ s.; $1155 \times 12 = 13860$ d.; $13860 \times 2 = 27720$ half-pence, Ans.
- (2.) $59 \times 12 = 708$ oz.; $708 \times 20 + 13 = 14173$ pwt.; $14173 \times 24 + 15 = 340167$ gr., Ans.
- (3.) $340167 \div 24 = 14173$ pwt. 15 gr.; $14173 \div 20 = 708$ oz. 13pwt.; $708 \div 12 = 59$ lb.; 59lb. 13pwt. 15gr., Ans.
- (4.) $761 \times 4 = 3044$ qr.; $3044 \div 5 = 608$ E.E. 4qr., Ans.
- (5.) $61 \times 3 = 183$ qr.; $183 \div 4 = 45$ yd. 3qr., Ans.
- (6.) $63 \times 4 = 252$ qt.; $252 \times 2 = 504$ pt.; $504 \div 3 = 168$ bottles, Ans.
- (7.) 2ft. 8in. = 32in.; $15 \times 8 \times 40 \times 16\frac{1}{2} \times 12 = 950400$ in.; $950400 \div 32 = 29700$ times, Ans.
- (8.) 2oz. 12pwt. = 52pwt.; 5lb. 2oz. 8pwt. = 1248pwt.; $1248 \div 52 = 24$ spoons, Ans.

- (9.) 14ft. 9in. = 177in.; $436 \times 8 \times 40 \times 16\frac{1}{2} \times 12 = 276-24960$ in.; $27624960 \div 177 = 156073\frac{32}{177}$ times, Ans.
- (10.) $123 \times 78 \times 4 = 38376$ ears; $38376 \div 8 = 4797$ qt.; 4797 qt. $\div 8 = 599$ pk. 5qt.; 599 pk. $\div 4 = 149$ bu. 3pk. 5qt., Ans.
- (11.) 5yd. 2qr. 3na. = 91na.; $182 \times 4 \times 4 = 2912$ na.; $2912 \div 91 = 32$ suits, Ans.
- (12.) 5pwt. 10gr. = 130gr.; 3lb. 1oz. 2pwt. 2gr. = 17810gr.; $17810 \div 130 = 137$ rings, Ans.
- (13.) 18 in. $\div 3 = 6$ in.; 6 in. $\times 4 = 24$ in.; 56 ft. $\times 60 = 3360$ ft.; $3360 \times 144 = 483840$ in.; $483840 \div 24 = 20160$ shingles, Ans.
- (14.) $56 \times 25 \times 2 = 2800$ sq. ft.; $2800 \times 6 = 16800$ shingles, Ans.
- (15.) 22m. 3fur. 17rd. = 7177rd.; $25000 \times 320 = 8000000$ rd.; $8000000 \div 7177 = 1114\frac{122}{177}$ da., Ans.
- (16.) 7lb. 10oz. = 122oz.; 10cwt. 3qr. 16lb. = 1091lb.; $1091 \times 16 = 17456$ oz.; $17456 \div 122 = 143\frac{5}{11}$ weeks, Ans.
- (17.) $7 \times 63 = 441$ gal.; $441 \times 4 = 1764$ qt.; $1764 \times 0.09 = \$158.76$, Ans.
- (18.) $15 \times 54 = 810$ gal.; $810 \times 4 = 3240$ qt.; $3240 \times 2 = 6480$ pt.; $6480 \times 0.03 = \$194.40$, Ans.
- (19.) $73 \times 32 = 2336$ qt.; $2336 \times 0.02 = \$46.72$, Ans.
- (20.) $29 \times 57 = 1653$ yd.; $1653 \times 0.15 = \$247.95$, Ans.
- (21.) $120 \times 4\frac{1}{2} \times 4 = 2160$ cubic feet; $\$4.00 \div 128 = \0.03125 ; $0.03125 \times 2160 = \$67.50$, Ans.
- (22.) $33 \times 18 = 594$ sq. ft.; $594 \div 9 = 66$ sq. yd.; $\$0.20 \times 66 = \13.20 , Ans.
- (23.) $20 \times 25 = 500$ pills; $6 + 5 + 4 = 15$ gr.; $500 \times 15 = 7500$ gr.; Ans.
- (24.) $87 \times 63 = 5481$ gal.; $5481 \times 0.33 = \$1808.73$, Ans.
- (25.) 128 ft. $\div 2 = 64$ ft.; $64 \times 880 = 56320$ oz.; $128 \times 480 = 61440$ oz.; 56320 oz. $+ 61440$ oz. = 117760oz.; $117760 \div 16 = 7360$ lb., Ans.

- (26.) $24 \times 0.43 = \$ 10.32$; $\$ 1548.00 \div 10.32 = \$ 150.00$,
Ans.
- (27.) From 1837 to 1857 there are 5 leap years, viz., 1840, 1844, 1848, 1852, and 1856. Therefore, $\$ 10.00 \times 5 = \$ 50.00$; $\$ 200.00 - \$ 50.00 = \$ 150.00$, Ans.
- (28.) $1,000,000 \times 25\frac{1}{4} = 25,800,000\text{gr.}$; $25,800,000 \div 7000 = 3685\frac{1}{2}\text{lb.}$, Ans.
- (29.) $188 \times 0.50 = \$ 94.00$; $188\text{gal.} \div 2 = 94\text{gal.}$; $94 \times 231 = 21714$; $21714 \div 282 = 77\text{gal.}$; $77 \times 4 = 308\text{qt.}$; $308 \times 0.12\frac{1}{2} = \$ 38.50$; $94 \times 4 = 376\text{qt.}$; $376 \times 0.14 = \$ 52.64$; $\$ 52.64 + \$ 38.50 = \$ 91.14$; $\$ 94.00 - \$ 91.14 = \$ 2.86$, Ans.

ADDITION OF COMPOUND NUMBERS.

(ART. 145, p. 112.)

2. 324T. 15cwt. 2qr. 15lb. 12oz. 9dr.
3. 233cwt. 1qr. 6lb. 4oz. 5dr.
4. 209lb. 7oz. 15pwt. 8gr.
5. 385lb. 7oz. 5pwt. 10gr.
6. 183~~lb~~ 6~~z~~ 3~~z~~ 1~~o~~ 19gr.
7. 246~~lb~~ 10~~z~~ 3~~z~~ 2~~o~~ 2gr.
8. 195deg. 55m. 5fur. 24rd. 1ft. 1in.
9. 317m. 3fur. 18rd. 4yd. 2ft. 3in.
10. 229yd. 3qr. 3na. 1 $\frac{1}{4}$ in.
11. 2513E.E. 3qr. 2na. 0in.
12. 2038A. 1R. 13p. 2ft. 95in.
13. 317A 0R. 24p. 13 $\frac{3}{4}$ ft.
14. 160m. 0fur. 1ch. 1p. 12l.
15. 194m. 6fur. 9ch. 0p. 12l.
16. 335T. 23ft. 1173in.
17. 378Q. 96ft. 1460in.
18. 354tu 0hhd. 30gal. 1qt.
19. 286hhd. 42gal. 1qt. 1pt.
20. 784hhd. 24gal. 2qt. 0pt.

21. 298bu. 0pk. 4qt. 1pt.
22. 128bu. 2pk. 7qt.
23. 227y. 7m. 16da. 21h. 28min. 2sec.
24. 240w. 1da. 23h. 33min. 8sec.
25. 5S. 10° 35' 16".
26. 11S. 2° 22' 42".
27. 20£. 17s. 10d.
28. 5lb. 9oz. 4pwt. 20gr.
29. 7℔ 10³ 3³ 1⁹ 4gr.
30. 117cwt. 1qr. 22lb.
31. 175yd. 2qr. 2na.
32. 1060m. 5fur. 16rd. 5yd. 1ft.
33. 588A. 1R. 31p.
34. 7S. 29° 33' 56".

SUBTRACTION OF COMPOUND NUMBERS.

(ART. 146, p. 116.)

1. 435£. 16s. 3d.
2. 67£. 18s. 5³/₄d.
3. 435£. 15s. 9¹/₄d.
4. 51T. 18cwt. 2qr. 21lb. 9oz. 8dr.
5. 53cwt. 3qr. 20lb. 14oz.
6. 54lb. 4oz. 14pwt. 19gr.
7. 38lb. 7oz. 14pwt. 13gr.
8. 52℔ 6³ 3³ 1⁹ 15gr.
9. 5℔ 5³ 0³ 1⁹ 17gr.
10. 6m. 7fur. 38rd. 2ft. 11in.
11. 9deg. 4m. 5fur. 37rd. 4yd. 1ft. 4in.
12. 48yd. 2qr. 2na. 1¹/₄in.
13. 31E.E. 4qr. 2na.
14. 38A. 2R. 5p. 264ft. 33in.
15. 3A. 1R. 38p. 18yd. 7ft. 36in.
16. 1m. 7fur. 1ch. 2p. 23l.
17. 12m. 6fur. 8ch. 0p. 24l.

18. 30T. 37ft. 1620in.
19. 163C. 53ft. 1289in.
20. 68tu. 1hhd. 53gal. 3qt.
21. 577hhd. 52gal.
22. 17ch. 35bu.
23. 12bu. 3pk. 4qt.
24. 4mo. 23da. 20h. 45min. 42sec.
25. 28y. 1mo. 15da. 21h. 48min. 5sec.
26. 9S. $2^{\circ} 56' 52''$.
27. 2S. $27^{\circ} 21' 54''$.
28. 67A. Ord. 38p. $265\frac{1}{4}$ ft.
29. 99£. 16s. $5\frac{1}{2}$ d.
30. 24956m. 7fur. 22rd. 15ft. 9in.
31. 4C. 50ft.
32. 11yd. 1qr. 3na.
33. 33cwt. 2q. 20lb.
34. 54gal. 0qt. 1pt.
35. 6A. 2R. 32p.
36. $12^{\circ} 11' 27''$.
37. 86bu. 1pk. 0qt. 1pt.
38. 1T. 19ft. 1418in.

MULTIPLICATION OF COMPOUND NUMBERS.

(ART. 147, p. 119.)

2. 19£. 10s. 4d.
3. 557T. 19cwt. 1qr. 11lb. 15oz. 7dr.
4. 39lb. 10oz. 13pwt. 3gr.
5. 261lb. 11oz. 0pwt. 0gr.
6. 427lb 10 $\frac{3}{4}$ 0 $\frac{3}{4}$ 2 $\frac{1}{2}$ 14gr.
7. 858deg. 44m. 4fur. 6rd. 7ft. 0in.
8. 215m. 7fur. 9ch. 3p. 1l.
9. 181A. 3R. 11p. 6yd. 4ft. 4lin.
10. 57yd. 2qr. 3na.
11. 123tu. 3hhd. 36gal. 3qt.
12. 209hhd. 55gal. 3qt. 0pt. 1gi.

13. 118bu. 1pk. 5qt.
14. 110y. 343d. 3h. 41m. 12sec.
15. 149deg. 9m. 5fur. 13rd. 3yd. 1ft.
16. 214A. 3R. 12p.
17. 3d. 16h. 54m.
18. 29bu. 0p. 0qt.

(ART. 148, p. 121.)

2. 56T. 14cwt. 3qr. 15lb.
3. 5£. 4s. 3d.
4. 10£. 8s. 3d.
5. 12£. 13s. 0d.
6. 1290£. 4s. 0d.
7. 32lb. 9oz. 15pwt.
8. 746m. 5fur.
9. 15° 0' 0''.
10. 1275A. 2R. 16p. 22yd. 8ft. 32in.
11. 2y. 68d. 19h. 54m.

(ART. 149, p. 122.)

2. 89hhd. 52gal. 3qt. 1pt.
3. 179bu. 3pk. 5qt. 0pt. 1gi.
4. 275ch. 19bu. 2pk.
5. 23£. 11s. 5½d.
6. 3419A. 2R. 23p.
7. 3375yd. 3qr. 1na. 0¼in.
8. 44hhd. 52gal. 2qt. 1pt.
9. 6357m. 5fur. 14rd. 11½ft.
10. 175gal. 2qt. 0pt.
11. 15T. 38ft. 1074in.
12. 132C. 2ft.

DIVISION OF COMPOUND NUMBERS.

(ART. 150, p. 123.)

2. 2£. 8s. 9½d.
3. 61T. 19cwt. 3qr. 17lb. 15oz. 15dr.
4. 7lb. 11oz. 14pwt. 15gr.
5. 32lb. 8oz. 17pwt. 12gr.
6. 38℔ 10⅓ 5⅓ 2Θ 14gr.
7. 71deg. 38m. 2fur. 13rd. 14ft. 4in.
8. 17m. 7fur. 9ch. 3p. 23l.
9. 16A. 2R. 4p. 19yd. 7ft. 79in.
10. 10yd. 3qr. 3na.
11. 17tun. 2hhd. 50gal. 1qt.
12. 29hhd. 61gal. 3qt. 1pt. 3gi.
13. 19bu. 2pk. 7qt. 1pt.
14. 13y. 316d. 15h. 27m. 39sec.
15. 16deg. 39m. 3fur. 39rd. 5yd. 2ft.
16. 23A. 3R. 19½p.
17. 7h. 24m. 30sec.
18. 3bu. 2pk. 4qt.

(ART. 151, p. 124.)

2. 12cwt. 2qr. 11lb.
3. 0£. 5s. 9½d.
4. 0£. 9s. 11d.
5. 0£. 11s. 6d.
6. 13£. 8s. 9½d.
7. 3oz. 5pwt. 15gr.
8. 24m. 7fur. 4rd.
9. 0° 15'.
10. 39A. 3R. 17p. 30yd. 8ft. 100in.
11. 2d. 5h. 17m. 19sec.

(ART. 152, p. 125.)

2. 2hhd. 19gal 0qt. 1pt.
3. 3bu. 1pk. 4qt. 1pt. 1gi.

4. 16ch. 7bu. 2pk. 0qt. 0pt.
5. 0£. 8s. 3½d.
6. 117A. 3R. 27p.
7. 37yd. 3qr. 2na. 2in.
8. 47gal. 3qt. 1pt.
9. 17m. 3fur. 13rd. 14ft.
10. 3gal. 1qt. 1pt.
11. 37ft. 978in.
12. 7C. 98ft.
13. 4T. 15cwt. 2qr. 10¼½lb.
14. 19m. 3fur. 39rd. 13ft. 2¼½in.
15. 24A. 3R. 6¾p.
16. 9A. 1R. 19p. 139¾ft.
17. 13A. 0R. 27p. 3yd. 0ft. 45½in.

PRINCIPLES AND APPLICATIONS.

(ART. 153, p. 127.)

(2.)

y.	mo.	d.
1857	3	19
1854	5	3
<hr/>		
2	10	16

(3.)

y.	mo.	d.
1857	5	12
1856	9	26
<hr/>		
0	7	16

(4.)

y.	mo.	d.
1776	6	4
1620	11	22
<hr/>		
155	6	12

(5.)

y.	mo.	d.
1799	11	14
1732	1	22
<hr/>		
67	9	22

(ART. 154, p. 128.)

2. From Dec. to Feb. = 2 mo.,
For Nov. 14d., Feb. 12d.,

$$\begin{array}{rcl}
 31 \times 2 & = & 62 \text{ days.} \\
 16 + 12 & = & 28 \\
 \hline
 & & 90 \text{ days.}
 \end{array}$$

3. From July to May = 10mo., $10 \times 31 = 310$ days.
 For June 13 days, $\begin{array}{r} 13 \\ \hline 323 \end{array}$
 For Sept. 1d., Nov. 1d., Feb. 3d., April 1, $\begin{array}{r} 6 \\ \hline 317 \text{ days.} \end{array}$
4. From April to May = 1mo., 30 days.
 For March 28 days, May 2 days, $\begin{array}{r} 30 \\ \hline 60 \text{ days.} \end{array}$
5. From Dec. to April = 4mo., $4 \times 31 = 124$ days.
 For Nov. 27 days, April 3 days, $\begin{array}{r} 30 \\ \hline 154 \end{array}$
 For Feb. 3 days, $\begin{array}{r} 3 \\ \hline 151 \text{ days.} \end{array}$
6. From Feb. to Dec. = 10 mo., $10 \times 31 = 310$ days.
 For Jan. 25 days, Dec. 10 days, $\begin{array}{r} 35 \\ \hline 345 \end{array}$
 For Feb. 2 days, April 1, June 1, Sept. 1, Nov. 1, $\begin{array}{r} 6 \\ \hline \text{Ans. 339 days.} \end{array}$

(ART. 155, p. 129.)

2. From April 2 to July 4 = 93 days.
 $93 \div 7 = 13$ weeks, 2 days.
 2-days after Wednesday = Friday, Ans.
3. From Jan. 1 to June 17 = 168 days.
 $168 \div 7 = 24$ weeks.
 Tuesday, Ans.
4. From Jan. 1 to Dec. 25 = 358 days.
 $358 \div 7 = 51$ weeks, 1 day.
 1 day before Tuesday = Monday, Ans.

5. From Nov. 4 to Feb. 1 = 89 days.

$89 \div 7 = 12$ weeks, 5 days, therefore February will commence 5 days after Tuesday = Sunday. If Sunday be the 1st day, Monday will be the 2nd and 9th days.

6. From Dec. 20 to March 1 = 72 days, it being *leap year*.
 $90 - 72 = 18$ days; the note will therefore become due March 19. $91 \div 7 = 13$ weeks. As there is no remainder, and as the note was given on Thursday, it will be payable on Wednesday, March 19.

NOTE. — In reckoning the interest on any note, it is not customary to charge interest on the day the note is paid, but it is reckoned on the day it is given.

DIFFERENCE OF LATITUDE.

(ART. 157, p. 130.)

2. Latitude of Quebec	= 46° 48' N.
Latitude of New Orleans	= 29° 57' N.
Difference of Latitude	= 16° 51', Ans.
3. Latitude of Washington City	= 38° 53' N.
Latitude of Cape Horn	= 55° 58' S.
Difference of Latitude	= 94° 51', Ans.
4. Latitude of Valparaiso	= 33° 2' S.
Latitude of San Francisco	= 37° 48' N.
Difference of Latitude	= 70° 50', Ans.
5. Latitude of New York City	= 40° 42' N.
Difference of Latitude N.	= 17° 33' N.
Latitude of Havana	= 23° 9' N., Ans.
6. Latitude of Mobile	= 30° 41' N.
Difference of Latitude	= 9° 15'
Latitude of Philadelphia	= 39° 56' N., Ans.

DIFFERENCE OF LONGITUDE.

(ART. 159. p. 131.)

- | | | |
|----------------------------------|---|---------------|
| 2. Longitude of Washington City | = | 77° 16' W. |
| Longitude of Paris | = | 2° 20' E. |
| Difference of Longitude | = | 79° 36', Ans. |
| 3. Longitude of Cape Flattery | = | 124° 43' W. |
| Longitude of St. Croix River | = | 67° 2' W. |
| Difference of Longitude | = | 57° 41', Ans. |
| 4. Longitude of Sacramento City | = | 120° 0' W. |
| Longitude of Raleigh | = | 78° 48' W. |
| Difference of Longitude | = | 41° 12', Ans. |
| 5. Longitude of Fort Leavenworth | = | 94° 44' W. |
| Longitude of Hartford, Ct. | = | 72° 40' W. |
| Difference of Longitude | = | 22° 4', Ans. |
| 6. Longitude of Honolulu | = | 157° 52' W. |
| Longitude of Canton | = | 113° 14' E. |
| | | 271° 6' |
| 360° — 271° 6' = 88° 54', Ans. | | |

LONGITUDE AND TIME.

(ART. 160, p. 132.)

- | | | |
|---------------------------|---|------------------|
| 2. Longitude of Galveston | = | 14° 43' |
| | | 4 |
| Difference of time | = | 0h. 58m. 52sec. |
| | | 12h. 0m. 0sec. |
| | | 12h. 58m. 52sec. |

3. Longitude of Valparaiso	= 71° 37' W.
Longitude of Rome	= 20° 30' E.
	<u>92° 7'</u>
	4
Difference of time	= 6h. 8m. 28sec.
	11h. 15m. 0s. A. M.
Time at Rome	= 5h. 23m. 28s. P. M.
4. Longitude of Jerusalem	= 35° 32' E.
Longitude of Baltimore	= 76° 37' W.
	<u>112° 9'</u>
	4
Difference of time	= 7h. 28m. 36sec.
Time at Jerusalem	= 9h. 0m. 0s. A. M.
	7h. 28m. 36s. A. M.
Time at Baltimore	= 1h. 31m. 24s. A. M., Ans.

(ART. 161, p. 133.)

(2.) 29m. ÷ 4 = 7° 15'	(3.) 2h. × 15 = 30°
36sec. ÷ 4 = 0° 9'	20m. ÷ 4 = 5°
<u>7° 24'</u>	40sec. ÷ 4 = 0° 10'
	<u>35° 10'</u>
(4.) 1h. × 15 = 15° 0'	(5.) 53m. ÷ 4 = 13° 15'
20sec. ÷ 4 = 0° 5'	30sec. ÷ 4 = 0° 7' 30"
<u>15° 5'</u>	<u>13° 22' 30"</u>
75° 10'	
<u>90° 15' W.</u>	
(6.) 1h. × 15 = 15° 0'	(7.) 7h. × 15 = 105° 0'
37m. ÷ 4 = 9° 15'	15m. ÷ 4 = 3° 45'
12sec. ÷ 4 = 0° 3'	40sec. ÷ 4 = 0° 10'
<u>Take 24° 18'</u>	<u>108° 55'</u>
From 93° 5'	30° 0'
<u>68° 47' West.</u>	<u>78° 55' West.</u>

$$\begin{array}{rcl}
 (8.) \text{ 4h.} \times 15 & = & 60^\circ 0' \\
 \text{44m.} \div 4 & = & 11^\circ 0' \\
 \text{32sec.} \div 4 & = & 0^\circ 8' \\
 & & \hline
 & & 71^\circ 8'
 \end{array}$$

MISCELLANEOUS EXAMPLES.

(PAGE 134.)

- (1.) 57 650 000 (2.) $755 - 125 = 630$; $630 \div 2 = 315$;
 263 517 496 $315 + 48 = 363$ bushels in the second;
 626 400 000 $315 - 48 = 267$ bushels in the third.
 100 000 000 (3.) $30 \times 3 = 90$ miles A travels. He has
 1 445 000 therefore travelled round the island three
 1 500 000 times. $30 \times 5 = 150$; $150 \div 30 = 5$;
 33) 1050 512 496 B has travelled round the island five
 31,833 712 times. A and B are therefore together.
- (4.) $\$1750 \times 2 = \3500 ; $\$103 \times 19 = \1957 ; $\$3500 +$
 $\$1957 + \$113 = \$5570$, Ans.
- (5.) $960 \div 2 = 480$; $480 \div 12 = 40$; $40 \div 2 = 20$ bushels
 each for those who left; $480 \div 8 = 60$; $60 \div 2 = 30$;
 $30 + 20 = 50$ bushels for each who remained.
- (6.) $652.00 \div .50 = 1304$ sheep, Ans.
- (7.) $27 \times 18 = 486$ sq. ft.; $486 \div 9 = 54$ yd.; 54×2.25
 $= \$121.50$, Ans.
- (8.) $365 \times 2\frac{1}{2} = 912\frac{1}{2}$ h.; $912\frac{1}{2}$ h. $\times 20 = 18250$ h. in 20
 years; $18250 \div 12 = 1520$ da. 10h. = 4y. 60da. 10h.,
 Ans.
- (9.) 7C. 76ft. 167lin. + 16C. 28ft. 56in. + 29C. 127ft. 1000in.
 + 29C. 10ft. 1216in. = 82C. 115ft. 487in.; 82C. 115ft.
 487in. - 45C. 96ft. 0in. = 37C. 19ft. 487in., Ans.
- (10.) $42^\circ 21' + 1^\circ 15' = 43^\circ 36'$; $43^\circ 36' - 10^\circ 40' = 32^\circ$
 $56'$, Ans.
- (11.) $2000\text{lb.} \times 2 = 4000\text{lb.}$; $4000 \div 54 = 74\frac{2}{3}$ cubic feet,
 Ans.
- (13.) $\$295,300,000 \div 250,000 = \1181.20 ; $\$295,300,000 \div$
 $4 = \$73,825,000$, Ans.

- (14.) $216 \times 1728 = 373248 \text{ in.}; 373248 \div 231 = 1615\frac{1}{2} \text{ gal.};$
 $1615\frac{1}{2} \div 63 = 25 \text{ hhd. } 40\frac{1}{2} \text{ gal., Ans.}$
- (15.) Ans. 11th day of May; 131st day of the year.
- (16.) 18T. 17cwt. 3qr. 20lb. = 37795lb.; $37795 \times 0.04 =$
 $\$ 1511.80$; 4T. 6cwt. 1qr. 14lb. = 8639lb.; $8639 \times$
 $0.05 = \$ 431.95$; 7T. 1cwt. 3qr. 10lb. = 14185lb.;
 $14185 \times 0.06 = \$ 851.10$; $8639 \text{ lb.} + 14185 \text{ lb.} =$
 $22824 \text{ lb.}; 37795 \text{ lb.} - 22824 \text{ lb.} = 14971 \text{ lb.}; 14971 \text{ lb.} \div$
 $4 = 3742\frac{3}{4} \text{ lb.}; 3742\frac{3}{4} \times 0.06 = \$ 224.56\frac{1}{2}$; 14971 lb.
 $- 3742\frac{3}{4} \text{ lb.} = 11228\frac{1}{4} \text{ lb.}; 11228\frac{1}{4} \text{ lb.} \div 2 = 5614\frac{1}{8} \text{ lb.};$
 $5614\frac{1}{8} \times 0.10 = \$ 561.41\frac{1}{4}$; $5614\frac{1}{8} \times 0.12 =$
 $\$ 673.69\frac{1}{2}$; $\$ 673.69\frac{1}{2} \div 2 = \$ 336.84\frac{3}{4}$; $\$ 431.95 +$
 $\$ 851.10 + \$ 224.56\frac{1}{2} + \$ 561.41\frac{1}{4} + \$ 336.84\frac{3}{4} =$
 $\$ 2405.87\frac{1}{2}$; $\$ 2405.87\frac{1}{2} - \$ 1511.80 = \$ 894.07\frac{1}{2}$, Ans.
- (17.) 3m. 7fur. 29rd. 15ft. + 5m. 0fur. 10rd. $1\frac{1}{2} \text{ ft.} = 9 \text{ m.}; 9 \text{ m.}$
 $\times 6\frac{1}{2} = 58\frac{1}{2} \text{ m.}$, the distance both travel each day;
 $2691 \div 58\frac{1}{2} = 46 \text{ da.}$, the number of days they travel
before they meet. 46 days from the 1st of January, omit-
ting 8 Sabbaths, as in duty bound, will be February 23d,
1857, at 2h. 30min. P. M. Swift will have travelled
1186m. 4fur. 22rd. 13ft. 6in.; and Fleet 1504m. 3fur.
17rd. 3ft. As January begins on Thursday, the 23d of
February will be on Monday, Ans.

EXAMPLES BY ANALYSIS.

(PAGE 136.)

- (1.) $\$ 8.75 \div 7 = \$ 1.25$; $\$ 1.25 \times 20 = \$ 25.00$, Ans.
- (2.) $\$ 85.00 \div 5 = \$ 17.00$; $\$ 17.00 \times 17 = \$ 289.00$, Ans.
- (3.) $\$ 0.75 \div 3 = \$ 0.25$; $\$ 0.25 \times 37 = \$ 9.25$, Ans.
- (4.) $\$ 1.92 \div 4 = \$ 0.48$; $\$ 0.48 \times 37 = \$ 17.76$, Ans.
- (5.) $\$ 1.08 \div 12 = \$ 0.09$; $\$ 0.09 \times 25 = \$ 2.25$, Ans.
- (6.) $\$ 63.00 \div 9 = \$ 7.00$; $\$ 7.00 \times 27 = \$ 189.00$, Ans.
- (7.) $20 \text{ m.} \div 5 = 4 \text{ m.}; 28 \div 4 = 7 \text{ h.}$, Ans.
- (8.) $49 \text{ crackers} \div 7 = 7 \text{ crackers}; 7 \text{ crackers} \times 19 = 133$
crackers, Ans.

- (9.) $\$ 20 \div 5 = \$ 4$; $40 \times 4 = \$ 160.00$, Ans.
- (10.) $\$ 0.36 \div 3 = \$ 0.12$; $\$ 0.12 \times 37 = \$ 4.44$, Ans.
- (11.) $\$ 0.72 \div 9 = \$ 0.08$; $\$ 0.08 \times 37 = \$ 2.96$, Ans.
- (12.) $2\text{£. } 17\text{s. } 5\text{d.} \div 52 = 0\text{£. } 1\text{s. } 1\frac{1}{4}\text{d.}$; $0\text{£. } 1\text{s. } 1\frac{1}{4}\text{d.} \times 76 = 4\text{£. } 3\text{s. } 11\text{d.}$, Ans.
- (13.) $4\text{£. } 3\text{s. } 11\text{d.} \div 76 = 0\text{£. } 1\text{s. } 1\frac{1}{4}\text{d.}$; $0\text{£. } 1\text{s. } 1\frac{1}{4}\text{d.} \times 52 = 2\text{£. } 17\text{s. } 5\text{d.}$, Ans.
- (14.) $17\text{m. } 4\text{fur. } 28\text{rd.} \div 6 = 2\text{m. } 7\text{fur. } 18\text{rd.}$; $2\text{m. } 7\text{fur. } 18\text{rd.} \times 100 = 293\text{m. } 1\text{fur.}$, Ans.
- (15.) $41\text{bu. } 3\text{pk. } 4\text{qt. } 1\text{pt.} \div 7 = 5\text{bu. } 3\text{pk. } 7\text{qt. } 1\text{pt.}$; $5\text{bu. } 3\text{pk. } 7\text{qt. } 1\text{pt.} \times 12 = 71\text{bu. } 3\text{pk. } 2\text{qt.}$; $71\text{bu. } 3\text{pk. } 2\text{qt.} \times 7 = 502\text{bu. } 2\text{pk. } 6\text{qt.}$, Ans.
- (16.) $39\text{A. } 2\text{R. } 5\text{p. } 8\text{yd. } 6\text{ft. } 108\text{in.} \div 8 = 4\text{A. } 3\text{R. } 30\text{p. } 20\text{yd.}$; $4\text{A. } 3\text{R. } 30\text{p. } 20\text{yd.} \times 9 = 44\text{A. } 1\text{R. } 35\text{p. } 28\text{yd. } 6\text{ft. } 108\text{in.}$; $44\text{A. } 1\text{R. } 35\text{p. } 28\text{yd. } 6\text{ft. } 108\text{in.} \times 8 = 355\text{A. } 3\text{R. } 7\text{p. } 18\text{yd. } 2\text{ft. } 36\text{in.}$, Ans.
- (17.) $63\text{gal. } 3\text{qt.} \div 4 = 15\text{gal. } 3\text{qt. } 1\text{pt. } 2\text{gi.}$; $15\text{gal. } 3\text{qt. } 1\text{pt. } 2\text{gi.} \times 37 = 589\text{gal. } 2\text{qt. } 1\text{pt. } 2\text{gi.}$, Ans.
- (18.) $5\text{yd. } 3\text{qr. } 1\text{na.} \div 4 = 1\text{yd. } 1\text{qr. } 3\frac{1}{4}\text{na.}$; $1\text{yd. } 1\text{qr. } 3\frac{1}{4}\text{na.} \times 36 = 52\text{yd. } 1\text{qr. } 1\text{na.}$, Ans.
- (19.) $11\text{T. } 3\text{cwt. } 2\text{qr.} \div 4 = 2\text{T. } 15\text{cwt. } 3\text{qr. } 12\text{lb. } 8\text{oz.}$; $2\text{T. } 15\text{cwt. } 3\text{qr. } 12\text{lb. } 8\text{oz.} \times 23 = 64\text{T. } 5\text{cwt. } 0\text{qr. } 12\text{lb. } 8\text{oz.}$, Ans.
- (20.) $286\text{da. } 4\text{h. } 33\text{min.} \times 12 = 3434\text{da. } 6\text{h. } 36\text{min.}$, the time one man would dig it; $3434\text{da. } 6\text{h. } 36\text{min.} \div 72 = 47\text{da. } 16\text{h. } 45\text{min. } 30\text{sec.}$, time 72 men would dig it, Ans.
- (21.) $27\text{yd. } 1\text{qr.} \div 21 = 1\text{yd. } 1\text{qr. } 0\text{na. } 1\frac{1}{2}\text{in.}$; $1\text{yd. } 1\text{qr. } 0\text{na. } 1\frac{1}{2}\text{in.} \times 11 = 14\text{yd. } 1\text{qr. } 0\text{na. } 0\frac{1}{2}\text{in.}$, Ans.
- (22.) $174\text{m. } 0\text{fur. } 26\text{rd.} \div 7 = 24\text{m. } 6\text{fur. } 38\text{rd.}$; $24\text{m. } 6\text{fur. } 38\text{rd.} \times 10 = 248\text{m. } 5\text{fur. } 20\text{rd.}$, Ans.
- (23.) $4 \times 60 = 240\text{ pair}$; $\$ 192.00 \div 240 = \$ 0.80$; $\$ 192.00 \div 4 = \$ 48.00$; $\$ 48.00 \times 25 = \$ 1200.00$, Ans.
- (24.) $3\text{A. } 2\text{R. } 20\text{p.} \div 4 = 0\text{A. } 3\text{R. } 25\text{p.}$, cost of one hoghead; $0\text{A. } 3\text{R. } 25\text{p.} \times 30 = 27\text{A. } 0\text{R. } 30\text{p.}$, Ans.

(25.) 20deg. 49m. 5fur. 35rd. 5yd. 0ft. 3in. $\div 9 =$ 2deg. 6m.
7fur. 6rd. 4yd. 1ft. 1lin.

20deg. 49m. 5fur. 35rd. 5yd. 0ft. 3in.
10

207deg. 12 $\frac{1}{2}$ m. 2fur. 39rd. $\frac{1}{2}$ yd. 2ft. 6in.
 $\frac{1}{2}$ yd. = 1ft. 6in.

207deg. 12 $\frac{1}{2}$ m. 2fur. 39rd. 1yd. 1ft. 0in.
 $\frac{1}{2}$ m. = 6fur. 26rd. 3yd. 2ft. 0in.

207deg. 13m. 1fur. 25rd. 5yd. 0ft. 0in., Ans.

FACTORING.

(PAGE 141.)

(2.)	(3.)	(4.)	(5.)
3)105	2)220	2)936	3)1953
5)35	2)110	2)468	3)651
7)7	11)55	2)234	7)217
1	5)5	3)117	31)31
Ans. 3, 5, 7.	1	3)39	1
	Ans. 2, 5, 11.	13)13	Ans. 3, 7, 31.
		1	
	Ans. 2, 3, 13.		

(6.)	(7.)	(8.)
2)12462	11)19987	3)225
3)6231	23)1817	3)435
31)2077	79)79	5)145
67)67	1	29
1	Ans. 11, 23, 79.	3)135
Ans. 2, 3, 31, 67.		3)45
	Ans. 3, 5.	3)15
		5)5
		1

(9.)	$2 \overline{)960}$	$2 \overline{)1568}$	$2 \overline{)5824}$
	$2 \overline{)480}$	$2 \overline{)784}$	$2 \overline{)2912}$
	$2 \overline{)240}$	$2 \overline{)392}$	$2 \overline{)1456}$
	$2 \overline{)120}$	$2 \overline{)196}$	$2 \overline{)728}$
	$2 \overline{)60}$	$2 \overline{)98}$	$2 \overline{)364}$
	$2 \overline{)30}$	$7 \overline{)49}$	$2 \overline{)182}$
	$3 \overline{)15}$	$7 \overline{)7}$	$7 \overline{)91}$
	$5 \overline{)5}$	1	$13 \overline{)13}$
	1	Ans. 2.	1
(10.)	$2 \overline{)2340}$	$2 \overline{)11934}$	$3 \overline{)12987}$
	$2 \overline{)1170}$	$3 \overline{)5967}$	$3 \overline{)4329}$
	$3 \overline{)585}$	$3 \overline{)1989}$	$3 \overline{)4953}$
	$3 \overline{)195}$	$3 \overline{)663}$	$13 \overline{)1651}$
	$5 \overline{)65}$	$13 \overline{)221}$	$127 \overline{)127}$
	$13 \overline{)13}$	$17 \overline{)17}$	1
	1	1	Ans. 3, 3, 13.
	(11.)	$3 \overline{)105}$	
		$5 \overline{)35}$	
		$7 \overline{)7}$	
		1	Ans. 3, 5, 7.

DIVISORS OR MEASURES.

(ART. 194, p. 145.)

$$(2.) \quad 72 = 2 \times 2 \times 2 \times 3 \times 3.$$

$$\text{Divisors. } \begin{cases} 1, 2, 4, 8 & = 2 \times 2 \times 4. \\ 3, 6, 12, 24 & = (2 \times 2 \times 2) \times 3. \\ 9, 18, 36, 72 & = (2 \times 2 \times 2) \times (3 \times 3). \end{cases}$$

$$(3.) \quad 105 = 3 \times 5 \times 7.$$

Exponent of 3 is $1 + 1 = 2$.

Exponent of 5 is $1 + 1 = 2$.

Exponent of 7 is $1 + 1 = 2$.

$2 \times 2 \times 2 = 8$, the number of divisors.

4. $1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$.
 2 (the exponent of factor 2) $+ 1 = 3$; $2 + 1 = 3$; $2 + 1 = 3$; $3 \times 3 \times 3 = 27$, Ans.
5. $3528 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 =$
 3 (the exponent of the factor 2) $+ 1 = 4$; $2 + 1 = 3$; $2 + 1 = 3$; $4 \times 3 \times 3 = 36$, Ans.
6. $5880 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 7$.
 $3 + 1 = 4$; $1 + 1 = 2$; $1 + 1 = 2$; $2 + 1 = 3$; $4 \times 2 \times 2 \times 3 = 48$, Ans.

COMMON DIVISORS.

(ART. 198, p. 146.)

2. $51 = 3 \times 17$.
 $153 = 3 \times 3 \times 17$.
 $255 = 3 \times 5 \times 17$.

The prime factors which are common are 1, 3, 17. Their products are found as in Art. 194.

Divisors 1, 3, 17, 51 = Ans. 1, 3, 17, 51.

3. $180 = 2 \times 2 \times 3 \times 3 \times 5$.
 $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$.
 2, 2, 3, 3, 5, are common. Hence,
 1, 2, 4,
 3, 6, 12 = $(2 \times 2) \times 3$.
 5, 10, 20 = $(2 \times 2) \times 5$.
 9, 18, 36 = $(2 \times 2) \times (3 \times 3)$.
 15, 30, 60 = $(2 \times 2) \times (3 \times 5)$.
 45, 90, 180 = $(2 \times 2) \times (3 \times 3 \times 5)$.

Ans. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45,
 [60, 90, 180.

4. $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$.
 $6075 = 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$.
 $8100 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$.

The factors $3 \times 3 \times 3 \times 3 \times 5 \times 5$ are common.

The exponent of 3 is $4 + 1 = 5$; the exponent of 5 is $2 + 1 = 3$; and $5 \times 3 = 15$, Ans.

$$5. 4500 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$$

$$9000 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$$

The *common* prime factors are $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$.

Their exponents are $2 + 1 = 3$; $2 + 1 = 3$; $3 + 1 = 4$; and

$$3 \times 3 \times 4 = 36, \text{ Ans.}$$

(ART. 200, p. 148.)

THE GREATEST COMMON DIVISOR OR MEASURE.

2.	56	6.	35	10.	15
3.	32	7.	1	11.	78
4.	57	8.	1	12.	4 acre lots.
5.	1	9.	12	13.	6 bushels.

LEAST COMMON MULTIPLE.

(ART. 203, p. 151.)

$$(2.) \quad 3 \quad 13 \quad 37 \quad 91 \qquad (3.) \quad 3 \overline{) 9 \quad 14 \quad 30 \quad 35 \quad 47}$$

$$5 \overline{) 3 \quad 14 \quad 10 \quad 35 \quad 47}$$

$$3 \times 37 \times 91 = 10101.$$

$$2 \overline{) 3 \quad 14 \quad 2 \quad 7 \quad 47}$$

$$7 \overline{) 3 \quad 7 \quad 1 \quad 7 \quad 47}$$

$$3 \quad 1 \quad 1 \quad 1 \quad 47$$

$$47 \times 3 \times 7 \times 2 \times 5 \times 3 = 29610, \text{ Ans.}$$

$$(4.) \quad \begin{array}{r} 6 \quad 8 \quad 10 \quad 18 \quad 20 \quad | \quad 24 \\ \hline 3 \quad 5 \end{array} \qquad (5.) \quad \begin{array}{r} 14 \quad 19 \quad 38 \quad | \quad 57 \\ \hline 14 \quad 2 \end{array}$$

$$3 \times 5 \times 24 = 360, \text{ Ans.}$$

$$14 \times 57 = 798, \text{ Ans.}$$

$$(6.) \quad \begin{array}{r} 20 \quad 36 \quad 48 \quad | \quad 50 \\ \hline 2 \quad 18 \quad | \quad 24 \\ \hline 3 \end{array} \qquad (7.) \quad \begin{array}{r} 15 \quad 25 \quad 35 \quad 45 \quad | \quad 100 \\ \hline 7 \quad 9 \end{array}$$

$$3 \times 24 \times 50 = 3600, \text{ Ans.} \qquad 7 \times 9 \times 100 = 6300, \text{ Ans.}$$

$$(8.) \quad \begin{array}{r} 100 \quad 200 \quad 300 \quad 400 \quad | \quad 575 \\ \hline 12 \quad | \quad 16 \\ \hline 3 \end{array}$$

$$3 \times 16 \times 575 = 27600, \text{ Ans.}$$

$$(9.) \begin{array}{r} \cancel{1} \ \cancel{2} \ \cancel{3} \ \cancel{4} \ 5 \ 6 \ 8 \ | \ 9 \\ \hline \phantom{\cancel{1} \ \cancel{2} \ \cancel{3} \ \cancel{4}} 5 \ \cancel{2} \ 8 \end{array}$$

$$5 \times 8 \times 9 = 360; 2520 \div 360 = 7, \text{ Ans.}$$

$$(10.) \begin{array}{r} \cancel{18} \ \cancel{24} \ 36 \ 20 \ 48 \ | \ 126 \\ \hline \phantom{\cancel{18} \ \cancel{24} \ 36} \cancel{2} \ \cancel{10} \ | \ 8 \\ \hline \phantom{\cancel{18} \ \cancel{24} \ 36} 5 \end{array}$$

$$5 \times 8 \times 126 = 5040, \text{ Ans.}$$

$$(11.) \begin{array}{r} \cancel{4} \ \cancel{6} \ 10 \ | \ 12 \\ \hline \phantom{\cancel{4} \ \cancel{6} \ 10} 5 \end{array}$$

$$5 \times 12 = 60\text{qt.}, \text{ Ans.}$$

$$(12.) \begin{array}{r} 50 \ 40 \ | \ 75 \\ \hline \cancel{2} \ 8 \end{array}$$

$$8 \times 75 = 600, \text{ Ans.}$$

MISCELLANEOUS EXAMPLES.

(Page 152.)

$$(1.) \begin{array}{r} 7 \overline{)6174} \\ \underline{7} 882 \\ 7 \overline{)126} \\ \underline{2} 18 \\ 3 \overline{)9} \\ \underline{3} 8 \\ 3 \overline{)8} \\ \underline{1} \end{array}$$

Ans. 3 times.

$$(2.) \begin{array}{r} 5 \overline{)5775} \\ \underline{5} 1155 \\ 5 \overline{)1155} \\ \underline{7} 231 \\ 3 \overline{)33} \\ \underline{11} 11 \\ 1 \overline{)11} \\ \underline{1} \end{array}$$

11, Ans.

$$(3.) \begin{array}{r} 3 \overline{)19929} \\ \underline{7} 6643 \\ 13 \overline{)949} \\ \underline{73} 73 \\ \underline{1} \end{array}$$

$$73 \times 13 \times 7 = 6643, \text{ Ans.}$$

$$(4.) \begin{array}{r} 2 \overline{)2338} \\ \underline{7} 1169 \\ 167 \overline{)167} \\ \underline{1} \end{array}$$

$$\left. \begin{array}{l} 2338 \div 2 = 1169 \\ 2338 \div 7 = 334 \end{array} \right\} \text{ Ans.}$$

KEY TO

$$\begin{array}{r}
 (5.) \ 5 \overline{)17385} \\
 \underline{3 \overline{)3477}} \\
 19 \overline{)1159} \\
 \underline{61 \overline{)61}} \\
 1
 \end{array}$$

Ans. 3, 5, 19, 61.

$$\begin{array}{rcl}
 (6.) \ 2 \overline{)3000} & 5 \times 5 \times 5 \times 3 \times 2 \times 2 = 1500 & \\
 \ 3 \overline{)1500} & 5 \times 5 \times 5 \times 2 \times 2 \times 2 = 1000 & \left. \vphantom{\begin{array}{l} 2 \overline{)3000} \\ 3 \overline{)1500} \end{array}} \right\} \text{Bins.} \\
 \ 5 \overline{)500} & 5 \times 5 \times 5 \times 3 \times 2 = 750 & \\
 \ 2 \overline{)100} & & 1, 2, 3 = \text{Bags.} \\
 \ 2 \overline{)50} & & \text{Ans.} \\
 \ 5 \overline{)25} & & \\
 \ 5 \overline{)5} & & \\
 \ 1 & &
 \end{array}$$

$$\begin{array}{rcl}
 (7.) \ 136 \overline{)152(1} & 152 \div 8 = 19 \text{ classes of ladies.} & \\
 \underline{136} & 136 \div 8 = 17 \text{ classes of gentlemen.} & \\
 \ 16 \overline{)136(8} & & \text{Ans.} \\
 \underline{128} & & \\
 \ 8 \overline{)16(2} & & \\
 \underline{16} & &
 \end{array}$$

8. The velocity of the hands of a clock is as the times in which they perform one revolution. The second-hand performs a revolution in 60 seconds, the minute-hand in 3600 seconds, and the hour-hand in 43200 seconds.

$$\begin{array}{r}
 60 \overline{)43200} \quad 3600 \quad 60 \\
 60 \overline{)720} \quad 60 \quad 1 \\
 \underline{12} \quad 1 \quad 1
 \end{array}$$

$$60 \times 60 \times 12 = 43200 \text{ seconds} = 12 \text{ hours, Ans.}$$

9. Greatest common divisor of 348, 372, 444, $492 = 12$;
 $348 \div 12 = 29$ rails; $372 \div 12 = 31$ rails; $444 \div 12 = 37$ rails; $492 \div 12 = 41$ rails; $29 + 31 + 37 + 41 = 138$ rails; $138 \times 5 = 690$ rails; $12\text{ft.} + 9\text{in.} = 12\text{ft. } 9\text{in.}$, length of each rail, Ans.

10. 3A. 2R. 1p. = 561p.; 5A. 3R. 15p. = 985p.; 8A. 0R. 29p. = 1309p.; 12A. 3R. 17p. = 2057p.; 15A. 0R. 31p. = 2431p.; $1309 \div 187 = 7$; $2057 \div 187 = 11$; 187p. is therefore the common divisor, and each field will contain 187p. = 1A. 0R. 27p., Ans.

$$\begin{array}{r} 561)985(1 \\ \underline{561} \\ 374)561(1 \\ \underline{374} \\ 187)374(2 \\ \underline{374} \end{array}$$

$$\begin{array}{l} (11.) \quad 2772 = 2 \times 2 \times 3 \times 3 \times 7 \times 11 \\ \quad 12 = 2 \times 2 \times 3 \\ \quad \left. \begin{array}{l} 12 \times 3 = 36 \\ 12 \times 7 = 84 \\ 12 \times 11 = 132 \end{array} \right\} \text{Ans.} \end{array}$$

$$(12.) \quad 3 + 1 = 4; 5 + 1 = 6; 7 + 1 = 8$$

$$\begin{array}{r} 2)4 \quad 6 \quad 8 \\ \underline{2)2 \quad 3 \quad 4} \\ 1 \quad 3 \quad 2 \end{array}$$

$$2 \times 2 \times 3 \times 2 = 24; 24 - 1 = 23 \text{ days, Ans.}$$

COMMON FRACTIONS.

REDUCTION.

(ART. 220, p. 156.)

2.	$\frac{4}{21}$	5.	$\frac{7}{54}$	8.	$\frac{1}{7}$	11.	$\frac{41}{81}$
3.	$\frac{3}{4}$	6.	$\frac{3}{35}$	9.	$\frac{416}{875}$	12.	$\frac{408}{156}$
4.	$\frac{3}{7}$	7.	$\frac{81}{222}$	10.	$\frac{311}{1118}$	13.	$\frac{41}{152}$

(ART. 221, p. 157.)

2.	$11 \frac{2}{15}$	5.	$31 \frac{34}{81}$	8.	125
3.	$14 \frac{17}{18}$	6.	$111 \frac{1}{3}$	9.	1
4.	$7 \frac{1}{3}$	7.	$91 \frac{28}{117}$		

(ART. 222, p. 157.)

3.	$\frac{90}{4}$	7.	$\frac{190}{1}$	10.	$\frac{1191}{61}$	13.	$\frac{7}{1}$
4.	$\frac{2577}{16}$	8.	$\frac{85}{17}$	11.	$\frac{69691}{97}$	14.	$\frac{247}{13}$
5.	$\frac{10142}{68}$	9.	$\frac{2554}{97}$	12.	$\frac{20990}{189}$	15.	$\frac{485}{4}$
6.	$\frac{11}{1}$						

(ART. 223, p. 158.)

$$3. \quad \frac{2}{3} \times \frac{5}{6} \times \frac{7}{8} \times \frac{11}{12} = \frac{770}{1728} = \frac{385}{864}, \text{ Ans.}$$

$$4. \quad \frac{6}{7} \times \frac{11}{15} \times \frac{3}{5} \times \frac{1}{18} = \frac{11}{525}, \text{ Ans.}$$

$$5. \quad \frac{4}{7} \times \frac{5}{8} \times \frac{3}{8} \times \frac{11}{13} = \frac{165}{1456}, \text{ Ans.}$$

$$6. \quad \frac{6}{11} \times \frac{6}{7} \times \frac{1}{4} \times \frac{3}{1} = \frac{108}{44} = 2\frac{5}{11}, \text{ Ans.}$$

$$7. \quad \frac{7}{11} \times 15\frac{1}{8} \times 5\frac{1}{10} \times 100 = \frac{7}{11} \times \frac{127}{8} \times \frac{57}{10} \times \frac{100}{1} = \frac{506730}{88} = 5758\frac{13}{44}, \text{ Ans.}$$

$$8. \quad \frac{3}{4} \times \frac{4}{7} \times \frac{11}{24} = \frac{11}{56}, \text{ Ans.}$$

$$9. \quad \frac{7}{11} \times \frac{11}{25} \times \frac{25}{31} \times \frac{7.75}{1} = \frac{54.25}{31} = \$1.75, \text{ Ans.}$$

$$10. \quad \frac{4}{9} \times \frac{9}{17} \times \frac{17}{18} \times 3\frac{1}{2} = \frac{4}{9} \times \frac{9}{17} \times \frac{17}{18} \times \frac{18}{5} = \frac{4}{5} \text{ gal., Ans.}$$

$$11. \quad \frac{1}{4} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{8}, \text{ Ans.}$$

$$12. \frac{4}{7} \times \frac{9}{20} \times \frac{3}{15} \times \frac{2}{16} \times \frac{2}{17} \times \frac{108}{1} = \frac{108}{16} = \$6.75, \text{ Ans.}$$

COMMON DENOMINATORS.

(ART. 226, p. 160.)

- | | |
|---|--|
| 2. $\frac{5}{12}, \frac{7}{12}, \frac{9}{12}, \frac{3}{12}$. | 16. $\frac{120}{120}, \frac{96}{120}, \frac{125}{120}$. |
| 3. $\frac{7320}{1320}, \frac{704}{1320}, \frac{1354}{1320}, \frac{275}{1320}$. | 17. $\frac{360}{630}, \frac{580}{630}, \frac{188}{630}$. |
| 4. $\frac{12}{64}, \frac{12}{64}, \frac{32}{64}, \frac{15}{64}$. | 18. $\frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}$. |
| 5. $\frac{32}{42}, \frac{24}{42}, \frac{22}{42}, \frac{21}{42}$. | 19. $\frac{364}{624}, \frac{122}{624}, \frac{4836}{624}$. |
| 6. $\frac{218}{308}, \frac{84}{308}, \frac{176}{308}, \frac{231}{308}$. | 20. $\frac{1485}{2295}, \frac{1020}{2295}, \frac{612}{2295}$. |
| 7. $\frac{30}{60}, \frac{60}{60}, \frac{45}{60}, \frac{65}{60}$. | 21. $\frac{2040}{2295}, \frac{540}{2295}, \frac{26316}{2295}$. |
| 8. $\frac{584}{893}, \frac{308}{893}, \frac{313}{893}, \frac{441}{893}$. | 22. $\frac{21}{42}, \frac{28}{42}, \frac{74}{42}, \frac{336}{42}$. |
| 9. $\frac{30}{120}, \frac{105}{120}, \frac{20}{120}, \frac{456}{120}$. | 23. $\frac{528}{1188}, \frac{756}{1188}, \frac{6038}{1188}$. |
| 10. $\frac{70}{168}, \frac{63}{168}, \frac{26}{168}, \frac{798}{168}$. | 24. $\frac{24}{48}, \frac{36}{48}, \frac{8}{48}, \frac{816}{48}$. |
| 11. $\frac{24}{42}, \frac{15}{42}, \frac{22}{42}, \frac{71}{42}$. | 25. $\frac{120}{120}, \frac{176}{120}, \frac{2580}{120}$. |
| 12. $\frac{224}{252}, \frac{147}{252}, \frac{238}{252}, \frac{24}{252}$. | 26. $\frac{12012}{14014}, \frac{5086}{14014}, \frac{5390}{14014}$,
[$\frac{8008}{14014}, \frac{7007}{14014}$]. |
| 13. $\frac{578}{608}, \frac{140}{608}, \frac{165}{608}, \frac{12}{608}$. | 27. $\frac{28506818}{178431552}, \frac{37088064}{178431552}$,
[$\frac{722813}{178431552}$]. |
| 14. $\frac{218}{240}, \frac{240}{240}, \frac{120}{240}, \frac{84}{240}$. | |
| 15. $\frac{14}{63}, \frac{441}{63}, \frac{504}{63}, \frac{324}{63}$. | |

ADDITION OF COMMON FRACTIONS.

(ART. 228, p. 162.)

- | | | | |
|--------------------------|-----|---------------------|------------------------|
| 3. $3\frac{19}{11}$. | 11. | 1. 18 . | 9 $2\frac{15}{15}$. |
| 4. $2\frac{9}{3}$. | 12. | 3 $\frac{8}{8}$. | 18 $\frac{8}{8}$. |
| 5. $1\frac{4}{4}$. | 13. | 1 $\frac{2}{2}$. | 45 $\frac{1}{1}$. |
| 6. $1\frac{1}{1}$. | 14. | 6 $\frac{4}{4}$. | 40 $\frac{1}{1}$. |
| 7. $2\frac{425}{21}$. | 15. | 9 $\frac{2}{2}$. | 1068 $\frac{37}{37}$. |
| 8. $2\frac{4}{2}$. | 16. | 6 $\frac{1}{1}$. | 12 $\frac{881}{881}$. |
| 9. $2\frac{1}{1}$. | 17. | 17 $\frac{1}{20}$. | 13 $\frac{21}{21}$. |
| 10. $2\frac{188}{888}$. | | | |

SUBTRACTION OF COMMON FRACTIONS

(ART. 230, p. 165.)

3.	$\frac{7}{17}$	11.	$\frac{7}{40}$	19.	$\frac{2}{25}$
4.	$\frac{9}{19}$	12.	$\frac{269}{1728}$	20.	$\frac{33}{100}$
5.	$\frac{15}{37}$	13.	$\frac{13}{250}$	21.	$\frac{5}{54}$
6.	$\frac{2}{30}$	14.	$\frac{1}{34}$	22.	$\frac{3}{56}$
7.	$\frac{5}{20}$	15.	$\frac{1}{6}$	23.	$\frac{2}{28}$
8.	$\frac{15}{37}$	16.	$\frac{23}{32}$	24.	$\frac{11}{12}$
9.	$\frac{1}{4}$	17.	$\frac{3}{8}$	25.	$\frac{1}{12}$
10.	$\frac{5}{36}$	18.	$\frac{92}{231}$		

(ART. 231, p. 166.)

2.	$26\frac{5}{8}$	6.	$15\frac{139}{239}$	10.	$77\frac{2}{3}$
3.	$11\frac{1}{4}$	7.	$16\frac{1}{4}$	11.	$680\frac{3}{4}$
4.	$670\frac{9}{50}$	8.	$5\frac{1}{4}$	12.	$30\frac{1}{8}$
5.	$368\frac{2}{3}$	9.	$18\frac{1}{4}$	13.	$31\frac{1}{10}$

(ART. 232, p. 167.)

(2.)	(3.)	(4.)
$18\frac{3}{8} = 18\frac{2}{2}$	$73\frac{4}{5} = 73\frac{8}{8}$	$67\frac{1}{3} = 67\frac{2}{3}$
$9\frac{3}{8} = \frac{9\frac{6}{8}}{8\frac{3}{2}}$	$16\frac{1}{12} = \frac{16\frac{2}{3}}{56\frac{2}{3}}$	$16\frac{2}{3} = \frac{16\frac{4}{3}}{50\frac{2}{3}}$
Ans.	Ans.	Ans.
(5.)	(6.)	(7.)
$29\frac{1}{9} = 29\frac{1\frac{2}{3}}{1\frac{2}{3}}$	$144\frac{3}{4} = 144\frac{3}{2}$	$171\frac{1}{2} = 171\frac{2}{2}$
$15\frac{1}{8} = \frac{15\frac{1}{4}}{1\frac{1}{4}}$	$99\frac{1}{3} = \frac{99\frac{2}{3}}{44\frac{2}{3}}$	$91\frac{9}{10} = \frac{91\frac{18}{10}}{79\frac{3}{5}}$
Ans.	Ans.	Ans.
(8.)	(9.)	(10.)
$101\frac{1}{8} = 101\frac{5}{8}$	$165\frac{1}{2} = 165\frac{2}{2}$	$77\frac{1}{2} = 77\frac{2}{2}$
$93\frac{1}{8} = \frac{93\frac{2}{4}}{8\frac{2}{4}}$	$98\frac{1}{8} = \frac{98\frac{1}{4}}{66\frac{1}{4}}$	$19\frac{1}{8} = \frac{19\frac{2}{4}}{57\frac{2}{4}}$
Ans.	Ans.	Ans.

(11.)	(12.)	(13.)
$84\frac{2}{3} = 84\frac{2}{3}$	$671\frac{1}{2} = 671\frac{1}{2}$	$17\frac{1}{2} = 17\frac{1}{2}$
$15\frac{1}{2} = 15\frac{1}{2}$	$183\frac{1}{2} = 183\frac{1}{2}$	$8\frac{1}{2} = 8\frac{1}{2}$
$69\frac{1}{2}$, Ans.	$487\frac{1}{2}$, Ans.	Ans. $8\frac{1}{2}$

14. $63 - 7\frac{3}{4} = 55\frac{1}{4}$ gal., Ans.
 15. $30 - 5\frac{1}{2} = 24\frac{1}{2}$ days, Ans.
 16. $17\frac{5}{8} + 28\frac{7}{8} = 46\frac{1}{2}$; $144 - 46\frac{1}{2} = 97\frac{1}{2}$ lb, Ans.
 17. $34 - 9\frac{7}{8} = 24\frac{1}{8}$ yd., Ans.
 18. $40 \times \frac{2}{7} = 11\frac{4}{7}$; $40 \times \frac{3}{7} = 17\frac{4}{7}$; $11\frac{4}{7} + 17\frac{4}{7} = 28\frac{8}{7}$ miles, Ans.
 19. $\frac{1}{4}$ of a square yard $= \frac{1}{4}$ of 9 feet $= 2\frac{1}{4}$ square feet. 1 yard $= 3$ feet; $\frac{1}{4}$ of 3 feet $= 6$ inches. 6 inches squared $= 36$ square inches $= \frac{1}{4}$ of a square foot. $2\frac{1}{4}$ feet $- \frac{1}{4}$ foot $= 2$ square feet, Ans.

MISCELLANEOUS EXAMPLES IN ADDITION AND SUBTRACTION OF FRACTIONS.

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1. $\frac{3}{4}, \frac{1}{2}, \frac{1}{8} = \frac{6}{8}, \frac{4}{8}, \frac{1}{8}$; $6 + 4 + 1 = 11$; $11 \div 8 = 1\frac{3}{8}$, Ans.
 2. $\frac{5}{8} + \frac{1}{2} + \frac{1}{4} + \frac{5}{8} = \frac{5}{8} + \frac{4}{8} + \frac{2}{8} + \frac{5}{8} = \frac{16}{8} = 2$, Ans.
 3. $20\frac{7}{10}, 19\frac{1}{2}, 22\frac{1}{10} = 20\frac{7}{10}, 19\frac{5}{10}, 22\frac{1}{10}$; $20\frac{7}{10} + 19\frac{5}{10} + 22\frac{1}{10} = 62\frac{3}{10}$ miles, Ans.
 4. $6\frac{1}{2}, 19\frac{1}{2}, 3\frac{1}{2} = 6\frac{1}{2}, 19\frac{1}{2}, 3\frac{1}{2}$; $6\frac{1}{2} + 19\frac{1}{2} + 3\frac{1}{2} = 29$ tons, Ans.
 5. $\frac{1}{5}, \frac{1}{5} = \frac{2}{5}$; $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$; $1 - \frac{4}{5} = \frac{1}{5}$; $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$, Ans.
 6. $1 - \frac{2}{3} = \frac{1}{3}$; $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$; $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$, Ans.
 7. $11\frac{5}{8}, 9\frac{1}{8} = 11\frac{5}{8}, 9\frac{1}{8}$; $11\frac{5}{8} + 9\frac{1}{8} = 21\frac{6}{8}$; $31\frac{1}{2}, 21\frac{1}{2} = 31\frac{1}{2}, 21\frac{1}{2}$; $31\frac{1}{2} - 21\frac{1}{2} = 10$ yards, Ans.

8. $4\frac{3}{4}$, 11, $26\frac{1}{2} = 4\frac{6}{14}$, 11, $26\frac{7}{14}$; $4\frac{6}{14} + 11 + 26\frac{7}{14} = 41\frac{13}{14}$;
 $84\frac{3}{8} - 41\frac{13}{14} = 42\frac{23}{56}$, $+ 3\frac{3}{4} = 46\frac{11}{56}$; $46\frac{11}{56} - 13\frac{8}{11} =$
 $46\frac{121}{616} - 13\frac{448}{616} = 32\frac{73}{616}$; $32\frac{73}{616} + 3\frac{1}{2} = 32\frac{73}{308} +$
 $3\frac{99}{308} = 35\frac{172}{308}$ gallons, Ans.
9. $19\frac{3}{4}$, $36\frac{1}{2}$, $33\frac{3}{8} = 19\frac{6}{8}$, $36\frac{4}{8}$, $33\frac{3}{8} = 19\frac{6}{8} + 36\frac{4}{8} +$
 $33\frac{3}{8} = 89\frac{11}{8}$; $89\frac{11}{8} - 71\frac{3}{8} = 89\frac{11}{8} - 71\frac{3}{8} = 18\frac{8}{8}$
yards, Ans.

MULTIPLICATION OF COMMON FRACTIONS.

(ART. 235, p. 171.)

5. $12 \times \frac{5}{7} = \frac{12}{1} \times \frac{5}{7} = \frac{60}{7} = 8\frac{4}{7}$, Ans.
6. $\frac{2}{7} \times 12 = \frac{2}{7} \times \frac{12}{1} = \frac{24}{7} = 3\frac{3}{7}$, Ans.
7. $\frac{14}{17} \times \frac{17}{42} = \frac{\cancel{14}}{\cancel{17}} \times \frac{\cancel{17}}{\cancel{42}} = \frac{1}{3}$, Ans.
8. $\frac{3}{7} \times \frac{19}{20} = \frac{57}{140}$, Ans.
9. $\frac{4}{5} \times \frac{6}{11} = \frac{24}{55}$, Ans.
10. $\frac{7}{12} \times \frac{11}{16} = \frac{77}{192}$, Ans.
11. $\frac{8}{11} \times \frac{4}{7} = \frac{32}{77}$, Ans.
12. $\frac{3}{4} \times \frac{15}{16} = \frac{45}{64}$, Ans.
13. $\frac{16}{17} \times \frac{11}{12} = \frac{\cancel{16}}{\cancel{17}} \times \frac{11}{\cancel{12}} = \frac{44}{51}$, Ans.

$$14. \frac{9}{10} \times \frac{14}{1} = \frac{9}{\cancel{10}^5} \times \frac{\cancel{14}^7}{1} = \frac{63}{5} = 12\frac{3}{5}, \text{ Ans.}$$

$$15. 13 \times \frac{4}{7} = \frac{13}{1} \times \frac{4}{7} = \frac{52}{7} = 7\frac{3}{7}, \text{ Ans.}$$

$$16. 16 \times \frac{3}{19} = \frac{16}{1} \times \frac{3}{19} = \frac{48}{19} = 2\frac{10}{19}, \text{ Ans.}$$

$$17. 11 \times \frac{4}{7} = \frac{11}{1} \times \frac{4}{7} = \frac{44}{7} = 6\frac{2}{7}, \text{ Ans.}$$

$$18. \frac{9}{10} \times \frac{14}{1} = \frac{9}{\cancel{10}^5} \times \frac{\cancel{14}^7}{1} = \frac{63}{5} = 12\frac{3}{5}, \text{ Ans.}$$

$$19. \frac{8}{9} \times \frac{19}{1} = \frac{152}{9} = 16\frac{8}{9}, \text{ Ans.}$$

$$20. \frac{9}{11} \times \frac{22}{27} = \frac{\cancel{9}^1}{\cancel{11}^1} \times \frac{\cancel{22}^2}{\cancel{27}^3} = \frac{2}{3}, \text{ Ans.}$$

$$21. \frac{4}{9} \times \frac{18}{19} = \frac{\cancel{4}^2}{\cancel{9}^1} \times \frac{\cancel{18}^2}{19} = \frac{8}{19}, \text{ Ans.}$$

$$22. \frac{3}{19} \times \frac{38}{39} = \frac{\cancel{3}^1}{\cancel{19}^1} \times \frac{\cancel{38}^2}{\cancel{39}^{13}} = \frac{2}{13}, \text{ Ans.}$$

$$23. \frac{7}{9} \times \frac{9}{11} = \frac{7}{\cancel{9}} \times \frac{\cancel{9}}{11} = \frac{7}{11}, \text{ Ans.}$$

$$24. \frac{1}{99} \times \frac{99}{100} = \frac{1}{\cancel{99}} \times \frac{\cancel{99}}{100} = \frac{1}{100}, \text{ Ans.}$$

$$25. \frac{3}{7} \times \frac{7}{11} \times \frac{11}{24} + 100 = \frac{1}{7} \times \frac{7}{11} \times \frac{11}{24} \times \frac{100}{1} = \frac{100}{8} = 12\frac{1}{2},$$

[Ans.]

$$26. \frac{1}{2} \times \frac{2}{3} \times \frac{7}{8} \times \frac{11}{1} = \frac{1}{2} \times \frac{2}{3} \times \frac{7}{8} \times \frac{11}{1} = \frac{77}{24} = 3\frac{5}{24}, \text{ Ans.}$$

$$27. \frac{7}{12} \times \frac{17}{1} = \frac{119}{12} = \$9\frac{1}{12}, \text{ Ans.}$$

$$28. \frac{9}{20} \times \frac{37}{1} = \frac{333}{20} = \$16\frac{13}{20}, \text{ Ans.}$$

$$29. \frac{5}{8} \times \frac{8}{1} \times \frac{7}{1} = \$35, \text{ Ans.}$$

$$30. 161\frac{1}{15} \times 19\frac{1}{15} = \frac{2426}{15} \times \frac{543}{28} = \frac{1317318}{420} = 3136\frac{3}{7}, \text{ Ans.}$$

$$31. \frac{3}{7} \times 8\frac{3}{4} = \frac{3}{7} \times \frac{35}{4} = \frac{105}{28} = 3\frac{3}{4}, \text{ Ans.}$$

$$32. \frac{9}{10} \times 17\frac{3}{11} = \frac{9}{10} \times \frac{190}{11} = \frac{1710}{110} = 15\frac{6}{11}, \text{ Ans.}$$

$$33. \frac{8}{9} \times 71\frac{4}{5} = \frac{8}{9} \times \frac{359}{5} = \frac{2872}{45} = 63\frac{32}{45}, \text{ Ans.}$$

$$34. \frac{3}{4} \times 9\frac{1}{4} \times \frac{2}{3} \times 17 = \frac{3}{4} \times \frac{37}{4} \times \frac{2}{3} \times \frac{17}{1} = \frac{629}{8} = 78\frac{5}{8},$$

[Ans.]

$$35. \frac{9}{10} \times 7 \times \frac{11}{15} \times 87\frac{3}{11} = \frac{9}{10} \times 7 \times \frac{11}{15} \times \frac{960}{11} = \frac{2016}{5} =$$

[403\frac{3}{5}, Ans.]

$$36. 8 \times \frac{7}{9} = \frac{8}{1} \times \frac{7}{9} = \frac{56}{9} = 6\frac{2}{9}, \text{ Ans.}$$

$$37. 12 \times \frac{5}{7} = \frac{12}{1} \times \frac{5}{7} = \frac{60}{7} = 8\frac{4}{7}, \text{ Ans.}$$

$$38. 15 \times \frac{6}{11} = \frac{15}{1} \times \frac{6}{11} = \frac{90}{11} = 8\frac{2}{11}, \text{ Ans.}$$

$$39. \frac{7}{8} \times \frac{4}{11} = \frac{7}{22}, \text{ Ans.}$$

$$40. 3\frac{4}{5} \times 10\frac{4}{7} = \frac{34}{9} \times \frac{74}{7} = \frac{2516}{63} = 39\frac{28}{63}, \text{ Ans.}$$

$$41. \frac{2}{3} \times 7\frac{1}{4} \times \frac{7}{8} \times 11\frac{3}{4} = \frac{2}{3} \times \frac{29}{4} \times \frac{7}{8} \times \frac{47}{4} = \frac{9541}{192} = 49\frac{133}{192},$$

[Ans.]

$$42. \frac{2}{7} \times 9 \times \frac{3}{5} \times 17 = \frac{2}{7} \times \frac{9}{1} \times \frac{3}{5} \times \frac{17}{1} = \frac{918}{35} = 26\frac{8}{35}, \text{ Ans.}$$

$$43. \frac{4}{7} \times 8\frac{3}{10} \times \frac{4}{7} \times 9\frac{1}{4} = \frac{4}{7} \times \frac{83}{10} \times \frac{4}{7} \times \frac{37}{4} = \frac{12284}{490} = 25\frac{17}{49},$$

[Ans.]

(ART. 236, p. 172.)

(3.)

$$\begin{array}{r} 8\frac{2}{5} \\ 7 \\ \hline \frac{2}{5} \times 7 = 4\frac{1}{5} \\ 7 \times 8 = 56 \\ \hline 60\frac{1}{5} \end{array}$$

(4.)

$$\begin{array}{r} 3\frac{1}{2} \\ 17 \\ \hline \frac{1}{2} \times 17 = 2\frac{1}{2} \\ 3 \times 17 = 41 \\ \hline 43\frac{1}{2} \end{array}$$

(5.)

$$\begin{array}{r} 8\frac{2}{7} \\ 13 \\ \hline \frac{2}{7} \times 13 = 5\frac{4}{7} \\ 8 \times 13 = 104 \\ \hline 109\frac{4}{7} \end{array}$$

(6.)

$$\begin{array}{r} 13\frac{2}{11} \\ 37 \\ \hline \frac{2}{11} \times 37 = 26\frac{4}{11} \\ 37 \times 13 = 481 \\ \hline 507\frac{4}{11} \\ 6* \end{array}$$

(7.)

$$\begin{array}{r} 11\frac{2}{7} \\ 8 \\ \hline \frac{2}{7} \times 8 = 6\frac{4}{7} \\ 8 \times 11 = 88 \\ \hline 94\frac{4}{7} \end{array}$$

(8.)

$$\begin{array}{r} 7\frac{2}{11} \\ 5 \\ \hline \frac{2}{11} \times 5 = 2\frac{4}{11} \\ 5 \times 7 = 35 \\ \hline \$0.37\frac{4}{11} \end{array}$$

(9.)

$$\begin{array}{r}
 23\frac{7}{12} \\
 6 \\
 \hline
 7\frac{7}{12} \times 6 = 31\frac{1}{2} \\
 6 \times 23 = 138 \\
 \hline
 \$141\frac{1}{2}
 \end{array}$$

(10.)

$$\begin{array}{r}
 8\frac{3}{5} \\
 5 \\
 \hline
 \frac{3}{5} \times 5 = 1\frac{3}{5} \\
 8 \times 5 = 40 \\
 \hline
 \$41\frac{3}{5}
 \end{array}$$

(11.)

$$\begin{array}{r}
 6\frac{3}{9} \\
 9 \\
 \hline
 \frac{3}{9} \times 9 = 3\frac{3}{9} \\
 9 \times 6 = 54 \\
 \hline
 \$57\frac{3}{9}
 \end{array}$$

(12.)

$$\begin{array}{r}
 \$6.37\frac{1}{2} \\
 12 \\
 \hline
 \frac{1}{2} \times 12 = 6 \\
 12 \times 6.37 = 76.44 \\
 \hline
 \$76.50
 \end{array}$$

(13.)

$$\begin{array}{r}
 9\frac{3}{11} \\
 11 \\
 \hline
 \frac{3}{11} \times 11 = 4\frac{3}{11} \\
 11 \times 9 = 99 \\
 \hline
 \$103\frac{3}{11}
 \end{array}$$

(14.)

$$\begin{array}{r}
 4\frac{3}{1.75} \\
 1.75 \\
 \hline
 \frac{3}{1.75} \times 1.75 = 65\frac{3}{1.75} \\
 1.75 \times 4 = 700 \\
 \hline
 \$7.65\frac{3}{1.75}
 \end{array}$$

(15.)

$$\begin{array}{r}
 11\frac{7}{7} \\
 7 \\
 \hline
 \frac{7}{7} \times 7 = 6\frac{1}{7} \\
 7 \times 11 = 77 \\
 \hline
 \$88\frac{1}{7}
 \end{array}$$

(16.)

$$\begin{array}{r}
 \$10\frac{9}{9} \\
 9 \\
 \hline
 \frac{9}{9} \times 9 = 5\frac{9}{9} \\
 9 \times 10 = 90 \\
 \hline
 \$95\frac{9}{9}
 \end{array}$$

(17.)

$$\begin{array}{r}
 \$3\frac{5}{5} \\
 5 \\
 \hline
 \frac{5}{5} \times 5 = 0\frac{5}{5} \\
 5 \times 3 = 15 \\
 \hline
 \$15\frac{5}{5}
 \end{array}$$

(18.)

$$\begin{array}{r}
 \$7.62\frac{1}{15} \\
 15 \\
 \hline
 \frac{1}{15} \times 15 = 7\frac{1}{15} \\
 15 \times 7.62 = 114.30 \\
 \hline
 \$114.37\frac{1}{15}
 \end{array}$$

(19.)

$$\begin{array}{r}
 \$8.37\frac{1}{40} \\
 40 \\
 \hline
 \frac{1}{40} \times 40 = 20 \\
 40 \times 8.37 = 334.80 \\
 \hline
 \$335.00
 \end{array}$$

DIVISION OF COMMON FRACTIONS.

(ART. 239, p. 175.)

$$6 \frac{7}{11} \div 18 = \frac{7}{11} \times \frac{1}{18} = \frac{7}{198}, \text{ Ans.}$$

$$7. \frac{4}{9} \div \frac{7}{8} = \frac{4}{9} \times \frac{8}{7} = \frac{32}{63}, \text{ Ans.}$$

$$8. 18 \div \frac{7}{11} = \frac{18}{1} \times \frac{11}{7} = \frac{198}{7} = 28\frac{2}{7}, \text{ Ans.}$$

$$9. \frac{5}{27} \div \frac{5}{3} = \frac{5}{27} \times \frac{3}{5} = \frac{15}{135} = \frac{1}{9}, \text{ Ans.}$$

$$10. \frac{16}{21} \div \frac{2}{3} = \frac{8}{7} = 1\frac{1}{7}, \text{ Ans.}$$

$$11. \frac{15}{22} \div 28 = \frac{15}{22} \times \frac{1}{28} = \frac{15}{616}, \text{ Ans.}$$

$$12. \frac{1}{17} \div 27 = \frac{1}{17} \times \frac{1}{27} = \frac{1}{459}, \text{ Ans.}$$

$$13. \frac{2}{15} \div 128 = \frac{\overset{1}{\cancel{2}}}{15} \times \frac{1}{\underset{64}{\cancel{128}}} = \frac{1}{960}, \text{ Ans.}$$

$$14. \frac{11}{17} \div 98 = \frac{11}{17} \times \frac{1}{98} = \frac{11}{1666}, \text{ Ans.}$$

$$15. \frac{14}{23} \div 19 = \frac{14}{23} \times \frac{1}{19} = \frac{14}{437}, \text{ Ans.}$$

$$16. \frac{5}{6} \div 167 = \frac{5}{6} \times \frac{1}{167} = \frac{5}{1002}, \text{ Ans.}$$

$$17. \frac{16}{29} \div 49 = \frac{16}{29} \times \frac{1}{49} = \frac{16}{1421}, \text{ Ans.}$$

$$18. \frac{1}{15} \div 15 = \frac{1}{15} \times \frac{1}{15} = \frac{1}{225}, \text{ Ans.}$$

$$19. 27 \div \frac{1}{17} = \frac{27}{1} \times \frac{17}{1} = 459, \text{ Ans.}$$

$$20. 128 \div \frac{2}{15} = \frac{\overset{64}{\cancel{128}}}{1} \times \frac{15}{\underset{1}{\cancel{2}}} = 960, \text{ Ans.}$$

$$21. 98 \div \frac{11}{17} = \frac{98}{1} \times \frac{17}{11} = \frac{1666}{11} = 151\frac{5}{11}, \text{ Ans.}$$

$$22. 19 \div \frac{14}{28} = \frac{19}{1} \times \frac{28}{14} = \frac{476}{14} = 34, \text{ Ans.}$$

$$23. 167 \div \frac{15}{18} = \frac{167}{1} \times \frac{18}{15} = \frac{3006}{15} = 200\frac{2}{3}, \text{ Ans.}$$

$$24. 49 \div \frac{16}{29} = \frac{49}{1} \times \frac{29}{16} = \frac{1421}{16} = 88\frac{13}{16}, \text{ Ans.}$$

$$25. 15 \div \frac{1}{15} = \frac{15}{1} \times \frac{15}{1} = 225, \text{ Ans.}$$

$$26. \frac{36}{51} \div \frac{3}{17} = \frac{12}{3} = 4, \text{ Ans.}$$

$$27. \frac{31}{97} \div \frac{31}{301} = \frac{31}{97} \times \frac{301}{31} = \frac{301}{97} = 3\frac{1}{97}, \text{ Ans.}$$

$$28. \frac{5}{6} \div \frac{8}{9} = \frac{5}{6} \times \frac{9}{8} = \frac{45}{48} = \frac{15}{16}, \text{ Ans.}$$

$$29. \frac{11}{15} \div \frac{7}{11} = \frac{11}{15} \times \frac{11}{7} = \frac{121}{105} = 1\frac{16}{105}, \text{ Ans.}$$

$$30. \frac{8}{25} \div \frac{14}{17} = \frac{8}{25} \times \frac{17}{14} = \frac{68}{175}, \text{ Ans.}$$

$$31. \frac{16}{21} \div \frac{1}{15} = \frac{16}{21} \times \frac{15}{1} = \frac{240}{21} = 11\frac{4}{7}, \text{ Ans.}$$

$$32. \frac{9}{25} \div 7\frac{2}{3} = \frac{9}{25} \times \frac{4}{31} = \frac{36}{775}, \text{ Ans.}$$

$$33. \frac{8}{11} \div \frac{164}{9} = \frac{8}{11} \times \frac{9}{148} = \frac{72}{1628} = \frac{18}{407}, \text{ Ans.}$$

$$34. 11\frac{2}{3} \div \frac{4}{7} = \frac{80}{3} \times \frac{7}{4} = 20, \text{ Ans.}$$

$$35. 21\frac{4}{5} \div 18\frac{4}{5} = \frac{109}{5} \times \frac{7}{130} = \frac{763}{650} = 1\frac{113}{650}, \text{ Ans.}$$

$$36. 17\frac{3}{11} \div 28\frac{1}{11} = \frac{190}{11} \times \frac{26}{739} = \frac{4940}{8129}, \text{ Ans.}$$

$$37. 161\frac{3}{7} \div 14\frac{2}{5} = \frac{2740}{17} \times \frac{5}{73} = \frac{13700}{1241} = 11\frac{42}{1241}, \text{ Ans.}$$

$$38. \frac{7}{11} \times \frac{4}{5} = \frac{28}{55}; \frac{3}{5} \times \frac{8}{11} = \frac{24}{55}; \frac{28}{55} \times \frac{55}{24} = \frac{7}{6} = 1\frac{1}{6}, \text{ Ans.}$$

$$39. \frac{5}{9} \times 7\frac{2}{11} = \frac{400}{99}; \frac{4}{11} \times 17\frac{2}{11} = \frac{488}{77}; \frac{400}{99} \times \frac{77}{488} = \frac{700}{1098}$$

$$[= \frac{350}{549}, \text{ Ans.}]$$

$$40. \frac{6}{17} \times 15 = \frac{90}{17}; \frac{7}{15} \times 22 = \frac{154}{15}; \frac{90}{17} \times \frac{15}{154} = \frac{1350}{2618} =$$

$$[= \frac{675}{1309}, \text{ Ans.}]$$

$$41. \frac{7}{7} - \frac{5}{7} = \frac{2}{7}; \frac{2}{3} \times \frac{2}{7} = \frac{4}{21}; \frac{2}{7} - \frac{4}{21} = \frac{2}{21}; \frac{2}{21} \times 3675 =$$

$$[350; \div 7 = \$50, \text{ Ans.}]$$

$$42. 1 - \frac{1}{3} = \frac{2}{3}; \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}; \frac{1}{3} + \frac{4}{9} = \frac{7}{9}; 1 - \frac{7}{9} = \frac{2}{9}; \frac{2}{9} \times$$

$$[\frac{1}{7} = \frac{2}{63}; \frac{2}{63} \times 5000 = \$158\frac{4}{9}, \text{ Ans.}]$$

(ART. 240, p. 176.)

$$2. \begin{array}{r} 9 \overline{)293} \\ 3, \text{ rem. } 23; 23 = 19; 19 \times 3 = 57; 3 + 19 = 22, \text{ Ans.} \end{array}$$

$$3. \begin{array}{r} 7 \overline{)14\frac{1}{2}} \\ 2, \text{ rem. } \frac{1}{2}; \frac{1}{2} \times 7 = 1\frac{1}{2}; 2 + 1\frac{1}{2} = 3\frac{1}{2}, \text{ Ans.} \end{array}$$

$$4. \begin{array}{r} 8 \overline{)133} \\ 1, \text{ rem. } 53; 53 = 43; 43 \times 8 = 344; 1 + 43 = 44, \text{ Ans.} \end{array}$$

5. $\begin{array}{r} 6 \overline{)14\frac{2}{3}} \\ 2, \text{ rem. } 2\frac{2}{3}; 2\frac{2}{3} = \frac{1^2}{3}; \frac{1^2}{3} \times 6 = 1\frac{2}{3}; 2 + 1\frac{2}{3} = 2\frac{2}{3}, \text{ Ans.} \end{array}$
6. $\begin{array}{r} 9 \overline{)37\frac{2}{3}} \\ 4, \text{ rem. } 1\frac{2}{3}; 1\frac{2}{3} = \frac{1^2}{3}; \frac{1^2}{3} \times 9 = 1\frac{2}{3}; 4 + 1\frac{2}{3} = 4\frac{2}{3}, \text{ Ans.} \end{array}$
7. $\begin{array}{r} 11 \overline{)96\frac{2}{3}} \\ 8, \text{ rem. } 8\frac{2}{3}; 8\frac{2}{3} = \frac{2^2}{3}; \frac{2^2}{3} \times 11 = 3\frac{2}{3}; 8 + 3\frac{2}{3} = 8\frac{2}{3}, \\ \text{[Ans.} \end{array}$
8. $\begin{array}{r} 8 \overline{)167\frac{7}{11}} \\ 20, \text{ rem. } 7\frac{7}{11}; 7\frac{7}{11} = \frac{9}{11}; \frac{9}{11} \times 8 = \frac{8}{11} = \frac{2}{11}; 20 + \frac{2}{11} \\ \text{[} = 20\frac{2}{11} \text{ cwt., Ans.} \end{array}$
9. $\frac{7}{8} \times \frac{1722}{1} = \frac{12054}{8} = 1507\frac{3}{8}; \begin{array}{r} 12 \overline{)1507\frac{3}{8}} \\ 125, \text{ rem. } 7\frac{3}{8}; 7\frac{3}{8} = \frac{61}{8}; \\ \text{[} \frac{61}{8} \times 12 = \frac{91}{4} = 22\frac{3}{4}; \$125 + \frac{91}{4} = \$125\frac{3}{4}, \text{ Ans.} \end{array}$
10. $\begin{array}{r} 19 \overline{)107\frac{3}{8}} \\ 5, \text{ rem. } 12\frac{3}{8}; 12\frac{3}{8} = \frac{99}{8}; \frac{99}{8} \times 19 = \frac{1881}{8}; 5 + \frac{1881}{8} = \\ 5\frac{1881}{8}; \frac{1881}{8} \times 100 = \frac{235125}{4} = .65\frac{5}{8}; \$5 + .65\frac{5}{8} = \\ \text{[} \$5.65\frac{5}{8}, \text{ Ans.} \end{array}$
11. $\frac{2}{7}$ of \$1 = $\frac{2}{7} \times 100 = 29\frac{1}{7}$; $29\frac{1}{7} = .28\frac{1}{7}$; $\$3 + .28\frac{1}{7} = 3.28\frac{1}{7}$;
 $\begin{array}{r} 15 \overline{)3.28\frac{1}{7}} \\ \$0.21, \text{ rem. } 13\frac{1}{7}; 13\frac{1}{7} = \frac{95}{7}; \frac{95}{7} \times 15 = \frac{1425}{7} = 203\frac{4}{7} = 203\frac{1}{2}; \\ \text{[} \$0.21 + \frac{1}{2} = \$0.21\frac{1}{2}, \text{ Ans.} \end{array}$
12. $\begin{array}{r} 12 \overline{)\$3.75\frac{3}{4}} \\ \$.31, \text{ rem. } 3\frac{3}{4}; 3\frac{3}{4} = \frac{15}{4}; \frac{15}{4} \times 12 = 1\frac{1}{2} = \frac{1}{2}; \\ \text{[} \$0.31 + \frac{1}{2} = \$0.31\frac{1}{2}, \text{ Ans.} \end{array}$
13. $\begin{array}{r} 19 \overline{)375\frac{1}{16}} \\ 19, \text{ rem. } 14\frac{1}{16}; 14\frac{1}{16} = \frac{225}{16}; \frac{225}{16} \times 19 = \frac{4275}{16} = 267\frac{3}{16}; \\ \text{[} 19 + 267\frac{3}{16} = 286\frac{3}{16} \text{ acres, Ans.} \end{array}$

REDUCTION OF COMPLEX FRACTIONS.

(ART. 242, p. 178.)

$$4. \frac{4\frac{2}{3}}{\frac{2}{3}} = \frac{3\frac{1}{3}}{\frac{2}{3}} = 3\frac{1}{3} \times \frac{3}{2} = 5\frac{1}{2} = 6\frac{1}{2}, \text{ Ans.}$$

$$5. \frac{2}{5\frac{2}{3}} = \frac{2}{\frac{17}{3}} = 2 \times \frac{3}{17} = \frac{6}{17}, \text{ Ans.}$$

$$6. \frac{7}{4\frac{2}{3}} = \frac{7}{\frac{14}{3}} = 7 \times \frac{3}{14} = \frac{21}{14} = 1\frac{1}{2}, \text{ Ans.}$$

$$7. \frac{7\frac{4}{8}}{8} = \frac{\frac{58}{8}}{8} = \frac{58}{64} = \frac{29}{32}, \text{ Ans.}$$

$$8. \frac{6\frac{2}{3}}{8\frac{2}{3}} = \frac{\frac{20}{3}}{\frac{26}{3}} = \frac{20}{26} \times \frac{3}{3} = \frac{10}{13} = \frac{20}{26}, \text{ Ans.}$$

$$9. \frac{2}{\frac{3}{5}} = 2 \times \frac{5}{3} = \frac{10}{3}, \text{ Ans.}$$

$$10. \frac{8}{\frac{1}{3}} = \frac{8}{\frac{1}{3}} = 8 \times \frac{3}{1} = 24, \text{ Ans.}$$

$$11. \frac{\frac{4}{2}}{\frac{1}{2}} = \frac{\frac{4}{2}}{\frac{1}{2}} = \frac{4}{2} \times \frac{2}{1} = \frac{4}{1} = 4, \text{ Ans.}$$

$$12. \frac{5\frac{1}{3}}{\frac{3}{7}} = \frac{\frac{16}{3}}{\frac{3}{7}} = \frac{16}{3} \times \frac{7}{3} = \frac{112}{9} = 12\frac{4}{9}, \text{ Ans.}$$

$$13. \frac{\frac{1}{4}}{6\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{13}{2}} = \frac{1}{4} \times \frac{2}{13} = \frac{2}{52} = \frac{1}{26}, \text{ Ans.}$$

$$14. \frac{3}{2\frac{1}{2}} = \frac{3}{\frac{5}{2}} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}, \text{ Ans.}$$

$$15. \frac{3\frac{1}{4}}{9} = \frac{\frac{13}{4}}{9} = \frac{13}{4} \times \frac{1}{9} = \frac{13}{36}, \text{ Ans.}$$

$$16. \frac{11\frac{2}{3}}{12\frac{5}{6}} = \frac{\frac{35}{6}}{\frac{25}{6}} = \frac{35}{25} \times \frac{5}{63} = \frac{175}{1575} = \frac{25}{207}, \text{ Ans.}$$

$$17. \frac{\frac{77}{11\frac{2}{3}}}{7} = \frac{\frac{70}{5\frac{2}{3}}}{7} = \frac{70}{9} \times \frac{5}{59} \times \frac{1}{7} = \frac{350}{3717} = \frac{50}{531}, \text{ Ans.}$$

$$18. \frac{\frac{7}{\frac{3}{5}}}{\frac{1}{\frac{1}{2}}} = \frac{7}{9} \times \frac{5}{3} \times \frac{5}{1} = \frac{175}{27} = 6\frac{13}{27}, \text{ Ans.}$$

(ART. 243, p. 179.)

$$1. \quad 28\frac{39}{47} = 28\frac{27}{37}; \frac{27}{37} = \frac{27}{7} \times \frac{9}{30} = \frac{9}{10}; 28 + \frac{9}{10} = 28\frac{9}{10} =$$

$$\frac{289}{10}; \frac{1}{8} \times \frac{2}{7} \times \frac{289}{10} = \frac{578}{210} = 2\frac{118}{105}; 3\frac{39\frac{1}{2}}{105} = 3\frac{79}{190};$$

$$\frac{79}{190} = \frac{79}{2} \times \frac{1}{105} = \frac{79}{210}; 3 + \frac{79}{210} = 3\frac{79}{210}; 2\frac{118}{105} +$$

$$[3\frac{79}{210} = 6\frac{9}{70}, \text{Ans.}]$$

$$2. \quad \frac{1}{9} = \frac{1}{9}; 2\frac{5}{8} = \frac{21}{8}; \frac{45}{94\frac{1}{11}} = \frac{45}{1041} = \frac{1}{1} \times \frac{11}{1041} = \frac{495}{1041};$$

$$\frac{47\frac{5}{8}}{314\frac{3}{8}} = \frac{428}{1573} = \frac{428}{9} \times \frac{5}{1573} = \frac{2140}{14157}; \frac{1}{9} + \frac{21}{8} +$$

$$\frac{495}{1041} + \frac{2140}{14157} \quad \begin{array}{r} 3) 8 \ 1041 \ 14157 \\ 8 \ 347 \ 4719 \end{array}$$

$$3 \times 8 \times 347 \times 4719 = 39299832.$$

$$9 \overline{) 39299832}$$

$$8 \overline{) 4366648} \times 1 = 4366648$$

$$1041 \overline{) 4912479} \times 21 = 103162059$$

$$14157 \overline{) 37752} \times 495 = 18687240$$

$$2776 \times 2140 = 5940640$$

$$\frac{132156587}{39299832} = 3\frac{11255331}{39299832}, \text{Ans.}$$

$$3. \quad \frac{49\frac{5}{8}}{97} = \frac{287}{97} = \frac{397}{8} \times \frac{1}{97} = \frac{397}{776}; \frac{34\frac{3}{8}}{145\frac{3}{11}} = \frac{173}{1598} = \frac{173}{5} \times$$

$$\frac{11}{1598} = \frac{1903}{7990}; \frac{397}{776} - \frac{1903}{7990} = \frac{847651}{8100120}, \text{Ans.}$$

$$\begin{aligned}
 4. \quad \frac{27}{37\frac{1}{2}} &= \frac{27}{\frac{75}{2}} = \frac{27}{1} \times \frac{2}{75} = \frac{2}{5}; \\
 \frac{87\frac{2}{3}}{98\frac{1}{2}} &= \frac{174}{197} = \frac{785}{9} \times \frac{8}{785} = \frac{8}{9}; \\
 \frac{7}{2\frac{1}{3}} &= \frac{7}{\frac{4}{3}} = \frac{7}{8} \times \frac{3}{7} = \frac{3}{8}; \\
 \frac{81\frac{5}{11}}{128} &= \frac{896}{128} = \frac{896}{11} \times \frac{1}{128} = \frac{7}{11}; \\
 \frac{5}{7} \times \frac{8}{9} \times \frac{3}{8} \times \frac{7}{11} &= \frac{5}{33}, \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{2}{3} \times 7\frac{3}{4} &= \frac{2}{3} \times \frac{31}{4} = \frac{62}{12} = \frac{31}{6}; \quad \frac{4}{5} \times 11\frac{1}{4} = \frac{4}{5} \times \frac{125}{11} = \\
 &= \frac{500}{55} = \frac{100}{11}; \quad \frac{31}{6} \times \frac{11}{100} = \frac{341}{600}, \text{ Ans.} \\
 6. \quad \frac{4}{9} \times \frac{91}{1} &= \frac{364}{9}; \quad \frac{9}{10} \times \frac{87}{1} = \frac{783}{10}; \quad \frac{364}{9} \times \frac{10}{783} = \frac{3640}{7047}, \\
 &[\text{Ans.}]
 \end{aligned}$$

MISCELLANEOUS EXAMPLES IN MULTIPLICATION AND DIVISION
OF FRACTIONS.

(PAGE 179.)

$$\begin{aligned}
 1. \quad 2\frac{1}{2} &= \frac{17}{7}; \quad 7\frac{1}{4} = \frac{123}{17}; \quad \frac{123}{17} \times \frac{17}{17} = \frac{123}{7} = 17\frac{1}{7}, \text{ Ans.} \\
 2. \quad 8\frac{1}{5} &= \frac{41}{5}; \quad 4.68\frac{1}{4} = \frac{3280}{7}; \quad \frac{3280}{7} \times \frac{5}{41} = \frac{16400}{287}; \quad \frac{16400}{287} \\
 &= \$0.57\frac{1}{4}, \text{ Ans.} \\
 3. \quad 96\frac{3}{8} &= \frac{771}{8}; \quad 1 - \frac{5}{8} = \frac{8}{8} - \frac{5}{8} = \frac{3}{8}; \quad \frac{771}{8} \times \frac{3}{8} = \frac{2313}{64}; \\
 - \quad \frac{2313}{64} \div 2 &= \frac{2313}{64} \times \frac{1}{2} = \frac{2313}{128} = 18\frac{3}{128} \text{ yd., Ans.}
 \end{aligned}$$

$$4. 18\frac{1}{2} \times \frac{3}{4} = \frac{55}{3} \times \frac{3}{4} = \frac{165}{12}; 6\frac{1}{2} \times \frac{2}{3} = \frac{13}{2} \times \frac{2}{3} = \frac{26}{6}; \frac{165}{12}$$

$$[\div \frac{26}{6} = \frac{165}{12} \times \frac{6}{26} = \frac{165}{52} = 3\frac{3}{52}, \text{ Ans.}]$$

$$5. 1\frac{1}{3} = 1 + \frac{1}{3}; \frac{1}{3} = \frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}; 1 + \frac{1}{6} = 1\frac{1}{6} = \frac{7}{6}; 1 \div$$

$$[\frac{7}{6} = \frac{1}{1} \times \frac{6}{7} = \frac{6}{7}, \text{ Ans.}]$$

$$6. 1 + \frac{11}{12} = \frac{12}{12} + \frac{11}{12} = \frac{23}{12}; 250 \times 12 = 3000; 3000 \div$$

$$[23 = \$130\frac{1}{3}, \text{ horse.}]$$

$$250 \times 11 = 2750; 2750 \div 23 = \$119\frac{1}{3}, \text{ chaise.}$$

$$71 \times 130\frac{1}{3} = 71 \times 233\frac{1}{3} = 233\frac{1}{3} = \$83\frac{1}{3}, \text{ harness.}$$

$$7. \$200 \times \frac{3}{4} = \$150; \$150 - \$144 = \$6; \$6 \times 4 =$$

$$[\$24.00, \text{ Ans.}]$$

$$8. \frac{1}{2} \times \frac{3}{4} \times 3\frac{1}{2} = \frac{1}{2} \times \frac{3}{4} \times 2\frac{5}{8} = 1\frac{5}{8} = \frac{13}{8}; \frac{13}{8} \times \frac{1}{2} = \frac{13}{16};$$

$$\frac{13}{16} \times \frac{1}{64} = \frac{225}{2048}; \frac{225}{2048} \div \frac{1}{2} = \frac{225}{1024} \times \frac{2}{1} = \frac{450}{1024} =$$

$$[\frac{225}{1024}, \text{ Ans.}]$$

$$9. 13\frac{7}{8} \times 25\frac{3}{4} = 11\frac{1}{8} \times 19\frac{3}{4} = 11\frac{433}{512}; 11\frac{433}{512} \div 2\frac{3}{8} = 11\frac{433}{32}$$

$$[\div 1\frac{3}{8}; 11\frac{433}{32} \times \frac{1}{13} = 17\frac{165}{416} = 137\frac{173}{416} \text{ bushels, Ans.}]$$

$$10. 3\frac{1}{2} \times 9\frac{7}{8} = \frac{7}{2} \times 7\frac{7}{8} = 5\frac{53}{16}; \frac{553}{16} \div \frac{553}{16} = \frac{553}{1} \times \frac{16}{553} =$$

$$[16 \text{ days, Ans.}]$$

$$11. \$0.90 \div 4 = \$0.22\frac{1}{2}; \text{ that is, he will receive } \frac{1}{2} \text{ more.}$$

$$12. 9\frac{3}{4} \times 16\frac{3}{4} = 4\frac{3}{4} \times 6\frac{7}{8} = 32\frac{3}{8} = 164\frac{3}{8} \text{ miles per day.}$$

$$164\frac{3}{8} \times 4 = 656\frac{3}{2} \text{ miles in 4 days.}$$

$$164\frac{3}{8} \times 10 = 1641\frac{1}{2} \text{ miles in 10 days.}$$

$$24\text{h.} - 16\frac{3}{4}\text{h.} = 7\frac{1}{4}\text{h.}; 7\frac{1}{4} \times 2 = 14\frac{1}{2} \text{ miles to deduct for drift in the night.}$$

$$164\frac{3}{8} - 14\frac{1}{2} = 149\frac{1}{8} \text{ miles gained per day.}$$

$149\frac{1}{2} \times 10 = 1496\frac{1}{2}$ miles gained in 10 days.

$656\frac{3}{4} \text{ m.} + 1496\frac{1}{2} \text{ m.} = 2153\frac{1}{10}$ miles' distance sailed in 14 days.

$2317\frac{1}{4} \text{ m.} - 2153\frac{1}{10} \text{ m.} = 164\frac{3}{20}$ miles remaining, which distance will be sailed the last day; therefore the boat will arrive at her port in 15 days, Ans.

A PROPOSED NUMERATOR OR DENOMINATOR.

(ART. 244, p. 181.)

3. $34 \div 17 = 2$; $\frac{17 \times 2}{20 \times 2} = \frac{34}{40}$, Ans.

4. $.3\bar{6} = \frac{27}{7}$; $9 \div 27 = \frac{1}{3}$; $\frac{27 \times \frac{1}{3}}{7 \times \frac{1}{3}} = \frac{9}{2\frac{1}{3}}$, Ans.

5. $5 \div 4 = 1\frac{1}{4}$; $\frac{4 \times 1\frac{1}{4}}{1 \times 1\frac{1}{4}} = \frac{5}{1\frac{1}{4}}$, Ans.

6. $12 \div 16 = \frac{3}{4}$; $\frac{15 \times \frac{3}{4}}{16 \times \frac{3}{4}} = \frac{11\frac{1}{4}}{12}$, Ans.

7. $15 \div 5 = 3$; $\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$, Ans.

8. $2 \div 3 = \frac{2}{3}$; $\frac{7 \times \frac{2}{3}}{3 \times \frac{2}{3}} = \frac{4\frac{2}{3}}{2}$, Ans.

9. $35 \div 20 = 1\frac{7}{4}$; $\frac{19 \times 1\frac{7}{4}}{20 \times 1\frac{7}{4}} = \frac{33\frac{1}{4}}{35}$, Ans.

10. $\frac{1}{3} = \frac{1}{3}$; $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$; $\frac{6}{20} = \frac{3}{10}$; $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$, Ans.

COMMON NUMERATOR.

(ART. 246, p. 182.)

2. 24, the least common multiple of the numerators, = common numerator.

$\frac{24}{8}$ of 9 = 27, new denominator.	$\frac{8}{8} = \frac{24}{27}$	} Ans.
$\frac{24}{4}$ of 7 = 42, " "	$\frac{4}{4} = \frac{24}{42}$	
$\frac{24}{3}$ of 4 = 32, " "	$\frac{3}{3} = \frac{24}{32}$	
$\frac{24}{6}$ of 7 = 28, " "	$\frac{6}{6} = \frac{24}{28}$	

3. $\frac{5}{8}, 2\frac{1}{2}, 1\frac{1}{7} = \frac{5}{8}, \frac{5}{2}, \frac{10}{7}.$

10, the least common multiple of the numerators, = common numerator.

$$\left. \begin{array}{l} \frac{10}{5} \text{ of } 8 = 16, \text{ new denominator.} \\ \frac{10}{5} \text{ of } 2 = 4, \quad \quad \quad \text{"} \\ \frac{10}{7} \text{ of } 7 = 7, \quad \quad \quad \text{"} \end{array} \right\} \begin{array}{l} \frac{5}{8} = \frac{10}{16} \\ \frac{5}{2} = \frac{10}{4} \\ \frac{10}{7} = \frac{10}{7} \end{array} \text{ Ans.}$$

4. $4\frac{4}{15}, 6\frac{2}{3}, 6\frac{3}{5} = \frac{64}{15}, \frac{32}{5}, \frac{20}{3}.$

320, the least common multiple of numerators, = common numerator.

$$\left. \begin{array}{l} \frac{320}{64} \text{ of } 15 = 75, \text{ new denominator.} \\ \frac{320}{32} \text{ of } 5 = 50, \quad \quad \quad \text{"} \\ \frac{320}{20} \text{ of } 3 = 48, \quad \quad \quad \text{"} \end{array} \right\} \begin{array}{l} \frac{64}{15} = \frac{320}{75} \\ \frac{32}{5} = \frac{320}{50} \\ \frac{20}{3} = \frac{320}{48} \end{array} \text{ Ans.}$$

From the above process, it is evident they will meet in 320 days. A will have gone round the island 75 times, and have travelled $75 \times 50 = 3750$ miles. B will have gone round the island 50 times, and have travelled $50 \times 50 = 2500$ miles. C will have gone round the island 48 times, and have travelled $48 \times 50 = 2400$ miles.

GREATEST COMMON DIVISOR OF FRACTIONS.

(ART. 248, p. 183.)

2. Greatest common divisor of the numerators = $\frac{4}{1}$ } Ans.
Least common denominator of the fractions = $\frac{315}{1}$

3. $12\frac{3}{8}, 9\frac{3}{4}, 8\frac{1}{4} = \frac{99}{8}, \frac{39}{4}, \frac{33}{4}.$
Greatest common divisor of the numerators = $\frac{3}{1}$ } Ans.
Least common denominator of the fractions = $\frac{8}{1}$

NOTE. — $\frac{3}{8}$ will divide each of the fractions, $12\frac{3}{8}$, $9\frac{3}{4}$, and $8\frac{1}{4}$, without a remainder.

4. Greatest common divisor of the numerators = $\frac{1}{1}$ } Ans.
Least common denominator of the fractions = $\frac{60}{1}$

5. $3\frac{4}{5}, 5\frac{7}{10}, 2\frac{8}{15} = 1\frac{2}{5}, \frac{57}{10}, \frac{38}{15}$.
 Greatest common divisor of the numerators = 19 }
 Least common denominator of the fractions = 30 } Ans.
6. $33\frac{3}{4}, 67\frac{1}{2}, 70\frac{7}{8} = 1\frac{35}{4}, 1\frac{35}{2}, 8\frac{57}{8}$.
 Greatest common divisor of the numerators = 27 }
 Least common denominator of the fractions = $\frac{27}{8} = 3\frac{3}{8}$ } Ans.
 $33\frac{3}{4} \div 3\frac{3}{8} = 10$ bags; $67\frac{1}{2} \div 3\frac{3}{8} = 20$ bags; $70\frac{7}{8} \div 3\frac{3}{8} = 21$ bags. $10 + 20 + 21 = 51$ bags, Ans.
7. $73\frac{7}{11}, 38\frac{4}{11}, 139\frac{10}{11} = 2\frac{10}{11}, 2\frac{7}{11}, 12\frac{39}{11}$.
 Greatest common divisor of numerators = 81
 Least common denominator of the fractions = 11
 $2\frac{10}{11} + 2\frac{7}{11} + 12\frac{39}{11} = 33\frac{21}{11}$; $33\frac{21}{11} \div 3\frac{1}{11} = 41$ lots.
 $81 \div 11 = 7\frac{4}{11}$ acres, the size of each lot.

LEAST COMMON MULTIPLE OF FRACTIONS.

(ART. 250, p. 184.)

2. $\frac{4}{5}, \frac{8}{7},$ and $\frac{9}{10}$.
 Least common mult. of the numerators = 24
 Greatest com. div. of the denominators = $\frac{24}{1} = 24$, Ans.
3. $3\frac{15}{16}, 7\frac{7}{8}, 5\frac{1}{4} = 3\frac{15}{16}, 8\frac{3}{8}, 2\frac{1}{4}$.
 Least common mult. of the numerators = 63
 Greatest com. div. of the denominators = $\frac{63}{4} = 15\frac{3}{4}$, Ans.
4. $\frac{3}{8}, \frac{6}{7}, \frac{9}{10}$.
 Least common mult. of the numerators = 18
 Greatest com. div. of the denominators = $\frac{18}{1} = 18$, Ans.

NOTE. — By this operation we find that 18 is the least whole number that can be divided by the fractions $\frac{3}{8}, \frac{6}{7},$ and $\frac{9}{10}$, without a remainder.

5. $2\frac{1}{4}, 4\frac{1}{2}, 9\frac{3}{8} = 2\frac{1}{4}, 4\frac{1}{2}, 1\frac{5}{8}$.
 Least common mult. of the numerators = 225
 Greatest com. div. of the denominators = $\frac{225}{2} = 112\frac{1}{2}$.
 $2\frac{1}{4} \div 2\frac{1}{2} = 50$ sheep; $4\frac{1}{2} \div 2\frac{1}{2} = 25$ calves; $9\frac{3}{8} \div 7\frac{5}{8}$
 = 12 yearlings.

$$6. \quad 3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4} = \frac{7}{2}, \frac{14}{3}, \frac{21}{4}.$$

$80 \div \frac{7}{2} = 22\frac{2}{7}$ hours, the time A will go round the island. $80 \div \frac{14}{3} = 17\frac{1}{3}$ hours, the time B will go round the island. $80 \div \frac{21}{4} = 15\frac{1}{3}$ hours, the time C will go round the island.

$$22\frac{2}{7}, 17\frac{1}{3}, 15\frac{1}{3} = 16\frac{2}{3}, 17\frac{1}{3}, 15\frac{1}{3}.$$

Least common mult. of 160, 120, 320 = 960

Greatest common divisor of 7, 7, 21 = $\frac{7}{7} = 137\frac{1}{7}$ hours.

In $137\frac{1}{7} \div 8 = 17\frac{1}{7}$ days, they will all meet at the point from which they started.

$26\frac{2}{3} \div 16\frac{2}{3} = 6$ times, A will go round the island.

$26\frac{2}{3} \div 17\frac{1}{3} = 8$ times, B will go round.

$26\frac{2}{3} \div 15\frac{1}{3} = 9$ times, C will go round.

$80 \times 6 = 480$ miles, A travels. $80 \times 8 = 640$ miles, B travels. $80 \times 9 = 720$ miles, C travels.

$$7. \quad 3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4} = \frac{7}{2}, \frac{14}{3}, \frac{21}{4}.$$

Least common multiple of 7, 35, 21 = 105

Greatest common divisor of 2, 8, 4 = $\frac{2}{2} = 52\frac{1}{2}$.

As $52\frac{1}{2}$ can be divided by each of the fractions $3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4}$, it is certain that twice $52\frac{1}{2} = 105$ may be divided by them; and no less whole number than this is a multiple of $52\frac{1}{2}$.

REDUCTION OF DENOMINATE FRACTIONS.

(ART. 253, p. 185.)

$$2. \quad \frac{1200}{1} \times \frac{20}{1} \times \frac{12}{1} \times \frac{1}{1} = \frac{28800}{1} = 28800, \text{ Ans.}$$

$$3. \quad \frac{510}{1} \times \frac{12}{1} \times \frac{20}{1} \times \frac{24}{1} = \frac{30720}{1} = 30720, \text{ Ans.}$$

$$4. \quad \frac{2592}{1} \times \frac{12}{1} \times \frac{8}{1} \times \frac{3}{1} = \frac{77760}{1} = 77760, \text{ Ans.}$$

$$5. \quad \frac{8800}{1} \times \frac{4}{1} \times \frac{25}{1} \times \frac{18}{1} = \frac{158400}{1} = 158400, \text{ Ans.}$$

$$6. \quad \frac{8000}{1} \times \frac{20}{1} \times \frac{4}{1} \times \frac{25}{1} = \frac{160000}{1} = 160000, \text{ Ans.}$$

$$7. \quad \frac{1}{225} \times \frac{5}{1} \times \frac{4}{1} \times \frac{21}{1} = \frac{42}{225} = \frac{14}{75}, \text{ Ans.}$$

$$8. \quad \frac{1}{110880} \times \frac{8}{1} \times \frac{40}{1} \times \frac{161}{1} \times \frac{12}{1} = \frac{63360}{110880} = \frac{1}{2}, \text{ Ans.}$$

9. $\frac{1}{38016} \times \frac{3}{1} \times \frac{8}{1} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} \times \frac{12}{1} = \frac{570240}{1140480} = \frac{1}{2}$, Ans.
10. $\frac{3}{25090560} \times \frac{4}{1} \times \frac{40}{1} \times \frac{272\frac{1}{4}}{1} \times \frac{144}{1} = \frac{18817920}{25090560} = \frac{3}{4}$,
[Ans.]
11. $\frac{1}{1152} \times \frac{4}{1} \times \frac{6\frac{3}{4}}{1} \times \frac{4}{1} = \frac{1008}{1152} = \frac{7}{8}$, Ans.
12. $\frac{3}{320} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{192}{320} = \frac{3}{5}$, Ans.
13. $\frac{1}{4207680} \times \frac{365\frac{1}{4}}{1} \times \frac{24}{1} \times \frac{60}{1} = \frac{525960}{4207680} = \frac{1}{8}$, Ans.
14. $\frac{3}{3200} \times \frac{4}{1} \times \frac{2\frac{1}{2}}{1} \times \frac{1\frac{1}{2}}{1} = \frac{4800}{3200} = \frac{3}{2}$, Ans.

(ART. 254, p. 186.)

2. $\frac{4}{5} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{1200}$, Ans.
3. $\frac{3}{8} \times \frac{1}{24} \times \frac{1}{20} \times \frac{1}{12} = \frac{1}{28800} = \frac{1}{28800}$, Ans.
4. $\frac{5}{8} \times \frac{1}{3} \times \frac{1}{8} \times \frac{1}{12} = \frac{1}{2880}$, Ans.
5. $\frac{6}{11} \times \frac{1}{16} \times \frac{1}{20} \times \frac{1}{4} = \frac{1}{17600} = \frac{1}{17600}$, Ans.
6. $\frac{3}{4} \times \frac{1}{25} \times \frac{1}{4} \times \frac{1}{20} = \frac{1}{8000}$, Ans.
7. $\frac{1}{5} \times \frac{1}{24} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{2400}$, Ans.
8. $\frac{4}{7} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{\frac{2}{8}} = \frac{1}{110880}$, Ans.
9. $\frac{1}{2} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{8} \times \frac{1}{3} = \frac{1}{38016}$, Ans.
10. $\frac{3}{4} \times \frac{1}{144} \times \frac{1}{272\frac{1}{4}} \times \frac{1}{40} \times \frac{1}{4} = \frac{3}{25090560}$, Ans.
11. $\frac{7}{8} \times \frac{1}{4} \times \frac{1}{\frac{66}{9}} \times \frac{1}{4} = \frac{1}{1152}$, Ans.
12. $\frac{3}{8} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} = \frac{1}{320}$, Ans.

$$13. \frac{1}{8} \times \frac{1}{60} \times \frac{1}{24} \times \frac{1}{365\frac{1}{4}} = \frac{1}{4207680}, \text{ Ans.}$$

$$14. \frac{3}{2} \times \frac{1}{16} \times \frac{1}{25} \times \frac{1}{4} = \frac{3}{3200}, \text{ Ans.}$$

(ART. 255, p. 187.)

(2.)			(3.)			(4.)			
s.	d.		s.	d.	qr.	cwt.	qr.	lb.	oz. dr.
1	0		28	0	0	1	0	0	0 0
	7			7					7
24	7	0	9	196	0 0	11	7	0 0 0 0	0
Ans.	3	$\frac{1}{2}$	Ans.	21	9 $\frac{1}{2}$	Ans.	2	13 10	2 $\frac{1}{2}$

(5.)			(6.)				(7.)				
lb.	oz.	dr.	lb.	oz.	pwt.	gr.	lb.	3	3	3	gr.
1	0	0	1	0	0 0	0	1	0	0	0 0	0
		4			8					4	
9	4	0 0	9	8	0 0 0	0	13	4	0 0 0 0	0	0
Ans.	7	1 $\frac{1}{2}$	Ans.	10	13	8	Ans.	0	3 5	1 12	$\frac{4}{3}$

(8.)				(9.)				(10.)			
E.E.	qr.	na.	in.	m.	fur.	rd.	ft. in.	fur.	rd.	ft.	in.
1	0	0	0	1	0	0 0	0 0	1	0	0 0	0
			5				11				8
9	5	0 0 0	0	13	11	0 0 0 0	0	9	8	0 0 0	0
Ans.	2	3	0 $\frac{1}{4}$	Ans.	6	30 12	8 $\frac{4}{3}$	Ans.	35	9	2

(11.)						(12.)			(13.)		
A.	B.	p.	yd.	ft.	in.	p.	ft.	in.	cord.	ft.	in.
1	0	0	0 0	0	0	1	0	0	1	0	0
					7			9			1
13	7	0 0 0 0	0	0	0	17	9	0 0	13	1	0
Ans.	2	6	4 5	127 $\frac{5}{13}$		Ans.	144	19 $\frac{1}{17}$	Ans.	9	1462 $\frac{2}{13}$

(14.)					(15.)		(16.)				
hhd.	gal.	qt.	pt.	gi.	hhd.	gal.	y.	da.	h.	m.	sec.
1	0	0	0	0	1	0	1	0	0	0	0
<hr/>					<hr/>		<hr/>				
2					7		11				
19)	2	0	0	0	9	7	23)	11	0	0	0
<hr/>					<hr/>		<hr/>				
Ans.	6	2	1	0 ₁₉	Ans.	42	Ans.	174	16	26	5 ₂₃

$$17. \$7\frac{3\frac{3}{4}}{4\frac{2}{5}} = \$7\frac{36}{22}; \frac{36}{11} \times \frac{5}{22} = \frac{180}{242}; \$7\frac{180}{242}, \text{Ans.}$$

(ART. 256, p. 188.)

$$2. \frac{3 \times 4 + 2 = 14}{12 \times 4 = 48} = \frac{14}{48} = \frac{7}{24}, \text{Ans.}$$

$$3. \left. \begin{array}{l} 21 \times 12 + 9 = 261; 261 \times 4 + 1 = 1045; \\ 1045 \times 3 + 1 = 3136 \\ 28 \times 12 \times 4 \times 3 = 4032 \end{array} \right\} = \frac{3136}{4032}, \text{Ans.}$$

$$4. \left. \begin{array}{l} 2 \times 25 + 13 = 63; 63 \times 16 + 10 = 1018; \\ 1018 \times 16 + 2 = 16290; 16290 \times 11 + 10 = 179200 \\ 4 \times 25 \times 16 \times 16 \times 11 = 281600 \end{array} \right\} = \frac{179200}{281600}, \text{Ans.}$$

$$5. \left. \begin{array}{l} 7 \times 16 + 1 = 113; 113 \times 9 + 7 = 1024 \\ 16 \times 16 \times 9 = 2304 \end{array} \right\} = \frac{1024}{2304} = \frac{1}{2}, \text{Ans.}$$

$$6. \left. \begin{array}{l} 10 \times 20 + 13 = 213; 213 \times 24 + 8 = 5120 \\ 12 \times 20 \times 24 = 5760 \end{array} \right\} = \frac{5120}{5760} = \frac{8}{9}, \text{Ans.}$$

$$7. \frac{268\frac{4}{5}}{231} = \frac{1344}{1155}, \text{Ans.}$$

$$8. \left. \begin{array}{l} 2 \times 4 = 8 \times 2\frac{1}{4} + 1 = 19 \times 13 + 5 = 252 \\ 4 \times 4 \times 2\frac{1}{4} \times 13 = 468 \end{array} \right\} = \frac{252}{468} = \frac{7}{13}, \text{Ans.}$$

$$9. \left. \begin{array}{l} 2 \times 4 + 3 = 11 \times 2\frac{1}{4} = 24\frac{3}{4} \times 4 + 1 = 100 \\ 5 \times 4 \times 2\frac{1}{4} \times 4 = 180 \end{array} \right\} = \frac{100}{180} = \frac{5}{9}, \text{Ans.}$$

$$10. \left. \begin{array}{l} 6 \times 40 + 30 = 270 \times 16\frac{1}{2} + 12 = 4467 \times 12 + 8 \\ = 53612 \times 3 = 160836 \times 13 + 12 \\ 8 \times 40 \times 16\frac{1}{2} \times 12 \times 3 \times 13 = 2471040 \end{array} \right\} = \frac{2090880}{2471040}, \text{Ans.}$$

11. $35 \times 16\frac{1}{2} + 9 = 586\frac{1}{2} \times 12 + 2 = 7040$
 $40 \times 16\frac{1}{2} \times 12 = 7920 = \frac{8}{9}, \text{ Ans.}$
12. $\left. \begin{aligned} 2 \times 40 + 6 &= 86 \times 30\frac{1}{4} + 4 = 2605\frac{1}{2} \times 9 + 5 \\ &= 23454\frac{1}{2} \times 144 + 127 = 3377575 \times 13 + 5 \end{aligned} \right\} = 43908480$
 $4 \times 40 \times 30\frac{1}{4} \times 9 \times 144 \times 13 = 81544320$
 $[= \frac{1}{13}, \text{ Ans.}]$
13. $144 \times 144 + 19 = 20755 \times 17 + 1 = 352836$
 $272\frac{1}{4} \times 144 \times 17 = 666468 = \frac{9}{17}, \text{ Ans.}$
14. $9 \times 1728 + 1462 = 17014 \times 13 + 2 = 221184$
 $128 \times 1728 \times 13 = 2875392 = \frac{1}{13}, \text{ Ans.}$
15. $6 \times 4 + 2 = 26 \times 2 + 1 = 53 \times 4 = 212 \times 19 + 4 = 4032$
 $63 \times 4 \times 2 \times 4 \times 19 = 38304$
 $[= \frac{2}{19}, \text{ Ans.}]$
16. 1lb. avoirdupois = 5760 grains
 1lb. troy = 7000 grains = $1\frac{1}{4}\frac{1}{5}, \text{ Ans.}$
17. $174 \times 24 + 16 = 4192 \times 60 + 26 = 251546 \times 60 + 5 = 15092765 \times 23 + 5 = 347133600$
 $365\frac{1}{4} \times 24 \times 60 \times 60 \times 23 = 725824800$
 $[= \frac{1}{2}\frac{1}{3}, \text{ Ans.}]$

ADDITION OF DENOMINATE FRACTIONS.

(ART. 257, p. 190.)

(2.)

	awt.	qr.	lb.
$\frac{7}{11}$ of a ton	= 12	2	$22\frac{8}{11}$
$\frac{17}{2}$ of a cwt.	=	3	$2\frac{3}{11}$
Ans.	13	2	0

(3.)

	qr.	na.	in.
$\frac{2}{3}$ of a yd.	= 2	1	$1\frac{7}{8}$
$\frac{1}{7}$ of an E. E.	=	2	$1\frac{1}{4}$
$\frac{5}{7}$ of a qr.	=	3	$0\frac{3}{8}$
Ans.	3	3	$1\frac{1}{4}\frac{3}{8}$

(4.)

	fur.	rd.	ft.	in.
$\frac{7}{11}$ of a mile	= 5	3	10	6
$\frac{4}{13}$ of a furlong	=	12	5	$0\frac{1}{3}$
$\frac{3}{22}$ of a yard	=		1	$2\frac{8}{11}$
	5	15	16	$9\frac{9}{143}$
Ans.	5	16	0	$3\frac{9}{143}$

(5.)

	A.	R.	p.	ft.	in.
$\frac{3}{4}$ of an A. =	0	2	34	77	113 $\frac{1}{2}$
$\frac{2}{3}$ of an A. =	0	2	26	181	72
$1\frac{1}{2}$ of an A. =	0	3	28	155	82 $\frac{1}{2}$
	2	1	9	141 $\frac{1}{2}$	123 $\frac{1}{2}$
Ans.	2	1	9	142	87 $\frac{1}{2}$

(6.)

	m.	fur.	rd.	ft.	in.
18 $\frac{3}{4}$ miles =	18	3	17	2	4 $\frac{3}{4}$
23 $1\frac{1}{4}$ miles =	23	6	11	7	0 $\frac{3}{4}$
19 $2\frac{1}{4}$ miles =	19	0	15	3	11 $\frac{1}{4}$
Ans.	61	2	3	13	4 $\frac{3}{4}$

(7.)

	gal.	qt.	pt.	gt.
$1\frac{1}{2}$ of a gal. =	0	3	1	1 $\frac{1}{2}$
$1\frac{1}{2}$ of a hhd. =	5	1	0	0
Ans.	6	0	1	1 $\frac{1}{2}$

(8.)

	d.	h.	m.
$\frac{5}{16}$ of a week =	2	4	30
$\frac{1}{5}$ of a day =	4	48	
Ans.	2	9	18

(9.)

	ft.	in.
$\frac{3}{4}$ of a square foot =	0	108
$\frac{1}{2}$ of a foot square =		36
Ans.	1	0

(10.)

	rd.	ft.	in.
	11	16	5
			6
	11	16	11 =
	12	0	5

SUBTRACTION OF DENOMINATE FRACTIONS.

(ART. 258, p. 190.)

(2.)

	qr.	na.	in.
$\frac{7}{8}$ E. E. =	4	1	1 $\frac{1}{8}$
$\frac{2}{3}$ of a yd. =	1	0	1 $\frac{2}{3}$
Ans.	3	0	2 $\frac{5}{8}$

(3.)

	fur.	rd.	ft.	in.
$\frac{2}{3}$ of a mile =	1	31	1	10
$\frac{7}{11}$ of a fur. =		25	7	6
	1	5	10 $\frac{1}{2}$	4 =
Ans.	1	5	10	10

(4.)

		m.	fur.	rd.	ft.	in.
$\frac{5}{7}$ deg.	=	49	3	9	8	$7\frac{1}{2}$
$\frac{3}{5}$ mile	=	0	4	32	0	0
Ans.		48	6	17	8	$7\frac{1}{2}$

(5.)

	R.	p.	yd.	ft.	in.
$\frac{1}{11}$ A. =	1	18	5	4	72
$\frac{1}{8}$ rod =	0	0	13	4	0
	1	17	22 $\frac{1}{4}$	0	72
				$\frac{1}{4}$ =	2.36
Ans.	1	17	22	2	108

(6.)

	ft.	in.
$\frac{9}{10}$ cord =	115	345 $\frac{3}{4}$
$\frac{2}{11}$ cord =	23	471 $\frac{3}{11}$
Ans.	91	1602 $\frac{18}{55}$

(7.)

$$\begin{array}{r} \frac{7}{13} \text{ hhd.} = 33 \text{ gal. } 3 \text{ qt. } 1 \text{ pt. } 1 \frac{1}{13} \text{ gi.} \\ \frac{4}{5} \text{ of } \frac{7}{13} \text{ hhd.} = \frac{28}{65} = 27 \text{ gal. } 0 \text{ qt. } 1 \text{ pt. } 0 \frac{28}{65} \text{ gi.} \\ \text{Ans. } 6 \text{ gal. } 3 \text{ qt. } 0 \text{ pt. } 1 \frac{7}{65} \text{ gi.} \end{array}$$

(8.)

		m.	fur.	rd.	ft.	in.
72m.	=	72	0	0	0	0
$\frac{1}{4}$ of 72m.	=	41	1	5	11	9 $\frac{3}{4}$
		30	6	34	4 $\frac{1}{2}$	2 $\frac{1}{2}$
					$\frac{1}{2}$	=6
Ans.		30	6	34	4	8 $\frac{1}{2}$

(9.)

	d.	h.	m.	sec.
$\frac{2}{7}$ year =	104	8	34	17 $\frac{1}{2}$
$\frac{4}{5}$ week =	3	2	40	0
Ans.	101	5	54	17 $\frac{1}{2}$

(10.)

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$ A. =	1	18	5	4	72
$\frac{1}{2}$ ft. =					72
Ans.	1	18	5	4	0

MISCELLANEOUS EXAMPLES IN FRACTIONS.

(PAGE 190.)

1. $17\frac{3}{11} = 1\frac{10}{11}$; $4\frac{2}{9} = \frac{42}{9}$; $1\frac{10}{11} \times \frac{42}{9} = 2\frac{170}{99} = 82\frac{2}{99}$ miles
= 82m. 4fur. 8rd. 1ft. 4in., Ans.
2. $29\frac{7}{13} = \frac{384}{13}$; $\frac{384}{13} \times \frac{384}{13} = \frac{147456}{169} = 872\frac{88}{169}$ poles =
5A. 1R. $32\frac{88}{169}$ p.; $\frac{88}{169}$ p. = 141ft. 109 $\frac{11}{169}$ in., Ans.
3. $17\frac{3}{4} = \frac{71}{4}$; $7\frac{1}{11} = \frac{78}{11}$; $4\frac{2}{3} = \frac{25}{3}$; $\frac{71}{4} \times \frac{78}{11} \times \frac{25}{3} = \frac{127374}{220}$
= 578 $\frac{107}{220}$ feet; $578\frac{107}{220} \div 128 = 4$ cords 66 $\frac{107}{220}$ cubic
feet, Ans.
4. $19\frac{7}{8} = 1\frac{15}{8}$; $6\frac{3}{4} = \frac{27}{4}$; $1\frac{15}{8} \times \frac{27}{4} = 4\frac{293}{32} = \$134.15\frac{3}{8}$,
Ans.

5. $376\frac{1}{3} = 67\frac{7}{8}$; $75\frac{3}{8} = 69\frac{3}{8}$; $67\frac{7}{8} \times 69\frac{3}{8} = 4687\frac{7}{16} = \$28387.06\frac{1}{4}$, Ans.
6. $17\frac{1}{12} = 19\frac{2}{12}$; $4.75 = 4\frac{7}{8}$; $19\frac{2}{12} \times 4\frac{7}{8} = 91\frac{34}{112} = \$81.55\frac{65}{112}$, Ans.
7. $1670\frac{7}{8} = 217\frac{17}{8}$; $12\frac{3}{4} = 5\frac{1}{4}$; $217\frac{17}{8} \times 5\frac{1}{4} = 1107\frac{567}{52} = \$212.99\frac{19}{52}$, Ans.
8. $28\frac{1}{4} = 31\frac{1}{4}$; $11\frac{3}{4} = 4\frac{7}{8}$; $31\frac{1}{4} \times 4\frac{7}{8} = 146\frac{64}{84} = \$333.27\frac{3}{11}$, Ans.
9. $37\frac{1}{3} = 23\frac{4}{3}$; $17.62\frac{1}{2} = 35\frac{25}{2}$; $23\frac{4}{3} \times 35\frac{25}{2} = 825\frac{550}{6} = \$655.20\frac{5}{6}$, Ans.
10. $\frac{7}{8} \times 57\frac{1}{2} = 40\frac{25}{8} = \$5.03\frac{1}{8}$, Ans.
11. $139\frac{1}{4} = 27\frac{1}{2}$; $38\frac{5}{8} = 19\frac{1}{4}$; $27\frac{1}{2} \times 19\frac{1}{4} = 189\frac{33}{8} = 5415\frac{1}{2}$ poles = 33A. 3R. $15\frac{1}{2}$ p., Ans.
12. $11\frac{3}{5} = 5\frac{3}{5}$; $5\frac{3}{5} \times 1\frac{1}{2} = 27\frac{9}{10}$; $27\frac{9}{10} \times \frac{1}{2} = 13\frac{9}{20} = 19\frac{1}{2}$ feet, Ans.
13. $18\frac{3}{8} = 14\frac{1}{2}$; $48.15\frac{3}{4} = 192\frac{63}{4}$; $192\frac{63}{4} \times 1\frac{8}{17} = 154\frac{104}{58}$ = \$2.62 $\frac{4}{9}$, Ans.
14. $98\frac{1}{3} = 19\frac{1}{3}$; $19\frac{1}{3} \times \frac{1}{6} = 7\frac{1}{6} = \$1.56\frac{1}{6}$, Ans.
15. $8\frac{3}{8} = 7\frac{1}{4}$; $7\frac{1}{4} \times \frac{1}{5} = 7\frac{1}{5} = \$1.64\frac{4}{5}$, Ans.
16. $11 \times 63 = 693$; $693 \div 17\frac{1}{2} = 710\frac{1}{2} = 142\frac{1}{2}$; $19\frac{1}{4} = 7\frac{9}{4}$; $142\frac{1}{2} \times 7\frac{9}{4} = 1122\frac{9}{8} = \$140.32\frac{3}{8}$, Ans.
17. $63 \times 4 \times 2 = 504$ pts.; $1\frac{1}{2} = \frac{3}{2}$; $504 \times \frac{3}{2} = 207\frac{16}{16} = 288$ bottles, Ans.
18. $18\frac{1}{2} = 22\frac{3}{2}$; $10\frac{5}{12} = 12\frac{5}{12}$; $7\frac{1}{12} = 9\frac{5}{12}$; $22\frac{3}{2} \times 12\frac{5}{12} \times 9\frac{5}{12} = 2648\frac{125}{1728} = 1532\frac{829}{1728}$; $1532\frac{829}{1728} \div 128 = 11$ cords, $124\frac{829}{1728}$ cubic feet, Ans.
19. $6\frac{7}{8} = 5\frac{5}{8}$; $65\frac{1}{4} = 26\frac{1}{4}$; $5\frac{5}{8} \times 26\frac{1}{4} = 144\frac{65}{32} = \$4.52\frac{1}{32}$, Ans.
20. $8\frac{3}{4} = 3\frac{3}{4}$; $57\frac{1}{2} = 67\frac{1}{2}$; $3 = \frac{3}{1}$; $3\frac{3}{4} \times 67\frac{1}{2} \times \frac{3}{1} = 793\frac{3}{2} = 146\frac{9}{16}$ feet, Ans.
21. $46\frac{5}{12} = 55\frac{7}{12}$; $17\frac{1}{2} = 3\frac{5}{2}$; $55\frac{7}{12} \times 3\frac{5}{2} = 194\frac{95}{24} = 812\frac{7}{24}$ feet, Ans.

22. $2\frac{1}{2} + 3\frac{1}{2} = 5\frac{6}{2}$; $5\frac{6}{2} \times \frac{2}{2} = 11$; $11 \times 5\frac{1}{2} = 60\frac{1}{2}$; $2\frac{5}{2} = \frac{5}{2}$; $3\frac{1}{2} = \frac{7}{2}$; $\frac{5}{2} \times \frac{7}{2} \times \frac{2}{2} = \frac{35}{2} = 17\frac{1}{2}$; $60\frac{1}{2} + 17\frac{1}{2} = 78$; $78 \times \frac{2}{2} = 156$ feet, Ans.
 $5\frac{6}{2} = \frac{66}{2}$; $2\frac{5}{2} = \frac{25}{2}$; $3\frac{1}{2} = \frac{37}{2}$; $\frac{66}{2} \times \frac{25}{2} \times \frac{37}{2} \times \frac{2}{2} = \frac{484875}{8} = 2862\frac{3}{8}$ cubic feet; second Ans.
23. $12 + 11\frac{1}{2} = 23\frac{1}{2}$; $23\frac{1}{2} \times 2 = 47$; $47 \times 7\frac{1}{2} = 352\frac{1}{2}$; $12 \times 11\frac{1}{2} = 138$; $352\frac{1}{2} + 138 = 490\frac{1}{2}$; $490\frac{1}{2} \times .02\frac{1}{4} = \$13.48\frac{7}{8}$, Ans.
24. $14\frac{1}{2} = \frac{29}{2}$; $10\frac{1}{4} = \frac{21}{2}$; $16\frac{1}{2} = \frac{33}{2}$; $\frac{29}{2} \times \frac{33}{2} = \frac{957}{2} = 239\frac{1}{2}$; $\frac{21}{2} \times \frac{33}{2} = \frac{693}{2} = 169\frac{1}{2}$; $239\frac{1}{2} + 169\frac{1}{2} = 408\frac{1}{2}$; $408\frac{1}{2} \times 2 = 816\frac{1}{2}$; $816\frac{1}{2} + 12 = 828\frac{1}{2}$; $828\frac{1}{2} \times 3 \times 4\frac{1}{2} \times 2 = \$223.76\frac{1}{4}$, Ans.
25. $14\frac{7}{2} = \frac{175}{2}$; $5\frac{1}{2} = \frac{11}{2}$; $4\frac{1}{4} = \frac{17}{4}$; $2150\frac{2}{5} = \frac{10752}{5}$; $\frac{175}{2} \times \frac{11}{2} \times \frac{17}{4} \times \frac{10752}{5} \times \frac{1}{10752} = \frac{1824984000}{6193152} = 294\frac{343}{8}$ bushels, Ans.
26. $10 \times 8 \times 6 = 480$; $8 \times 8 \times 8 = 512$; $512 - 480 = 32$ feet, Ans.
27. $7\frac{1}{2} \times 6 \times 5\frac{1}{2} = \frac{15}{2} \times \frac{6}{1} \times \frac{11}{2} = \frac{99}{1} = 99$ cubic feet.
 $9\frac{1}{2} \times 4\frac{1}{2} \times 5\frac{1}{2} = \frac{19}{2} \times \frac{9}{2} \times \frac{11}{2} = \frac{1881}{2} = 235\frac{1}{2}$; $247\frac{1}{2} - 235\frac{1}{2} = 12$; $1728 \times 12\frac{3}{8} = 21384$; $21384 \div 231 = 92\frac{4}{7}$ gallons, Ans.

The first cistern will contain $92\frac{4}{7}$ most gallons.

$$\begin{array}{r}
 28. \quad 31\text{rd. } 13\frac{3}{10}\text{ft.,} \quad 41\text{rd. } 1\frac{9}{10}\text{ft.,} \quad 38\text{rd. } 0\frac{1}{5}\text{ft.,} \quad 45\text{rd. } 12\frac{7}{10}\text{ft.} = \\
 \quad 524\frac{1}{5}\text{ft.,} \quad \quad \quad 678\frac{3}{5}\text{ft.,} \quad \quad \quad 627\frac{1}{5}\text{ft.,} \quad \quad \quad 755\frac{1}{5}\text{ft.} = \\
 \quad \frac{2624}{5} \quad \quad \quad \frac{3392}{5} \quad \quad \quad \frac{3136}{5} \quad \quad \quad \frac{3776}{5}
 \end{array}$$

The greatest common divisor of 2624, 3392, 3136, 3776, is = $\overline{64}$

The greatest common multiple of 5, 5, 5, 5, is = $\overline{5}$

Therefore $\frac{64}{5}$ will divide each of the fractions without a remainder; thus $\frac{2624}{5} \div \frac{64}{5} = 41$; $\frac{3392}{5} \div \frac{64}{5} = 53$; $\frac{3136}{5} \div \frac{64}{5} = 49$; $\frac{3776}{5} \div \frac{64}{5} = 59$. The number of rails will therefore be $41 + 53 + 49 + 59 = 202$; $202 \times 4 = 808$ rails.

$64 \div 5 = 12\frac{3}{5}$; $12\frac{3}{5} + 7\frac{1}{5} = 13\frac{4}{5}$ feet, length of the rails, Ans

29. $\frac{5}{8}, \frac{2}{3}, \frac{7}{9} = \frac{35}{72}, \frac{42}{72}, \frac{48}{72}$.

Least common multiple of the numerators = 1680
 Greatest common divisor of the denominators = $\frac{1680}{56} = 30$ yds., Ans.

We therefore find that 30 is the least whole number that can be divided by $\frac{5}{8}, \frac{2}{3}$, or $\frac{7}{9}$, without a remainder.

30. $100 \div 30 = 3\frac{1}{3}$ times.

31. 31A. 3R. 6p. = 5086p.; 39A. 2R. $37\frac{1}{2}$ p. = 6357 $\frac{1}{2}$ p.

$$\frac{5086}{6357\frac{1}{2}} \times 2 = \frac{10172}{12715} = \frac{4}{5}, \text{ Ans.}$$

32. $68 \times \$7\frac{1}{2} = \$538.33\frac{1}{3}$, Ans.

33. $8\frac{2}{3} \times \$42\frac{2}{3} = \369.20 , Ans.

34. $\frac{3-3}{3-6} = 0$, Ans.

35. $20 \times 15 = 300$ ft., contents of the upper part of the room.
 $20 + 15 = 35$; $35 \times 2 = 70$ ft., length round the room.
 $70 \times 8\frac{1}{2} = 595$ feet, contents of the upright ceiling of the room.

$2 \times 7 \times 3 = 42$ ft., contents of the doors.

$4 \times 5\frac{1}{2} \times 3\frac{1}{3} = 73\frac{1}{3}$ ft., contents of the windows.

$70 - 6 = 64$; $64 \times \frac{2}{3} = 42\frac{2}{3}$ ft., contents of the mop-boards.

NOTE. — We deduct 6 feet for the two doors.

$595 + 300 = 895$; $42 + 73\frac{1}{3} + 42\frac{2}{3} = 158$.

$895 - 158 = 737$ ft.; $737 \div 9 = 81\frac{8}{9}$ square yards.

$81\frac{8}{9} \times 6\frac{1}{4} = \frac{1337}{9} \times \frac{25}{4} = \frac{18425}{36} = \$5.11\frac{25}{36}$, for plastering.

$595 - 158 = 437$ ft.; $437 \div 9 = 48\frac{5}{9}$ square yards.

$48\frac{5}{9} \times .09 = \4.37 , for papering.

$48\frac{5}{9} \times 3 = 145\frac{5}{3} = \frac{437}{3}$; $2\frac{2}{3} = \frac{13}{6}$; $\frac{437}{3} \div \frac{13}{6} = \frac{437}{3} \times \frac{6}{13} = \frac{2182}{13} = \$2.80\frac{5}{13}$, for paper, Ans.

QUESTIONS TO BE PERFORMED BY ANALYSIS.

(Page 194.)

8. $30\frac{4}{15} = \frac{454}{15}$; $\frac{1728}{15} \times \frac{15}{454} = \frac{25920}{454}$; $7\frac{1}{3} = \frac{227}{30}$; $\frac{25920}{454} \times \frac{227}{30} = \frac{5883840}{13620} = \432 , Ans.

4. $7\frac{17}{30} = \frac{227}{30}$; $4\frac{12}{27} \times \frac{30}{227} = \frac{12220}{227}$; $17\frac{28}{127} \times \frac{227}{12220} = \frac{292256}{12220} = 30\frac{4}{15}$ tons, Ans.
5. $7\frac{17}{30} = \frac{227}{30}$; $4\frac{12}{27} \times \frac{30}{227} = \frac{12220}{227}$; $30\frac{4}{15} = \frac{454}{15}$; $\frac{454}{15} \times \frac{12220}{227} = \frac{5883840}{3405} = \1728 , Ans.
6. $30\frac{4}{15} = \frac{454}{15}$; $17\frac{28}{127} \times \frac{15}{454} = \frac{25920}{454}$; $4\frac{12}{27} \times \frac{454}{25920} = 7\frac{11}{30}$ tons, Ans.
7. $7\frac{17}{30} = \frac{227}{30}$; $6\frac{1}{8} = \frac{58}{9}$; $\frac{58}{9} \times \frac{227}{16} = \frac{4866}{4} = 492\frac{3}{4}$ bushels, Ans.
8. $\frac{7}{11}$ of $17 = \frac{119}{11} = 10\frac{9}{11}$; $10\frac{9}{11} + 15 = 25\frac{9}{11} = 23\frac{4}{11}$; $17\frac{28}{127} \times \frac{234}{284} = \frac{12802}{284} = \$66\frac{2}{7}$, each girl's share; $\frac{7}{11}$ of $66\frac{2}{7} = \frac{7}{11} \times \frac{12802}{284} = \frac{13321}{142} = \$42\frac{1}{2}$, each boy's share.
9. $14\frac{49}{12} \times \frac{2}{3} = \frac{13941}{12} = 18.63$; $4\frac{3}{4} = \frac{31}{4}$; $\frac{1863}{4} \times \frac{31}{4} = \frac{57753}{4} = \$82.50\frac{3}{4}$, Ans.
10. $82.50\frac{3}{4} = \frac{57753}{4}$; $4\frac{3}{4} = \frac{31}{4}$; $\frac{57753}{4} \times \frac{4}{31} = \frac{404271}{31} = 18.63$; $\frac{1449}{5} = \frac{7}{5}$, Ans.
11. $14\frac{49}{12} \times \frac{2}{3} = \frac{13941}{12} = 18.63$; $82.50\frac{3}{4} \div 18.63 = 4\frac{3}{4}$, Ans.
12. $82.50\frac{3}{4} = \frac{57753}{4}$; $4\frac{3}{4} = \frac{31}{4}$; $\frac{57753}{4} \times \frac{4}{31} = \frac{404271}{31} = 18.63$; $\frac{7}{5}$ of $18.63 = \frac{13941}{12} = \14.49 , Ans.
13. $14\frac{49}{12} = \frac{119}{12}$; $\frac{590}{119} \times \frac{8}{119} = \frac{4000}{119}$; $9\frac{11}{12} = \frac{119}{12}$; $\frac{4000}{119} \times \frac{119}{12} = \frac{4000}{12} = \frac{1000}{3} = \$333.33\frac{1}{3}$, Ans.
14. $9\frac{11}{12} = \frac{119}{12}$; $333.33\frac{1}{3} = \frac{100000}{3}$; $\frac{100000}{3} \times \frac{12}{119} = \frac{1200000}{119}$; $14\frac{49}{12} = \frac{119}{12}$; $\frac{1200000}{119} \times \frac{12}{119} = \frac{1428571428}{119} = \500 .
[Ans.]
15. $333.33\frac{1}{3} = \frac{100000}{3}$; $9\frac{11}{12} = \frac{119}{12}$; $\frac{100000}{3} \times \frac{12}{119} = \frac{1200000}{119}$;
 $\frac{50000}{1} \times \frac{357}{1200000} = \frac{1785}{120} = 14\frac{7}{8}$ tons, Ans.
16. $14\frac{49}{12} = \frac{119}{12}$; $\frac{50000}{119} \times \frac{8}{119} = \frac{400000}{119}$; $333.33\frac{1}{3} = \frac{100000}{3}$;
 $\frac{100000}{3} \times \frac{119}{400000} = \frac{11900000}{400000} = 9\frac{11}{12}$ tons, Ans.
17. $97\frac{7}{8} = \frac{783}{8}$; $\frac{2132}{783} \times \frac{783}{8} = \frac{25936}{8} = .32$; $763\frac{5}{8} \times .32 = \244.36 , Ans.
18. $763\frac{5}{8} = \frac{6109}{8}$; $\frac{24436}{6109} \times \frac{8}{6109} = \frac{195488}{6109} = .32$; $97\frac{7}{8} \times .32 = \31.32 , Ans.

19. $763\frac{3}{4} = \frac{6102}{8}$; $244\frac{36}{100} \times \frac{6102}{100} = \frac{125488}{100} = .32$; $31.32 \div .32 = 97\frac{1}{2}$ gal., Ans.
20. $1975 \div 40 = 49\frac{3}{8}$; $49\frac{3}{8} \times 144 = 7110\text{lb.}$, Ans.
21. $15.75 \div 17 = 92\frac{1}{17}$; $92\frac{1}{17} = \frac{1575}{17}$; $9\frac{1}{4} = \frac{37}{4}$; $\frac{1575}{17} \times \frac{37}{4} = \frac{58275}{68} = \$8.56\frac{17}{68}$, Ans.
22. $50\frac{1}{4} = \frac{201}{4}$; $\frac{1}{8} \times \frac{201}{4} = \frac{201}{32}$; $87\frac{3}{8} = \frac{435}{8}$; $\frac{201}{32} \times \frac{435}{8} = \frac{155052}{256} = \$492\frac{33}{32}$, Ans.
23. $78 \times 13 = 1014$; $13 + 7 = 20$; $1014 \div 20 = 50\frac{7}{10}$ days, Ans.
24. $10 \times 9 = 90$; $90 \div 15 = 6$ days, Ans.
25. $15 \times 6 = 90$; $90 \div 10 = 9$ days, Ans.
26. $10 \times 9 = 90$; $90 \div 6 = 15$ hours, Ans.
27. $17\frac{1}{11} = \frac{190}{11}$; $5\frac{3}{4} = \frac{21}{4}$; $\frac{190}{11} \times \frac{21}{4} = \frac{4185}{22}$; $97\frac{1}{2} = \frac{271}{2}$; $\frac{4185}{22} \times \frac{271}{2} = \frac{367004}{44} = \$302\frac{31}{44}$, Ans.
28. $9\frac{3}{5} = \frac{48}{5}$; $19\frac{7}{8} = \frac{158}{8}$; $\frac{48}{5} \times \frac{158}{8} = \frac{2544}{5} = 189\frac{4}{5}$; $189\frac{4}{5} + 7 = 196\frac{4}{5}$; $9\frac{3}{5}$ tons = 192 cwt.; $196\frac{4}{5} \div 192 = \$1\frac{788}{120}$, Ans.
29. $9\frac{3}{5}$ tons = 192 cwt.; $192 \times 1\frac{1}{4} = 336$; $336 - 7 = \$329$, Ans.
30. $47\frac{3}{11} = \frac{520}{11}$; $2\frac{3}{4} = \frac{9}{4}$; $\frac{520}{11} \times \frac{9}{4} = \frac{4160}{44} = 126\frac{2}{11}$; $0.75 = \frac{3}{4}$; $126\frac{2}{11} \div \frac{3}{4} = 168\frac{8}{11}$ bushels, Ans.
31. $57\frac{9}{11} = \frac{636}{11}$; $\frac{636}{11} \times \frac{1}{15} = \frac{636}{165}$; $19\frac{7}{8} = \frac{158}{8}$; $\frac{636}{165} \times \frac{158}{8} = \frac{101124}{110} = \$76\frac{67}{110}$, Ans.
32. $19\frac{1}{8} = \frac{152}{8}$; $76\frac{67}{100} = \frac{101324}{100}$; $\frac{101324}{100} \times \frac{8}{155} = \frac{209888}{155} = \frac{636}{11}$; $57\frac{9}{11} = \frac{636}{11}$; $\frac{636}{11} \times \frac{1}{155} = 15$ cords, Ans.
33. $7\frac{1}{10} = \frac{71}{10}$; $47\frac{1}{3} = \frac{142}{3}$; $\frac{71}{10} \times \frac{142}{3} = \frac{10086}{30} = 345\frac{8}{15}$. = 17£. 5s. 6½d., Ans.
34. $172\text{£. } 15\text{s. } 0\frac{1}{2}\text{d.} = 207\frac{302}{100}$; $47\frac{1}{3} = \frac{142}{3}$; $207\frac{302}{100} \times \frac{142}{3} = \frac{624106}{150} = 875\frac{288}{125}$ = 3£. 12s. 11½d., Ans.
35. $43\frac{1}{2} = \frac{297}{2}$; $\frac{297}{2} \times \frac{1}{13} = \frac{297}{26} = \$23.61\frac{1}{13}$, Ans.
36. $17\frac{3}{8} = \frac{138}{8}$; $2\frac{7}{11} = \frac{29}{11}$; $\frac{138}{8} \times \frac{29}{11} = \frac{2322}{88}$; $\frac{2322}{88} \times \frac{50}{1} = \frac{116000}{88} = \$7.58\frac{1013}{88}$, Ans.
37. $87\frac{1}{2} = \frac{612}{2}$; $\frac{612}{2} \times \frac{1}{612} = 7$; $14\frac{7}{10} \times 7 = \102.90 , Ans.

38. $4375 \times \frac{1}{5} = 87500 = 50$; $50 \times 10 = \$500$, Ans.
39. $500 \div 10 = 50$; $\frac{4375}{500} = \frac{7}{10}$ acre, Ans.
40. $71.87 \div 9 = 7.98\frac{1}{3} = 7\frac{29}{30}$; $\frac{1}{3} \times 7\frac{29}{30} = \frac{29}{90} = \$4.56\frac{2}{3}$, Ans.
41. $4.56\frac{2}{3} \times \frac{1}{3} = 7.98\frac{1}{3}$; $7.98\frac{1}{3} \times 9 = \71.87 , Ans.
42. $111 \times 19 = 2109$; $2109 \div 47 = 44\frac{1}{47}$ days, Ans.
43. $44\frac{1}{47} \times 47 = 2109$; $2109 \div 19 = 111$ days, Ans.
44. $\frac{230}{4} \times \frac{1}{4} = 10\frac{1}{4} = 25.30$; $25.30 \times 17 = \$430.10$, Ans.
45. $430.10 \div 17 = 25.30$; $\frac{1}{11} \times 25.30 = \frac{2530}{11} = \9.20 , Ans.
46. $715 \times 16 = 11440 = 16.34\frac{2}{7}$; $16.34\frac{2}{7} \times 7 = \114.40 , Ans.
47. $\frac{7}{8} \times 11440 = 11440 = \7.15 , Ans.
48. $19\frac{1}{2} = 19\frac{1}{2}$; $87\frac{1}{2} = 87\frac{1}{2}$; $\frac{136}{7} \times \frac{7}{612} = \frac{136}{612} = \frac{34}{153}$;

$$\left[\frac{34}{153} \times \frac{60}{1} \times \frac{7}{1} = \frac{14280}{153} = 93\frac{1}{3} \text{bu.}, \text{ Ans.} \right]$$
49. Smith will reap $\frac{1}{80}$ of the field in an hour; his wife will reap $\frac{1}{160}$ of the field in an hour. They will both reap $\frac{1}{80} + \frac{1}{160} = \frac{3}{160}$ of the field, in an hour. Then they will reap the whole field in $2\frac{1}{3}$ hours $= 45\frac{1}{3}$; $45\frac{1}{3} \div 8 = 5\frac{3}{4}$ days, Ans.

DECIMAL FRACTIONS.

(ART. 266, p. 199.)

1. Fifty-six thousandths.
2. One thousand three ten thousandths.
3. Two thousand seven hundred eighty-six ten thousandths.
4. Sixteen thousand three hundred two hundred thousandths.
5. Nine hundred seventy-five thousandths.
6. One, and six hundred thirty one thousandths.
7. Forty-eight, and seven hundredths.

8. One, and three hundred fifteen thousandths.
 9. Five, and six thousand one ten thousandths.
 10. Eighty-seven, and six ten thousandths.
 11. One, and seven millionths.
 12. Five, and one hundred one thousand sixteen millionths.
 13. One, and three hundred twenty-seven millionths.
 14. One millionth.
 15. Sixteen, and seven billionths.
- | | | | | | |
|-----|---------|-----|----------------------------|-----|-----------|
| 16. | .13 | 21. | .001031 | 26. | 465.14 |
| 17. | .006 | 22. | 7.0017 | 27. | 93.07 |
| 18. | .0019 | 23. | 333.003 | 28. | 24.000009 |
| 19. | .00406 | 24. | 1.000001 | 29. | 221.00009 |
| 20. | .000001 | 25. | 325.7 | 30. | 49000.049 |
| 31. | | | 79002000.105 | | |
| 32. | | | 69015.00015 | | |
| 33. | | | 80000.0083 | | |
| 34. | | | 9000019019.19 | | |
| 35. | | | 27.927 | | |
| 36. | | | 4900000000000.000000000001 | | |
| 37. | | | 21.0001 | | |
| 38. | | | 87000.000087 | | |
| 39. | | | 99099.000009009 | | |
| 40. | | | 17.0117 | | |
| 41. | | | 33.33 | | |
| 42. | | | 47000.0000029 | | |
| 43. | | | 15.04007 | | |
| 44. | | | 11000.11 | | |
| 45. | | | 17.0000000000000081 | | |
| 46. | | | 9.0000000000057 | | |
| 47. | | | 69000.349 | | |

ADDITION OF DECIMALS.

(ART. 268, p. 201.)

- | | | | |
|----|--------------|----|-----------|
| 2. | 7564.0052656 | 4. | 7234.0968 |
| 3. | 2071.449495 | 5. | 6913.5477 |

(6.)	(7.)	(8.)
\$ 15.06	\$ 137.50	23000010.
107.09	55.63	1000.00005
1.625	1.375	27.000019
93.765	.875	7.5
Ans. \$ 217.54	Ans. \$ 195.38	Ans. 23001044.500069

(9.)	(10.)	(11.)
59.059	25.000007	3.75
25000.0025	145.643	11.7
5.000005	175.89	16.125
205.05	17.00348	Ans. 31.575
Ans. 25269.111505	Ans. 363.536487	

(12.)	(13.)
73.29	209000.000046
87.047	98207.0015
3005.0106	15.08
28.03	.0049
29000.005	Ans. 307222.086446
Ans. 32193.3826	

SUBTRACTION OF DECIMALS.

(ART. 270, p. 202.)

6. 9.49989 | 7. 88.9429 | 8. .001

(9.)	(10.)	(11.)
97.7	315.0027	29004005.
27.028	115.07	29000.
Ans. 70.672	Ans. 199.9327	349200.00024
		378200.00024
		Ans. 28625804.99976

(12.)	(13.)	(14.)
1000000.	\$ 19.	\$ 400.
.000001	1.375	316.875
Ans. 999999.999999	Ans. \$ 17.625	Ans. \$ 83.125

	(16.) 1000.
	<u>93.45</u>
(15.)	124.
19000000.	• 244.285
.000000019	<u>216.136</u>
Ans. 18999999.999999981	<u>677.871</u>
	Ans. 322.129

MULTIPLICATION OF DECIMALS.

	(ART. 271, p. 203.)	10.	33.5175
3.	.438496	11.	41448651.06
4.	.0949416	12.	.000019737
5.	.003721061	13.	\$ 153.525
6.	26137.65	14.	\$ 18.4375
7.	152.2756	15.	\$ 46.95
8.	43910.073	16.	\$ 149.5125
9.	.00000081	17.	3616.175

CONTRACTIONS IN MULTIPLICATION OF DECIMALS.

1.	(ART. 272, p. 204.) 131634.	3.	1.
2.	347890.	4.	\$ 7000.

	(ART. 273, p. 206.)	(3.)
(2.) 325.701428		56.7534916
<u>3938127</u>		<u>8296735</u>
227990		<u>28376746</u>
6514	(4.) 843.7527	1702605
326	5714368	397274
261	<u>6750022</u>	34052
10	506252	5108
3	25313	114
Ans. 235.104	3375	<u>45</u>
	84	Ans. 305.15944
	59	
	4	
	<u>Ans. 7285109.</u>	

DIVISION OF DECIMALS.

(ART. 274, p. 207.)		12.	312.43
3.	321.2	13.	31243000000.
4.	.758	14.	.31243
5.	.561	15.	.31243
6.	13.861	16.	32000.
7.	749.084	17.	.000032
8.	3124.3	18.	.5403+
9.	.31243	19.	140yd.
10.	312430000.	20.	\$ 50.
11.	.000031243	21.	64753000000.

CONTRACTIONS IN DIVISION OF DECIMALS.

(ART. 275, p. 208.)		6.	.070461
1.	3.1675	7.	.70460
2.	9.1605	8.	.0000070460.
3.	.000070461	9.	.0001965.
4.	.00070461	10.	\$ 3.50
5.	.0070461	11.	\$ 0.1025

2.	(ART. 276, p. 209.) 4.95445	4.	8.7938+
3.	426.1043	5.	9876.54321

REDUCTION OF DECIMALS.

(ART. 277, p. 210.)		7.	$96\frac{24}{1000} = 96\frac{3}{125}$
2.	$\frac{875}{1000} = \frac{7}{8}$	8.	$163\frac{4}{100} = 163\frac{1}{25}$
3.	$\frac{9375}{10000} = \frac{15}{16}$	9.	$1001\frac{4375}{10000} = 1001\frac{7}{16}$
4.	$\frac{8125}{100000} = \frac{13}{160}$	10.	$1457\frac{322}{1000} = 1457\frac{11}{250}$
5.	$\frac{75}{100000} = \frac{3}{4000}$	11.	$19678\frac{36}{100} = 19678\frac{9}{25}$
6.	$31\frac{75}{100} = 31\frac{3}{4}$	12.	$9163\frac{8755}{10000} = 9163\frac{1751}{2000}$

2.	(ART. 278, p. 211.) .625	4.	.09375
3.	.5	5.	.076932

6.	19.125	11.	\$4.8125
7.	\$315.875	12.	\$60.1875
8.	1163.75	13.	.51
9.	.625	14.	2.9875
10.	.370625		

(ART. 279, p. 213.)

$$\begin{array}{r} (2.) \\ 20 \overline{) 9.00} \\ \underline{.45, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (8.) \\ 25 \overline{) 14.} \\ \underline{4} \quad 3.56 \\ 20 \overline{) 15.89} \\ \underline{.7945, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (4.) \\ 16 \overline{) 12.} \\ 16 \overline{) 8.75} \\ 25 \overline{) 21.546875} \\ 4 \overline{) 2.861875} \\ \underline{.71546875, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (5.) \\ 4 \overline{) 3.00} \\ 4 \overline{) 1.7500} \\ \underline{.4375, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (6.) \\ 12 \overline{) 9.00} \\ 3 \overline{) 2.75} \\ 51 \overline{) 2.91666666} \\ 40 \overline{) 35.53030303} \\ 8 \overline{) 5.88825757} \\ \underline{.73603219+, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (7.) \\ 2 \overline{) 1.} \\ 4 \overline{) 2.5} \\ 63 \overline{) 3.625} \\ \underline{.0575396+, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (8.) \\ 2 \overline{) 1.0} \\ 8 \overline{) 0.50} \\ 4 \overline{) 0.0625} \\ \underline{0.015625, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (9.) \\ 40 \overline{) 16.} \\ 4 \overline{) 2.4} \\ \underline{.6, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (10.) \\ 40 \overline{) 175} \\ \underline{4.375, \text{ Ans.}} \end{array}$$

$$\begin{array}{r} (11.) \\ 4 \overline{) 3.755} \\ \underline{\text{Ans. .93875}} \end{array}$$

$$\begin{array}{r} (12.) \\ 10 \overline{) 6.0} \\ 60 \overline{) 34.6} \\ 60 \overline{) 25.576} \\ \underline{\text{Ans., .4262+}} \end{array}$$

(13.)

$$\begin{array}{r} 25 \overline{) 20.0} \\ 4 \overline{) 2.8} \\ \hline 243.7, \text{ Ans.} \end{array}$$

(14.)

$$\begin{array}{r} 2 \overline{) 1.5} \\ 4 \overline{) 2.75} \\ \hline 156.6875, \text{ Ans.} \end{array}$$

(15.)

$$\begin{array}{r} 4 \overline{) 3.} \\ 12 \overline{) 11.75} \\ 20 \overline{) 19.97916+} \\ \hline .9989583+, \text{ Ans.} \end{array} \quad \begin{array}{r} 4 \overline{) 1.0} \\ 12 \overline{) 9.25} \\ 20 \overline{) 16.77083+} \\ \hline .8385416+, \text{ Ans.} \end{array} \quad \begin{array}{r} 4 \overline{) 2.} \\ 12 \overline{) 5.5} \\ 20 \overline{) 17.4583+} \\ \hline .8729166+, \\ \text{[Ans.} \end{array}$$

$$\begin{array}{r} .9989583+ \\ .8385416+ \\ .8729166+ \\ \hline 2.710416+ \end{array}$$

(ART. 280, p. 214.)

2. $.625 \times 12 = 7.5\text{d.}$; $.5 \times 4 = 2.0$; $7\frac{1}{2}\text{d.}$, Ans.
3. $.6725 \times 4 = 2.69$; $.69 \times 25 = 17.25$; $25 \times 16 = 4$;
2qr. 17lb. 4oz., Ans.
4. $.9375 \times 4 = 3.75$; $.75 \times 4 = 3$; 3qr. 3na., Ans.
5. $.7895 \times 8 = 6.316$; $.316 \times 40 = 12.64$; $.64 \times 16\frac{1}{2} =$
 10.56 ; $.56 \times 12 = 6.72$; $72 = 1\frac{2}{5}$; 6fur. 12rd. 10ft.
 $6\frac{1}{2}\frac{2}{3}\text{in.}$, Ans.
6. $.9378 \times 4 = 3.7512$; $.7512 \times 40 = 30.048$; $.048 \times$
 $272\frac{1}{4} = 13.068$; $.068 \times 144 = 9.792$; $.792 = 1\frac{2}{5}$;
3R. 30p. 13ft. $9\frac{2}{5}\frac{2}{3}\text{in.}$, Ans.
7. $.5615 \times 63 = 35.3745$; $.3745 \times 4 = 1.498$; $.498 \times 2$
 $= .996$; $.996 \times 4 = 3.984$; $.984 = 1\frac{2}{3}$; 35gal. 1qt.
Opt. $3\frac{1}{2}\frac{2}{3}\text{gi.}$, Ans.
8. $.367 \times 365\frac{1}{4} = 134.046\frac{3}{4}$; $.046\frac{3}{4} \times 24 = 1.122$; $.122 \times$
 $60 = 7.32$; $.32 \times 60 = 19.2$; $.2 = \frac{1}{5}$; 134da. 1h. 7m.
 $19\frac{1}{5}\text{sec.}$, Ans.

9. $.6923828125 \times 4 = 2.76953125$; $.76953125 \times 25 = 19.23828125$; $.23828125 \times 16 = 3.8125$; $.8125 \times 16 = 13$; 2qr. 19lb. 3oz. 13dr., Ans.
10. $.015625 \times 4 = .0625$; $.0625 \times 8 = .5$; $.5 \times 2 = 1$; 1p., Ans.
11. $.55 \times 5 = 2.75$; $.75 \times 4 = 3$; 2qr. 3na., Ans.
12. $.6 \times 4 = 2.4$; $.4 \times 40 = 16$; 2R. 16p., Ans.

MISCELLANEOUS EXERCISES.

$$\begin{array}{r}
 (1.) \\
 25 \overline{) 18.} \\
 4 \overline{) 2.72} \\
 \underline{7.68} \\
 11.75 \\
 \underline{3840} \\
 5376 \\
 768 \\
 768 \\
 \hline
 \$90.24, \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (2.) \\
 25 \overline{) 14.} \\
 4 \overline{) 3.56} \\
 \underline{19.89} \\
 9.25 \\
 \underline{9945} \\
 3978 \\
 17901 \\
 \hline
 \$183.9825 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (3.) \\
 40 \overline{) 15.000} \\
 4 \overline{) 2.37500} \\
 \underline{39.59375} \\
 87.375 \\
 \underline{19796875} \\
 27715625 \\
 11878125 \\
 27715625 \\
 \underline{31675000} \\
 3459.5030625 = \\
 \$3459.503\frac{1}{2} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (4.) \\
 40 \overline{) 15.000} \\
 8 \overline{) 3.375000} \\
 \underline{87.421875} \\
 578.75 \\
 \underline{437109375} \\
 611953125 \\
 699375000 \\
 611953125 \\
 \underline{437109375} \\
 50595.41015625 = \\
 \$50595.41\frac{1}{8} \text{ Ans.}
 \end{array}$$

(5.)

$$\begin{array}{r}
 12 \overline{) 9.00} \quad 12 \overline{) 3.50} \\
 \underline{18.75} \quad \underline{2.29\frac{1}{2}} \\
 2.29\frac{1}{2} \\
 \underline{16875} \\
 3750 \\
 3750 \\
 \underline{3125} \\
 42.96875 \\
 \underline{.053} \\
 12890625 \\
 21484375 \\
 \hline
 2.27734375 = \$2.277\frac{1}{2}, \text{ Ans.}
 \end{array}$$

(6.)

$$\begin{array}{r}
 12 \overline{) 7.168} \\
 \underline{1.597\frac{1}{2}} \\
 6.5 \\
 \underline{7985} \\
 9582 \\
 \underline{21\frac{1}{2}} \\
 10.3826\frac{1}{2} \\
 \underline{12} \\
 4.5920 \\
 10\text{ft. } 4.592\text{in.}, \text{ Ans.}
 \end{array}$$

(7.)

$$\begin{array}{r}
 29\frac{1}{2} = 29.5 \\
 4.316 \overline{) 29.500} (6\text{h. } 50\text{m. } 6+\text{sec.}, \\
 \underline{25896} \quad [\text{Ans.}] \\
 3604 \\
 \underline{60} \\
 4.316 \overline{) 216240} (50\text{m.} \\
 \underline{21580} \\
 440 \\
 \underline{60} \\
 4.316 \overline{) 26400} (6+\text{sec.} \\
 \underline{25896}
 \end{array}$$

(8.)

$$\begin{array}{r}
 4 \overline{) 2.0} \\
 4 \overline{) 1.500} \\
 \underline{5.375} \\
 5.625 \\
 \underline{26875} \\
 10750 \\
 \underline{32250} \\
 26875 \\
 \underline{30.234375} = \\
 \$30.234\frac{1}{2}, \text{ Ans.}
 \end{array}$$

(9.)

$$\begin{array}{r}
 25 \overline{) 7.00} \\
 4 \overline{) 3.28} \\
 \hline
 4.82 \\
 17 \\
 \hline
 81.94 \\
 5.875 \\
 \hline
 40970 \\
 57358 \\
 65552 \\
 \hline
 40970 \\
 481.39750 = \\
 \$ 481.39\frac{3}{4}, \text{ Ans.}
 \end{array}$$

(10.)

$$\begin{array}{r}
 40 \overline{) 25.000} \\
 4 \overline{) 3.625} \\
 \hline
 176.90625 \\
 75.375 \\
 \hline
 88453125 \\
 123884375 \\
 53071875 \\
 88453125 \\
 \hline
 123884375 \\
 13334.30859375 = \\
 \$13334.308\frac{1}{2}, \text{ Ans.}
 \end{array}$$

(12.)

$$\begin{array}{r}
 19\text{ft. 3in.} = 19.25 \\
 15\text{ft. 9in.} = 15.75 \\
 \hline
 9625 \\
 13475 \\
 9625 \\
 \hline
 1925 \\
 303.1875 \\
 144 \\
 \hline
 7500 \\
 7500 \\
 \hline
 1875 \\
 27.0000 \\
 303\text{ft. 27in., Ans.}
 \end{array}$$

(11.)

$$\begin{array}{r}
 17.625 \\
 12.75 \\
 \hline
 88125 \\
 123375 \\
 35250 \\
 \hline
 17625 \\
 224.71875 = \\
 \$ 224.718\frac{3}{4}, \text{ Ans.}
 \end{array}$$

13. $14\text{ft. 6in.} = 14.5$; $12\text{ft. 6in.} = 12.5$; $8\text{ft. 9in.} = 8.75$;
 $14.5 + 12.5 = 27$; $27 \times 2 = 54$; $8.75 \times 54 = 472.5$;
 $472.5 \div 9 = 52\frac{1}{2}$ yards, Ans.

$$14. 10\text{ft. } 7\text{in.} = 127\text{in.}; 5\text{ft. } 10\text{in.} = 70\text{in.}, 127 \times 70 \times 4 = 35560; 35560 \div 144 = 246\frac{1}{3}\text{ft., Ans.}$$

(15.)

$$\begin{array}{r} 4 \overline{) 3.00} \\ 4 \overline{) 2.75} \\ \hline 4.6875 \end{array}$$

$$112.5 \div 4.6875 = 24, \text{ Ans.}$$

(16.)

$$\begin{array}{r} 2 \overline{) 1.0} \\ 4 \overline{) 2.500} \\ \hline 1.625 \end{array}$$

$$1.82 \div 1.625 = 1.12$$

$$\$1.12 \times 63 = \$70.56, \text{ Ans.}$$

$$17. \$15.06 \div 125.5 = \$.12, \text{ Ans.}$$

$$18. \begin{array}{r} 4 \overline{) 3} \\ 17.75 \times 35.75 = \$634.562\frac{1}{2}, \text{ Ans.} \end{array}$$

$$19. \$87.25 \times 7\frac{1}{3} = \$675.84\frac{2}{3}, \text{ Ans.}$$

$$20. 34\text{ft. } 9\text{in.} = 34.75; 1\text{ft. } 3\text{in.} = 1.25; 1\text{ft. } 6\text{in.} = 1.5; 34.75 \times 1.25 \times 1.5 = 65.15625\text{ft., Ans.}$$

(21.)

$$\begin{array}{r} 4 \overline{) 1.00} \\ 18.25 \end{array}$$

$$36.50 \div 18.25 = \$2.00, \text{ Ans.}$$

(22.)

$$\$477.72 \div 9 = \$53.08, \text{ Ans.}$$

(23.)

$$5.375$$

$$1.78$$

$$1.1875$$

$$1.125$$

$$1.275$$

$$2.625$$

$$13.3675 =$$

$$\$13.367\frac{1}{2}, \text{ Ans.}$$

(24.)

$$2\frac{3}{8} = 2.375; 3\frac{1}{4} = 3.875; 1\frac{1}{8} = 1.0625;$$

$$2.375 + 3.875 + 1.0625 = 7.3125$$

$$17.625$$

$$365625$$

$$146250$$

$$438750$$

$$511875$$

$$73125$$

$$128.8828125 =$$

$$\$128.882\frac{1}{8}, \text{ Ans.}$$

(25.)

$$15.25 \times 8.4 \times 10 = 1281;$$

$$1281 \times 1728 = 2213568;$$

$$2213568 \div 231 = 9682\frac{6}{11}\text{gal.}$$

$$9682\frac{6}{11} \div 63 = 152\text{hhd. } 6\frac{6}{11}\text{gal., Ans.}$$

(26.)

$$\begin{array}{r} 25 \overline{) 7.00} \\ 4 \overline{) 2.28} \\ \underline{3.57} \\ 13.625 \\ \underline{1785} \\ 714 \\ 2142 \\ \underline{1071} \\ 357 \\ \underline{48.64125} = \end{array}$$

$$\$ 48.641\frac{1}{4}, \text{ Ans.}$$

(27.)

$$\begin{array}{r} 40 \overline{) 35.000} \\ 4 \overline{) 3.87500} \\ \underline{37.96875} \\ 125.75 \\ \underline{18984375} \\ 26578125 \\ \underline{18984375} \\ 7593750 \\ \underline{3796875} \\ 4774.5703125 = \end{array}$$

$$\$ 4774.570\frac{1}{16}, \text{ Ans.}$$

(28.)

$$\begin{array}{r} 25 \overline{) 21.00} \\ 4 \overline{) 2.84} \\ \underline{17.71} \\ 11.25 \\ \underline{8855} \\ 3542 \\ 1771 \\ \underline{1771} \end{array}$$

$$\$ 199.23\frac{3}{4}, \text{ Ans.}$$

(30.) 97.625

$$\begin{array}{r} 7\frac{4}{8} \\ \underline{683375} \\ 75930\frac{3}{8} \\ \underline{759.305\frac{3}{8}} = \end{array}$$

$$\$ 759.305\frac{3}{8}, \text{ Ans.}$$

(29.)

$$\begin{array}{r} 9.375 \\ \underline{3.37} \\ 65625 \\ 28125 \\ \underline{28125} \\ 31.59375 \\ \underline{7.75} \end{array}$$

$$15796875$$

$$22115625$$

$$22115625$$

$$\underline{244.8315625} =$$

$$\$ 244.851\frac{1}{16}, \text{ Ans.}$$

(31.)	(32.)	(33.)
4 3.00	25 3.50	40 15.000
4 3.7500	4 1.14	8 3.375000
<u>7.9375</u>	20 15.285	<u>17.421875</u>
4.75	<u>27.76425</u>	<u>1725.875</u>
<u>396875</u>	<u>183.62</u>	<u>87109375</u>
555625	5552850	121953125
<u>317500</u>	16658550	139375000
37.703125 =	8329275	87109375
\$ 37.703 $\frac{1}{8}$, Ans.	22211400	34843750
	<u>2776425</u>	121953125
	5098.0715850 =	<u>17421875</u>
	\$ 5098.071 $\frac{11}{16}$, Ans.	30067.978515625 =
		\$ 30067.978 $\frac{3}{4}$, Ans.

(34.)	(35.)
40 15.	25 12.5
4 3.375	4 2.5
A. 17.84375	20 15.625
\$ 624.53125 \div 17.84375 =	<u>19.78125</u>
\$ 35, Ans.	\$ 494.53125 \div 19.78125 =
	\$ 24.999 $\frac{172}{3287}$, Ans.

(36.) 40 1004.75
A. 25.11875
<u>4</u>
R. .47500
<u>40</u>
p. 19.00000
Ans. 25A. 0R. 19p.

37. $20.5 \times 12.75 \times 7.6 = 1986.45$ cubic feet;
 $1986.45 \div 128 = 15$ cords $66\frac{9}{16}$ feet.

38. $31 \times .08\frac{1}{4} = \$2.55\frac{1}{4}$; $\$2.75 \times 7\frac{1}{2} = \$20.62\frac{1}{2}$; $\$20.62\frac{1}{2}$
 $+ \$2.55\frac{1}{4} = \$23.18\frac{1}{4}$; $23.18\frac{1}{4} \div .62\frac{1}{2} = 37\frac{23}{25}$ yd.,
[Ans.]

39. $3.50 \div 40 = 8\frac{3}{4}$ bushels.

$\frac{20}{20}, \frac{10}{20} = \frac{20}{20}, \frac{10}{20}; \frac{20}{20} + \frac{10}{20} = \frac{30}{20}; \frac{30}{20} - \frac{10}{20} = \frac{20}{20} \text{ bu.}; 8\frac{3}{4} \div 5 = 1\frac{3}{4} \text{ bushels.}$

$1\frac{3}{4} \text{ bu.} \times 6 = 10\frac{1}{2} \text{ bu.}; 1\frac{3}{4} \text{ bu.} \times 9 = 15\frac{3}{4} \text{ bu.}$

Ans. $8\frac{3}{4}$ bushels of oats; $10\frac{1}{2}$ bushels corn; $15\frac{3}{4}$ bushels wheat.

Proof: $\$0.75 \times 10\frac{1}{2} = \$7.87,5; \$2 \times 15\frac{3}{4} = \$31.50;$
 $\$3.50 + \$7.87,5 + \$31.50 = \$42.87,5.$

40. $250.35 \div 2 = 125.175; 125.175 \div 8 = 15.646875;$
 $15.646875 + 1 = 16.646875 \text{ times, Ans.}$

41. $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}; \frac{5}{6} - \frac{1}{2} = \frac{1}{3}; \frac{5}{12} \times \frac{1}{5} = \frac{1}{12}, \text{ part for Mary.}$

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}; \frac{5}{6} - \frac{1}{2} = \frac{1}{3} = \frac{1}{3}, \text{ part for wife.}$

$\frac{1}{4} - \frac{1}{12} = \frac{1}{6} = \$2243.26, \text{ part Mary received less than James.}$

Therefore $\$2243.26 \times 6 = \$13459.56, \text{ amount.}$

$\$13459.56 \times \frac{1}{4} = \$3364.89, \text{ James' part.}$

$\$13459.56 \times \frac{1}{3} = \$4486.52, \text{ William's part.}$

$\$13459.56 \times \frac{1}{12} = \$1121.63, \text{ Mary's part.}$

$\$13459.56 \times \frac{1}{6} = \$4486.52, \text{ wife's part.}$

INFINITE OR CIRCULATING DECIMALS.

(ART. 296, p. 220.)

3. $.6 = \frac{6}{10} = \frac{3}{5}, \text{ Ans.}$

4. $1.62 = 1.621 = 1\frac{621}{1000} = 1\frac{27}{400}, \text{ Ans.}$

5. $.53 = \frac{53}{100} = \frac{53}{100}, \text{ Ans.}$

6. $.769230 = \frac{769230}{1000000} = \frac{76923}{100000}, \text{ Ans.}$

7. $.5925 = \frac{5925}{10000} \times \frac{10000}{10000} = \frac{5925}{10000} = \frac{237}{400}, \text{ Ans.}$

8. $31.62 = 31.621 = 31\frac{621}{1000} = 31\frac{27}{400}, \text{ Ans.}$

9. $.008497133 = \frac{8497133}{1000000000} = \frac{8497133}{1000000000} = \frac{8497133}{1000000000}, \text{ Ans.}$

(ART. 297, p. 222.)

3. $\frac{1}{11} = .\dot{0}\dot{9}$, Ans.
 4. $\frac{13}{1320} = \frac{13}{440} = 13 \div 440 = .029\dot{5}4$, Ans.
 5. $\frac{107}{253} = 107 \div 253 = .422924901185770750988\dot{1}$, Ans.
 6. $13\frac{17}{45}$; $17 \div 45 = .3\dot{7}$; $13.3\dot{7}$, Ans.
 7. $\frac{83}{19536} = \frac{83}{9768}$; $83 \div 9768 = .00849713\dot{3}$, Ans.
 8. $2 \div 29 = .06827586206896551724137931034\dot{8}$, Ans.

TRANSFORMATION OF REPETENDS.

(ART. 302, p. 223.)

(2.)	(3.)
3.67i = 3.67167167167i	1.52 = 1.525252
1.007i = 1.00710071007i	8.7156 = 8.71567i
8.52 = 8.525252525252	3.567 = 3.567777
7.616325 = 7.616325616325	1.378 = 1.378787
(4.)	(5.)
.0007 = .000707070	.3i23 = .3i23123
.1414i4 = .141414414	3.27 = 3.2727272
887.i = 887.111i1111i	5.02 = 5.0222222
(6.)	
17.0884 = 17.08848484848484848484848484848484	
1563.0929 = 1563.09299299299299299299299299299299	
15.12345 = 15.12345123451234512345123451234512	

ADDITION OF CIRCULATING DECIMALS.

(ART. 303, p. 224.)

(2.)	(3.)
3.5555555	$\frac{1}{3} = \dot{3} = .33333\dot{3}$
7.6516516	$\frac{1}{7} = .14285\dot{7} = .14285\dot{7}$
1.7657657	$\frac{1}{8} = \dot{1} = .11111\dot{1}$
6.1737373	Ans. <u>.587301</u>
51.7777777	
3.7000000	
27.6316316	
1.0030030	
Ans. <u>103.2591227</u>	

(4.)

$$27.5\dot{6} = 27.5\dot{6}7567567567\dot{5}$$

$$5.\dot{6}3\dot{2} = 5.6\dot{3}2632632632\dot{6}$$

$$6.\dot{7} = 6.7\dot{7}777777777\dot{7}$$

$$16.3\dot{5}\dot{6} = 16.3\dot{5}656565656\dot{5}$$

$$.7\dot{1} = .7\dot{1}111111111\dot{1}$$

$$6.\dot{1}234 = 6.1\dot{2}3412341234\dot{1}$$

$$\text{Ans. } 63.1\dot{6}906708688\dot{8}$$

(5.)

$$.165002 = .165002$$

$$31.64 = 31.64$$

$$1.\dot{6} = 1.666666\dot{6}$$

$$.3463\dot{4} = .346344\dot{4}$$

13.

13.

$$\text{Ans. } 46.818013\dot{1}$$

(6.)

$$.\dot{8}7 = .87878\dot{7}$$

$$.\dot{8} = .88888\dot{8}$$

$$.\dot{8}7\dot{6} = .87687\dot{6}$$

$$\text{Ans. } 2.\dot{6}4455\dot{3}$$

(7.)

$$.\dot{3} = .333333333\dot{3}$$

$$.45 = .45$$

$$.4\dot{5} = .45454545\dot{4}$$

$$.3\dot{5}1 = .351351351\dot{3}$$

$$.6468 = .6468$$

$$.646\dot{8} = .64688888\dot{8}$$

$$.646\dot{8} = .64686868\dot{8}$$

$$.646\dot{8} = .646846846\dot{8}$$

$$\text{Ans. } 4.176634561\dot{8}$$

(8.)

$$1.25 = 1.25000000\dot{0}$$

$$3.4 = 3.44444444\dot{4}$$

$$.63\dot{7} = .637\dot{3}7373\dot{7}$$

$$7.88\dot{5} = 7.88555555\dot{5}$$

$$7.875 = 7.87500000\dot{0}$$

$$7.87\dot{5} = 7.87587587\dot{5}$$

$$11.\dot{1} = 11.11111111\dot{1}$$

$$\text{Ans. } 40.07936072\dot{2}$$

(9.)

$$131.613 = 131.613$$

$$15.00\dot{1} = 15.001\dot{1}$$

$$67.13\dot{4} = 67.134\dot{4}$$

$$1000.6\dot{3} = 1000.633\dot{3}$$

$$\text{Ans. } 1214.381\dot{8}$$

(10.)

$$5.1634\dot{5} = 5.\dot{1}634516345163451634\dot{5}$$

$$8.\dot{6}38\dot{1} = 8.\dot{6}381638163816381638\dot{1}$$

$$3.7\dot{5} = 3.\dot{7}5757575757575757\dot{5}$$

$$\text{Ans. } 17.\dot{5}591912084737409030\dot{2}$$

SUBTRACTION OF CIRCULATING DECIMALS.

(ART. 304, p. 225.)

(2.)

$$7.\dot{1} = 7.\dot{1}\dot{1}$$

$$5.0\dot{2} = 5.0\dot{2}$$

$$\text{Ans. } 2.0\dot{8}$$

(3.)

$$31\dot{5}.8\dot{7} = 315.\dot{8}75875\dot{8}7587\dot{5}$$

$$78.\dot{0}37\dot{8} = 78.\dot{0}3780378037\dot{8}$$

$$\text{Ans. } 237.\dot{8}3807209549\dot{7}$$

(4.)

$$\frac{2}{9} = .\dot{2} = .\dot{2}2222\dot{2}$$

$$\frac{7}{9} = .\dot{7}7777\dot{7} = .\dot{7}7777\dot{7}$$

$$\text{Ans. } .\dot{0}7936\dot{5}$$

(5.)

$$16.134\dot{7} = 16.134\dot{7}$$

$$11.088\dot{4} = 11.088\dot{4}$$

$$\text{Ans. } 5.046\dot{2}$$

(6.)

$$18.167\dot{8} = 18.167\dot{8}$$

$$3.2\dot{7} = 3.27\dot{2}\dot{7}$$

$$\text{Ans. } 14.895\dot{1}$$

(7.)

$$3.12\dot{3} = 3.12312\dot{3}$$

$$.7\dot{1} = .71717\dot{1}$$

$$\text{Ans. } 2.40595\dot{1}$$

(8.)

$$\frac{3}{7} = .42857\dot{1} = .42857\dot{1}$$

$$\frac{1}{7} = .1\dot{4}2857 = .142857\dot{1}$$

$$\text{Ans. } .\dot{2}4675\dot{3}$$

(9.)

$$\frac{4}{9} = .\dot{4} = .44444\dot{4}$$

$$\frac{2}{9} = .\dot{2}28571\dot{4} = .228571\dot{4}$$

$$\text{Ans. } .\dot{1}5873\dot{0}$$

(10.)

$$\frac{2}{17} = .\dot{1}1764705882\dot{3}$$

$$\frac{3}{17} = .\dot{1}76470588235294\dot{1}$$

$$\text{Ans. } .\dot{1}76470588235294\dot{1}$$

(11.)

$$\begin{aligned} 5.12345 &= 5.1234512345123451234512345123451 \\ 2.3523456 &= 2.3523456523456523456523456523456 \\ \text{Ans. } 2.7711055821666927777988888599994 \end{aligned}$$

MULTIPLICATION OF CIRCULATING DECIMALS.

(ART. 305, p. 226.)

(2.)

$$\begin{array}{r} 87.3258\bar{6} \\ \quad 437 \\ \hline 61128106 \\ \cdot 26197759_7 \\ 34980346_{34} \\ \hline \text{Ans. } 381.6140\bar{3}3\bar{8} \end{array}$$

$$\begin{aligned} 3. \quad 582.\bar{8}47 &= 582\frac{347}{999} = \frac{581765}{999}; .0\bar{8} = \frac{8}{90} = \frac{4}{45}; \frac{581765}{999} \\ &\times \frac{4}{45} = \frac{581765}{59970} = \frac{116353}{11994}; \frac{116353}{11994} = 19.411578244\bar{9}, \\ &\text{Ans.} \end{aligned}$$

$$\begin{aligned} 4. \quad 3.1\bar{4}5 &= 3.1\frac{45}{99} = 3.1\frac{5}{11} = \frac{348}{11}; 4.29\bar{7} = 4\frac{297}{999} = 4\frac{1}{3} = \\ &\frac{158}{37}; \frac{348}{11} \times \frac{158}{37} = \frac{55014}{4070} = 13.516953\bar{3}, \text{ Ans.} \end{aligned}$$

(5.)

$$\begin{array}{r} .28571\bar{4} \\ \quad 21 \\ \hline 285714 \\ 571428_5 \\ \hline \text{Ans. } 8.000000 \end{array}$$

(6.)

$$\begin{array}{r} .46160714285\bar{7} \\ \quad 20 \\ \hline 9.23214285714\bar{2} \\ \quad 4 \\ \hline 0.92857142857\bar{1} \\ \quad 25 \end{array}$$

(7.) .284931506

$$\begin{array}{r} 365 \\ \hline 1424657534 \\ 1709589041_0 \\ 854794520_{54} \\ \hline \text{Ans. } 104.00000000 \end{array}$$

4642857142855

18571428571428

23.214285714283

Ans. 9cwt. 0qr. 23+lb.

DIVISION OF CIRCULATING DECIMALS.

(ART. 306, p. 226.)

2. $345.\dot{8} = 345\frac{8}{10}$; $\dot{6} = \frac{6}{10} = \frac{3}{5}$; $345\frac{8}{10} \div \frac{3}{5} = 518\frac{8}{3} = 518.8\dot{3}$, Ans.
3. $234.\dot{6} = 234\frac{6}{10}$; $\dot{7} = \frac{7}{10}$; $234\frac{6}{10} \div \frac{7}{10} = 301\frac{6}{7} = 301.71428\dot{5}$, Ans.
4. $13.516953\dot{3} = 13\frac{5}{10} + \frac{169533}{999999} = 13\frac{5}{10} + \frac{169533}{999999}$; $3.14\dot{5} = 3\frac{1}{10} + \frac{45}{999} = 3\frac{1140}{9990}$; $13\frac{5}{10} + \frac{169533}{999999} \div 3\frac{1140}{9990} = 4.297$, Ans.
5. $381.614033\dot{8} = 381\frac{6140}{10000} + \frac{338}{999999} = 381\frac{6140}{10000} + \frac{338}{999999}$; $4.37 = 4\frac{37}{100}$; $381\frac{6140}{10000} + \frac{338}{999999} \div 4\frac{37}{100} = 87.32586$, Ans.
6. $.42857\dot{1} = \frac{428571}{999999}$; $.625 = \frac{5}{8}$; $\frac{428571}{999999} \div \frac{5}{8} = .685714\dot{2}$, Ans.
7. $2.\dot{3}70 = 2\frac{370}{1000} = 2\frac{368}{999}$; $4.92307\dot{6} = 4\frac{923076}{999999} = 4\frac{923072}{999999}$; $2\frac{368}{999} \div 4\frac{923072}{999999} = .48\dot{1}$, Ans.
8. $\frac{9}{99} \div \frac{33799}{999999} = .3\dot{9}$, Ans.
9. $316.31015 \div \frac{3}{8} = 948.93045$, Ans.
10. $100006 \div \frac{8}{9} = 150009$, Ans.
11. $.3\dot{6} = \frac{36}{100} = \frac{9}{25}$; $.2\dot{5} = \frac{25}{100} = \frac{1}{4}$; $\frac{9}{25} \div \frac{1}{4} = 1.422924901185770750988\dot{1}$, Ans.

CONTINUED FRACTIONS.

(ART. 309, p. 229.)

- (3.) $\frac{281}{82} = 82)261(3$ First approx. val. $= \frac{3}{1} = 3$.
 $\frac{246}{15)82(5$ $3 \times 5 + 1 = 16$
 $\frac{75}{7)15(2$ $1 \times 5 + 0 = \frac{5}{5} = 3\frac{1}{5}$.
 $\frac{14}{1)7(7$ $16 \times 2 + 3 = 35$
 $\frac{7}{-}$ $\frac{5}{5} \times 2 + 1 = \frac{11}{11} = 3\frac{2}{11}$.
 $35 \times 7 + 16 = 261$
 $11 \times 7 + 5 = \frac{82}{82} = 3\frac{15}{82}$.
 Ans. $3, 3\frac{1}{5}, 3\frac{2}{11}, 3\frac{15}{82}$, real val.

(4.) $29\overline{)77}(2$

$$\begin{array}{r} 58 \\ 19\overline{)29}(1 \end{array}$$

$$\begin{array}{r} 19 \\ 10\overline{)19}(1 \end{array}$$

$$\begin{array}{r} 10 \\ 9\overline{)10}(1 \end{array}$$

$$\begin{array}{r} 9 \\ 1\overline{)9}(9 \end{array}$$

First approx. val. = $\frac{1}{2}$.

$$1 \times 1 + 0 = 1$$

$$\frac{1}{2} \times 1 + 1 = \frac{3}{2}$$

$$1 \times 1 + 1 = 2$$

$$\frac{3}{2} \times 1 + 2 = \frac{7}{2}$$

$$2 \times 1 + 1 = 3$$

$$\frac{5}{2} \times 1 + 3 = \frac{11}{2}$$

$$3 \times 9 + 2 = 29$$

$$\frac{8}{7} \times 9 + 5 = \frac{77}{7}, \text{ original val.}$$

Ans. $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{7}{77}$.

(5.) $1327\overline{)1327}(1$

$$\begin{array}{r} 1327 \\ 304\overline{)1327}(4 \end{array}$$

$$\begin{array}{r} 1216 \\ 111\overline{)304}(2 \end{array}$$

$$\begin{array}{r} 222 \\ 82\overline{)111}(1 \end{array}$$

$$\begin{array}{r} 82 \\ 29\overline{)82}(2 \end{array}$$

$$\begin{array}{r} 58 \\ 24\overline{)29}(1 \end{array}$$

$$\begin{array}{r} 24 \\ 5\overline{)24}(4 \end{array}$$

$$\begin{array}{r} 20 \\ 4\overline{)20}(5 \end{array}$$

$$\begin{array}{r} 4 \\ 1\overline{)4}(4 \end{array}$$

$$\begin{array}{r} 4 \\ 4\overline{)4} \end{array}$$

Ans. $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{7}{77}, \frac{847}{1041}$.

[$\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{7}{77}, \frac{847}{1041}$]

(6.)

$$3\frac{2}{3} = 347)829(2$$

$$\begin{array}{r} 694 \\ 135)347(2 \end{array}$$

$$\begin{array}{r} 270 \\ 77)135(1 \end{array}$$

$$\begin{array}{r} 77 \\ 58)77(1 \end{array}$$

$$\begin{array}{r} 58 \\ 19)58(3 \end{array}$$

$$\begin{array}{r} 57 \\ 1)19(19 \end{array}$$

$$\begin{array}{r} 19 \end{array}$$

 $\frac{2}{3} = 2$, first approx. val.

$$2 \times 2 + 1 = 5$$

$$\bar{1} \times 2 + 0 = \bar{2} = 2\frac{1}{2}$$

$$5 \times 1 + 2 = 7$$

$$\bar{2} \times 1 + 1 = \bar{3} = 2\frac{1}{3}$$

$$7 \times 1 + 5 = 12$$

$$\bar{3} \times 1 + 2 = \bar{5} = 2\frac{2}{3}$$

$$12 \times 3 + 7 = 43$$

$$\bar{5} \times 3 + 3 = \bar{18} = 2\frac{7}{5}$$

$$43 \times 19 + 12 = 819$$

$$\bar{18} \times 19 + 5 = \bar{347} =$$

$$[2\frac{1}{3}\frac{2}{3}\frac{5}{5}]$$

Ans. 2, 2 $\frac{1}{2}$, 2 $\frac{2}{3}$, 2 $\frac{7}{5}$, 2 $\frac{1}{3}\frac{2}{3}\frac{5}{5}$.

$$(7.) \frac{1}{13\frac{8}{1000}} = \frac{1000}{13568}$$

$$1000)13568(13$$

$$\begin{array}{r} 13000 \end{array}$$

$$\begin{array}{r} 568)1000(1 \end{array}$$

$$\begin{array}{r} 568 \end{array}$$

$$\begin{array}{r} 432)568(1 \end{array}$$

$$\begin{array}{r} 432 \end{array}$$

$$\begin{array}{r} 136)432(3 \end{array}$$

$$\begin{array}{r} 408 \end{array}$$

$$\begin{array}{r} 24)136(5 \end{array}$$

$$\begin{array}{r} 120 \end{array}$$

$$\begin{array}{r} 16)24(1 \end{array}$$

$$\begin{array}{r} 16 \end{array}$$

$$\begin{array}{r} 8)16(2 \end{array}$$

$$\begin{array}{r} 16 \end{array}$$

 $\frac{1}{13}$ = first approx. val.

$$1 \times 1 + 0 = 1$$

$$\bar{13} \times 1 + 1 = \bar{14}$$

$$1 \times 1 + 1 = 2$$

$$\bar{14} \times 1 + 13 = \bar{27}$$

$$2 \times 3 + 1 = 7$$

$$\bar{27} \times 3 + 14 = \bar{95}$$

$$7 \times 5 + 2 = 37$$

$$\bar{95} \times 5 + 27 = \bar{502}$$

$$37 \times 1 + 7 = 44$$

$$\bar{502} \times 1 + 95 = \bar{597}$$

$$44 \times 2 + 37 = 125$$

$$\bar{579} \times 2 + 44 = \bar{1396}$$

Ans. $\frac{1}{13}$, $\frac{1}{14}$, $\frac{2}{27}$, $\frac{7}{95}$, $\frac{37}{502}$, $\frac{44}{597}$, $\frac{125}{1396}$.

(8.) $1.27 = \frac{127}{100}$.

$100)127(1$

$\frac{100}{27)100(3$

$\frac{81}{19)27(1$

$\frac{19}{8)19(2$

$\frac{16}{3)8(2$

$\frac{6}{2)3(1$

$\frac{1}{2} \quad \frac{47}{26} \times 1 + 11 = \frac{127}{100} = 1.27$

$\frac{1}{2} \quad \frac{47}{26} \times 2 + 33 = 127$

$\frac{1}{2} \quad \frac{47}{26} \times 2 + 26 = \frac{100}{100} = 1$

First approx. val. = $\frac{1}{2}$.

$\frac{1}{2} \times 3 + 1 = 4$

$\frac{1}{2} \times 3 + 0 = \frac{3}{2} = 1\frac{1}{2}$.

$\frac{4}{3} \times 1 + 1 = 5$

$\frac{4}{3} \times 1 + 1 = \frac{8}{3} = 2\frac{2}{3}$.

$\frac{5}{4} \times 2 + 4 = 14$

$\frac{5}{4} \times 2 + 3 = \frac{11}{2} = 5\frac{1}{2}$.

$\frac{14}{11} \times 2 + 5 = 33$

$\frac{14}{11} \times 2 + 4 = \frac{26}{11} = 2\frac{4}{11}$.

$\frac{33}{26} \times 1 + 14 = 47$

$\frac{33}{26} \times 1 + 11 = \frac{37}{26} = 1\frac{11}{26}$.

Ans. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{11}{16}, \frac{23}{32}, \frac{47}{64}, \frac{127}{128} = 1, 1\frac{1}{2}, 1\frac{1}{4}, 1\frac{3}{8}, 1\frac{7}{8}, 1\frac{11}{16}, 1\frac{27}{32}$.

RATIO.

REDUCTION AND COMPARISON OF RATIOS.

(ART. 328, p. 232.)

2. $63 : 72 = \frac{7}{8} = \frac{7}{8}$, Ans.

3. $60 : 24 = \frac{5}{2} = \frac{5}{2}$, Ans.

4. $4 \times 6 \times 3 : 8 \times 9 \times 2 = 72 : 144 = \frac{1}{2} = \frac{1}{2}$, Ans.

5. $19 \times 5 \times 2 \times 3 : 15 \times 12 \times 38 = 570 : 684 = \frac{5}{58} = \frac{5}{58}$, Ans.

(ART. 329, p. 233.)

3. $\frac{5}{8} : \frac{7}{4} = \frac{5}{8} \times \frac{4}{7} = \frac{5}{14} = 35 : 24$, Ans.

4. $13\frac{1}{2} : 27 = \frac{13\frac{1}{2}}{27} = \frac{27}{54} = \frac{1}{2} = 1 : 2$, Ans.

5. $6.25 : 3.125 = \frac{6.25}{3.125} = \frac{2}{1} = 2 : 1$, Ans.

$$6. \left. \begin{array}{l} 4 : 16 \\ 25 : 10 \end{array} \right\} = 4 \times 25 : 16 \times 10 = 100 : 160 = 5 : 8 = \frac{5}{8} =$$

$$[\frac{5}{8} = 5 : 8, \text{ Ans.}]$$

$$7. \left. \begin{array}{l} 3 : 6 \\ 9 : 27 \\ 108 : 12 \end{array} \right\} = 3 \times 9 \times 108 = 2916 : 6 \times 27 \times 12 =$$

$$[1924 = \frac{2916}{1524} = \frac{3}{2} = 3 : 2, \text{ Ans.}]$$

$$8. \left. \begin{array}{l} 12\frac{1}{2} : 6\frac{1}{2} \\ 76.5 : 25.5 \end{array} \right\} = 12.5 \times 76.5 = 956.25 : 6.25 \times 25.5 =$$

$$[159.375 = \frac{956.25}{6.075} = 6 = 6 : 1, \text{ Ans.}]$$

(ART. 330, p. 233.)

$$3. 39 : 13 = \frac{39}{13} = 3, \text{ Ans.}$$

$$4. 2\frac{1}{2} : 9 = \frac{2\frac{1}{2}}{9} = \frac{5}{18}, \text{ Ans.}$$

$$5. 21 : 21 = \frac{21}{21} = 1, \text{ Ans.}$$

$$6. \frac{1}{3} \times \frac{1}{6} \times 240 = \frac{240}{18} = 13^{\circ} : \frac{1}{2} \times \frac{50}{1} = \frac{50}{2} = 25; 13^{\circ}$$

$$\times \frac{1}{25} = \frac{1}{2}, \text{ Ans.}$$

$$7. 24 : 6 = \frac{24}{6} = 4, \text{ Ans.}$$

$$8. 4 : 36 = \frac{4}{36} = \frac{1}{9}, \text{ Ans.}$$

$$9. 94A. 2R. 16p. = 15136p.; 11A. 3R. = 1880p.; 1880 :$$

$$15136 = \frac{1880}{15136} = \frac{235}{1892}, \text{ Ans.}$$

$$10. 17 : 9 = \frac{17}{9} = 1\frac{8}{9}; 39 : 19 = \frac{39}{19} = 2\frac{1}{19}; \text{ therefore the}$$

$$\text{ratio of 39 to 19 is the larger.}$$

$$11. 36 \times 4 \times 3 = 432 : 12 \times 16 \times 2 = 384 = \frac{432}{384} = \frac{9}{8};$$

$$60 \div (3 \times 5) = 4; (20 \times 2) \div 8 = 5; \frac{9}{8} - \frac{4}{5} =$$

$$= \frac{1}{40}, \text{ Ans.}$$

$$12. .02 = \frac{2}{100}; 2.503 = \frac{2503}{1000} = \frac{2503}{1000} = \frac{250}{100} \times \frac{2503}{1000} = \frac{225075}{100000}$$

$$= 112\frac{71}{111}, \text{ Ans.}$$

$$13. \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}; \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}; \frac{1}{10} \times \frac{1}{12} = \frac{1}{120} = \frac{1}{60}.$$

$$4 : 5 = \frac{4}{5}; \frac{4}{5} - \frac{1}{60} = \frac{49}{60}, \text{ Ans. That is, the first ratio is}$$

$$\text{the greater by } \frac{1}{60}.$$

$$14. 220 : 500 = \frac{220}{500} = \frac{11}{25}, \text{ Ans.}$$

$$15. 36 : 180 = \frac{36}{180} = \frac{1}{5}, \text{ Ans.}$$

16.) 4900)11283(2

9800

1483)4900(3

First approx. ratio = $\frac{1}{2}$. 4449

$$1 \times 3 + 0 = 3$$

$$451)1483(3$$

$$2 \times 3 + 1 = 7$$

$$1353$$

$$3 \times 3 + 1 = 10$$

$$130)451(3$$

$$7 \times 3 + 2 = 23$$

$$390$$

$$10 \times 3 + 3 = 33$$

$$61)130(2$$

$$23 \times 3 + 7 = 76$$

$$122$$

$$33 \times 2 + 10 = 76$$

$$8)61(7$$

$$76 \times 2 + 23 = 175$$

$$56$$

$$76 \times 7 + 33 = 565$$

$$5)8(1$$

$$175 \times 7 + 76 = 1301$$

$$5$$

$$565 \times 1 + 76 = 641$$

$$3)5(1$$

$$1301 \times 1 + 135 = 1436$$

$$3$$

$$641 \times 1 + 555 = 1196$$

$$2)3(1$$

$$1436 \times 1 + 1301 = 2737$$

$$2$$

$$1196 \times 1 + 641 = 1837$$

$$1)2(2$$

$$2737 \times 1 + 1436 = 4173$$

$$2$$

$$1837 \times 2 + 1196 = 4870$$

$$4173 \times 2 + 2637 = 11083$$

Ans. $\frac{1}{2}$, 3, 19, 33, 76, 565, 641, 1196, 1837, 4870.

10*

(17.) 1000000)3141592(3

3000000

141592)1000000(7

991144

8856)141592(15

First approx. ratio = $\frac{3}{1}$.8856 $3 \times 7 + 1 = 22$ 53032 $\bar{1} \times 7 + 0 = \bar{7}$ 44280 $\frac{22}{7} \times 15 + 3 = 333$

8752)8856(1

 $\frac{7}{7} \times 15 + 1 = \frac{106}{7}$ 8752 $\frac{333}{106} \times 1 + 22 = 355$

104)8752(84

 $\frac{106}{106} \times 1 + 7 = \frac{113}{106}$ 832 $355 \times 84 + 333 = 30153$ 432 $\frac{113}{113} \times 84 + 106 = \frac{9492}{113}$ 416 $30153 \times 6 + 355 = 181273$

16)104(6

 $\frac{9492}{9492} \times 6 + 113 = \frac{57065}{9492}$ 96 $181273 \times 2 + 30153 = 392699$

8)16(2

 $\frac{57065}{57065} \times 2 + 9492 = \frac{123622}{57065}$ 16Ans. $\frac{3}{1}, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{30153}{9492}, \frac{181273}{57065}, \frac{392699}{123622} =$ $3, 3\frac{1}{7}, 3\frac{15}{106}, 3\frac{16}{113}, 3\frac{677}{9492}, 3\frac{2873}{57065}, 3\frac{1833}{123622}.$

ANALYSIS BY RATIO.

(ART. 331, p. 235.)

(2.)

\$ 80

17

\$ 1360 = 17A.

(2R. : 1A. = $\frac{1}{2}$); $\frac{1}{2}$ of 80 = 40 = 2R.(1R. : 2R. = $\frac{1}{2}$); $\frac{1}{2}$ of 40 = 20 = 1R.(20r. : 1R. = $\frac{1}{2}$); $\frac{1}{2}$ of 20 = 10 = 20r.(10r. : 20r. = $\frac{1}{2}$); $\frac{1}{2}$ of 10 = 5 = 10r.(5r. : 10r. = $\frac{1}{2}$); $\frac{1}{2}$ of 5 = 2.50 = 5r.\$ 1437.50, Ans.

(3.)

\$ 2.50

16

\$ 40.00 = 16cwt.

(2qr. : 1cwt. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 2.50 = 1.25 = 2qr.(1qr. : 2qr. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 1.25 = .625 = 1qr.(10lb. : 1cwt. = $\frac{1}{10}$); $\frac{1}{10}$ of \$ 2.50 = .25 = 10lb.\$ 42.125, Ans.

(4.)

\$ 14.00

27

\$ 378.00 = 27cwt.

(1qr. : 1cwt. = $\frac{1}{4}$); $\frac{1}{4}$ of \$ 14 = 3.50 = 1qr.(20lb. : 1cwt. = $\frac{1}{5}$); $\frac{1}{5}$ of \$ 14 = 2.80 = 20lb.\$ 384.30, Ans.

(5.)

\$ 5.60

7

\$ 39.20 = 7yd.

(2qr. : 1yd. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 5.60 = 2.80 = 2qr.(1qr. : 2qr. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 2.80 = 1.40 = 1qr.(2na. : 1qr. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 1.40 = .70 = 2na.\$ 44.10, Ans.

(6.)

\$ 20

7

\$ 140.00 = 7ton.

(10cwt. : 1 ton = $\frac{1}{10}$); $\frac{1}{10}$ of \$ 20 = 2.00 = 10cwt.(2cwt. : 10cwt. = $\frac{1}{5}$); $\frac{1}{5}$ of \$ 10 = 2.00 = 2cwt.(1cwt. : 2cwt. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 2 = 1.00 = 1cwt.(2qr. : 1cwt. = $\frac{1}{2}$); $\frac{1}{2}$ of \$ 1 = .50 = 2qr.\$ 153.50, Ans.

$$\begin{array}{r}
 (8.) \qquad \qquad \qquad \$4.40 \\
 \qquad \qquad \qquad \underline{19} \\
 \qquad \qquad \qquad \$83.60 = 19\text{yd.} \\
 (2\text{qr.} : 1\text{yd.} = \tfrac{1}{2}); \tfrac{1}{2} \text{ of } \$4.40 = 2.20 = 2\text{qr.} \\
 (1\text{qr.} : 1\text{yd.} = \tfrac{1}{4}); \tfrac{1}{4} \text{ of } \$4.40 = 1.10 = 1\text{qr.} \\
 (2\text{na.} : 1\text{qr.} = \tfrac{1}{2}); \tfrac{1}{2} \text{ of } \$1.10 = .55 = 1\text{na.} \\
 \qquad \qquad \qquad \underline{\$87.45, \text{ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 (9.) \qquad \qquad \qquad \$32 \\
 \qquad \qquad \qquad \underline{24} \\
 \qquad \qquad \qquad \$768 \\
 (3\text{R.} : 1\text{A.} = \tfrac{3}{4}); \tfrac{3}{4} \text{ of } \$32 = 24 \\
 (20\text{p.} : 1\text{A.} = \tfrac{1}{8}); \tfrac{1}{8} \text{ of } \$32 = 4 \\
 \qquad \qquad \qquad \underline{\$796, \text{ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 (10.) \qquad \qquad \qquad 200 \\
 \qquad \qquad \qquad \underline{\$0.30} \\
 \qquad \qquad \qquad \$60.00 = 200\text{lb.} \\
 (4\text{oz.} : 1\text{lb.} = \tfrac{1}{4}) = \tfrac{1}{4} \text{ of } \$0.30 = .075 = 4\text{oz.} \\
 \qquad \qquad \qquad \underline{\$59.925, \text{ Ans.}}
 \end{array}$$

$$\begin{array}{r}
 (11.) \qquad \qquad \qquad 714 \\
 \qquad \qquad \qquad \underline{15\text{s.}} \\
 \qquad \qquad \qquad 3570 \\
 \qquad \qquad \qquad \underline{714} \\
 \qquad \qquad \qquad 10710\text{s.} \\
 (6\text{d.} : 1\text{s.} = \tfrac{1}{2}); \tfrac{1}{2} \text{ of } 714 = 357 \\
 \qquad \qquad \qquad 20 \overline{)11067} \\
 \qquad \qquad \qquad \underline{553\text{£. } 7\text{s., Ans.}}
 \end{array}$$

$$(12.) \quad \begin{array}{r} \$450 \\ 2 \end{array}$$

$$\begin{array}{l} \$900 = 2y. \\ (6mo. : 1y. = \frac{1}{2}); \frac{1}{2} \text{ of } \$450 = 225 = 6mo. \\ (3mo. : 6mo. = \frac{1}{2}); \frac{1}{2} \text{ of } \$225 = 112.50 = 3mo. \\ (15da. : 3mo. = \frac{1}{6}); \frac{1}{6} \text{ of } \$112.50 = 18.75 = 15da. \\ \hline \$1256.25, \text{ Ans.} \end{array}$$

$$(13.) \quad \begin{array}{l} \$80.50 \times 25 = \$2012.50 = 25A. \\ (2R. : 1A. = \frac{1}{2}); \frac{1}{2} \text{ of } \$80.50 = 40.25 = 2R. \\ (20r. : 2R. = \frac{1}{4}); \frac{1}{4} \text{ of } \$40.25 = 10.06+ = 20r. \\ (10r. : 20r. = \frac{1}{2}); \frac{1}{2} \text{ of } \$10.06+ = 5.03+ = 10r. \\ (5r. : 10r. = \frac{1}{2}); \frac{1}{2} \text{ of } \$5.03+ = 2.51+ = 5r. \\ \hline \$2070.35+, \text{ Ans.} \end{array}$$

$$(14.) \quad \begin{array}{r} 498 \\ (2s. : 1\text{£} = \frac{1}{10}); \frac{1}{10} \text{ of } 498 = 49\text{£} 16s. \\ (6d. : 2s. = \frac{1}{4}); \frac{1}{4} \text{ of } 49\text{£} 16s. = 12\text{£} 9s. \\ \hline 62\text{£} 5s., \text{ Ans.} \end{array}$$

$$(16.) \quad \begin{array}{l} 35\text{gal. } 2\text{qt. } 1\text{pt.} : 11\text{gal. } 3\text{qt. } 1\text{pt.} = 3; \\ \$5.83\frac{3}{4} \times 2 = \$17.51\frac{1}{4}, \text{ Ans.} \end{array}$$

$$(17.) \quad \begin{array}{l} 12\text{yd. } 1\text{qr. } 2na. : 24\text{yd. } 3\text{qr.} = \frac{1}{2}; \\ \$49.50 \times \frac{1}{2} = \$24.75, \text{ Ans.} \end{array}$$

$$(18.) \quad \begin{array}{l} 73\text{bu. } 3\text{pk.} : 14\text{bu. } 3\text{pk.} = 5; \\ 17\text{bu. } 2\text{pk. } 4\text{qt.} \times 5 = 88\text{bu. } 0\text{pk. } 4\text{qt.}, \text{ Ans.} \end{array}$$

$$(19.) \quad \begin{array}{l} \$9.75 : \$3.25 = \frac{1}{3}; \\ 1\text{T. } 2\text{cwt. } 2\text{qr. } 15\text{lb.} \times \frac{1}{3} = 7\text{cwt. } 2\text{qr. } 5\text{lb.}, \text{ Ans.} \end{array}$$

$$(20.) \quad \begin{array}{l} 4h. : 1h. } 20m. = 3; \\ 27m. } 3\text{fur. } 20\text{rd.} \times 3 = 82m. } 2\text{fur. } 20\text{rd.}, \text{ Ans.} \end{array}$$

SIMPLE PROPORTION.

(ART. 338, p. 240.)

3. $16A. : 197A. :: \$720 : \8865 , Ans.

$$\frac{197 \times \overset{45}{\cancel{720}}}{\cancel{16}} = \$8865, \text{ Ans.}$$

4. $\$8865 : \$720 :: 137A. : 16A.$, Ans.

$$\frac{720 \times 137}{8865} = 16A., \text{ Ans.}$$

5. $15\text{hhd.} : 84\text{hhd.} :: \$17595 : \$985.32$, Ans.

$$\frac{84 \times 17595}{15} = \$985.32, \text{ Ans.}$$

6. $\$12 : \$40 :: \$6 : \12 .

$$\frac{\overset{20}{\cancel{40}} \times \cancel{6}}{\cancel{12}} = \$20, \text{ Ans.}$$

7. $15 \text{ men} : 10 \text{ men} :: 45 \text{ days} : 30 \text{ days}$, Ans.

$$\frac{10 \times \overset{3}{\cancel{45}}}{\cancel{15}} = 30 \text{ days, Ans.}$$

8. $7 + 9 = 16 : 8 + 4 = 12 :: 12 : 9$.

$$\frac{12 \times 12}{16} = 9, \text{ Ans.}$$

9. $3 \text{ men} : 9 \text{ men} :: 17 \text{ days} : 51 \text{ days}$, Ans.

10. $17 \text{ days} : 51 \text{ days} :: 3 \text{ men} : 9 \text{ men}$; $9 - 3 = 6 \text{ men}$, Ans.

11. $5\frac{1}{2} \text{ rods} : 160 \text{ rods} :: 1 \text{ rod} : 29\frac{1}{11} \text{ rods}$, Ans.

12. $\$100 : \$850 :: \$6 : \51 , Ans.

13. $\$6 : \$32 :: \$100 : \$533.33\frac{1}{3}$, Ans.

14. $20\text{gal.} : 180\text{gal.} :: 167\text{lb.} : 1503\text{lb.}$, Ans.

15. $2\text{ft.} : 3\text{ft.} :: 75\text{ft.} : 112\frac{1}{2}\text{ft.}$, Ans.

16. $\$4.75 : \$160 :: 36 \text{ miles} : 1212\frac{2}{9} \text{ miles}$, Ans.

17. 8 days : 12 days :: 100 men : 150 men, Ans.
18. $\frac{7}{12}$ yd. : $\frac{1}{4}$ yd. :: \$ $\frac{7}{20}$: \$ 0.48, Ans.
19. 36A. 3R. : 21A. 3R. 20p. :: \$ 1260 : \$ 750, Ans.
20. 10pwt. 18gr. : 2000lb. :: \$ 10 : \$ 4465 $11\frac{2}{3}$, Ans.
21. $4\frac{1}{2}$ yd. : $13\frac{1}{2}$ yd. :: \$ 9.75 : \$ 29.25, Ans.
22. $2\frac{1}{2}$ in. : 144 in. :: 1 in. : $57\frac{2}{3}$ in., Ans.
23. $\frac{7}{16}$: $\frac{3}{32}$:: 51£. : 10£. 18s. 6½d., Ans.
24. 13° 10' 35" : 360° :: 1 day : 27da. 7h. 43m. +, Ans.
25. 7lb. : 12lb. :: \$ $\frac{3}{4}$: \$ 1.28 $\frac{2}{3}$, Ans.
26. \$ 1.75 : \$ 213.50 :: 7lb. : 8cwt. 2qr. 4lb., Ans.
27. 7oz. : 7lb. 11oz. :: 30£. : 407£. 2s. 10½d., Ans.
28. \$ 600 : \$ 500 :: 6 months : 5 months, Ans.
29. \$ 7.50 : \$ 8.00 :: 7oz. : $7\frac{7}{15}$ oz., Ans.
30. 1 man : 1000 men :: $3\frac{3}{4} \times 1\frac{1}{5} = 22\frac{5}{10}$ yd. : 7031 $\frac{1}{4}$ yd.
 $1\frac{1}{4}$ yd. : 1 yd. :: 7031 $\frac{1}{4}$ yd. : 5625 yd., Ans.
31. 10h. : 14h. :: 9 days : $12\frac{3}{8}$ days, Ans.
32. 75 — 40 = 35gal. : 500gal. :: 1h. : 14h. 17m. 8½sec., Ans.
33. \$ 0.56 : \$ 120.96 :: 1 glove : 216 gloves.
 $216 \div 12 = 18$ doz., Ans.
34. 20m. : 1m. :: 1cist. : $\frac{1}{20}$ cist.
 40m. : 1m. :: 1cist. : $\frac{1}{40}$ cist.
 76 m. : 1m. :: 1cist. : $\frac{1}{76}$ cist.
 $\frac{1}{20} + \frac{1}{40} + \frac{1}{76} = \frac{53}{680}$.
 $\frac{53}{680}$ cist. : 1cist. :: 1m. : 11m. $19\frac{1}{3}$ sec., Ans.
35. 5da. : 1da. :: 1 field : $\frac{1}{5}$ field; 6da. : 1da. :: 1 field : $\frac{1}{6}$ field.
 $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ field; $\frac{11}{30}$ field : 1 field :: 1da. : $2\frac{8}{11}$ da., Ans.
36. 8ft. : 24ft. :: 6 men : 18 men.
 Then 6 days : 12 days :: 18 men : 36 men, Ans.
37. A can do $\frac{1}{27}$ of the labor in 1 day, A and C can do $\frac{1}{12}$ of it; therefore, C alone can do $\frac{1}{12} - \frac{1}{27} = \frac{1}{36}$ of it.
 Then $\frac{1}{36}$ work : 1 work :: 1da. : 30da., Ans.
38. 700 men : 1 man :: 184000lb. : 262½lb.
 Then 5lb. : 262½lb. :: 1 week : 52 weeks 4 days, Ans.
39. $\frac{3}{4} \times 3 \times 25 = 56\frac{1}{4}$ lb.; $56\frac{1}{4}$ lb. : 3150lb. :: 1 week : 56 weeks, Ans.

40. $8 \times 8 = 64\text{in.}$; $20 \times 16 \times 144 = 46080\text{in.}$
 $64\text{in.} : 46080\text{in.} :: 1 \text{ tile} : 720 \text{ tiles, Ans.}$
41. $10 \times 9 \times 4 = 360 \text{ cubic inches in each stone.}$
 $80 \times 20 \times 2\frac{1}{2} \times 1728 = 6220800 \text{ cubic inches in the wall.}$
 $360\text{in.} : 6220800\text{in.} :: 1 \text{ stone} : 17280 \text{ stones, Ans.}$
42. 1T. 7cwt. 3qr. 20lb. = 2795lb.; 13T. 5cwt. 2qr. = 26550lb.
 $2795\text{lb.} : 26550\text{lb.} :: \$9.50 : \$90.24+, \text{ Ans.}$
43. $61.3\text{lb.} : 1\text{lb.} :: \$44.9942 : \$0.734, \text{ Ans.}$
44. 1hhd. : .15hhd. : : $\$2.39 : \$0.3585, \text{ Ans.}$
45. .75 ton : 1 ton : : $\$15 : \$20, \text{ Ans.}$
46. .5yd. : 6yd. : : $10\text{yd.} : 120\text{yd., Ans.}$
47. 10h. : 12h. : : $15\text{da.} : 18\text{da., Ans.}$
48. 9 months : 5 months : : $450 \text{ men} : 250 \text{ men.}$
 $450 - 250 = 200 \text{ men, Ans.}$
49. As the hour and minute hand pass each other 11 times in 12 hours, and as they are together at 12 o'clock, it is evident they will next pass each other in $\frac{1}{11}$ of 12 hours = 1h. 5m. $27\frac{3}{11}\text{sec., Ans.}$
 $11\text{h.} : 12\text{h.} :: 1\text{h.} : 1\text{h. } 5\text{m. } 27\frac{3}{11}\text{sec., Ans.}$

BY ANALYSIS.

50. If A and B can perform a piece of labor in $5\frac{5}{11}$ days, it is evident that in 1 day they would do $\frac{1}{5\frac{5}{11}} = \frac{1}{6\frac{1}{2}}$ of the work. If B and C can do the work in $6\frac{2}{3}$ days, in 1 day they would perform $\frac{1}{6\frac{2}{3}} = \frac{1}{8}$ of the work. If A and C can do the work in 6 days, in 1 day they would perform $\frac{1}{6}$ of it. It then appears that A, B, and C, by laboring each 2 days, will perform $\frac{1}{6\frac{1}{2}} + \frac{1}{8} + \frac{1}{6} = \frac{3}{8} = \frac{1}{2}$ of the work; and therefore, by laboring 1 day each, they would do $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$ of it. And if $\frac{1}{4}$ of the labor be performed in 1 day, it is evident that the whole work will be performed in 4 days, Ans.

If A, B, and C, can do $\frac{1}{4}$ of the work in one day, and A and B can do $\frac{1}{6\frac{1}{2}}$, it is evident that C can do $\frac{1}{4} - \frac{1}{6\frac{1}{2}} =$

$\frac{4}{30} = \frac{1}{15}$ of it in 1 day, and therefore will be 15 days in performing the whole. B and C can do $\frac{2}{30}$ of it in a day; therefore A can do $\frac{1}{4} - \frac{2}{30} = \frac{1}{15}$ of it in a day, or he will be 10 days in doing the whole. A and C can do $\frac{1}{4}$ of the work in a day; therefore B can do $\frac{1}{4} - \frac{1}{10} = \frac{1}{20}$ in a day, or, in performing the whole labor, he will be 20 days, Ans.

FORM OF STATEMENT.

5 $\frac{1}{15}$ days : 1 day :: 1 work : $\frac{1}{15}$ work = A and B.

6 $\frac{2}{30}$ days : 1 day :: 1 work : $\frac{2}{30}$ work = B and C.

6 days : 1 day :: 1 work : $\frac{1}{6}$ work = A and C.

$$\frac{1}{2} \text{ work} = 2A, 2B, 2C =$$

$$\frac{1}{4} \text{ work} = A, B, \text{ and } C.$$

$\frac{1}{4} - \frac{1}{15} = \frac{1}{60}$ work : 1 work :: 1 day : 15 days = C.

$\frac{1}{4} - \frac{2}{30} = \frac{1}{15}$ work : 1 work :: 1 day : 10 days = A.

$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ work : 1 work :: 1 day : 12 days = B.

$\frac{1}{4}$ work : 1 work :: 1 day : 4 days = A, B, and C.

(ART. 339, p. 243.)

(2.)

246A. 1R. 32p. = 39432p.; 3 + 4 + 5 = 12.

12 : 3 :: 39432p. : 61A. 2R. 18p. }
 12 : 4 :: 39432p. : 82A. 0R. 24p. } Ans.
 12 : 5 :: 39432p. : 102A. 2R. 30p. }

(3.)

$$4\frac{1}{4} = 4\frac{5}{20} = \frac{85}{20};$$

$$6\frac{1}{2} = 6\frac{10}{20} = \frac{130}{20};$$

$$6\frac{3}{5} = 6\frac{12}{20} = \frac{132}{20};$$

$$7 = 7 = \frac{140}{20};$$

$$24\frac{7}{20} = \frac{487}{20}.$$

$\frac{487}{20} : \frac{85}{20} :: 319 : 55\frac{339}{487}$
 $\frac{487}{20} : \frac{130}{20} :: 319 : 85\frac{75}{487}$
 $\frac{487}{20} : \frac{132}{20} :: 319 : 86\frac{256}{487}$
 $\frac{487}{20} : \frac{140}{20} :: 319 : 91\frac{343}{487}$ } Ans.

4. Gold 9 + silver $\frac{1}{2}$ + copper $\frac{1}{2}$ = 10.

$$\left. \begin{array}{l} 10 : 9 :: \text{loz. 1pwt. 12gr.} : 19\text{pwt. } 8\frac{2}{3}\text{gr.} = \text{gold.} \\ 10 : \frac{1}{2} :: \text{loz. 1pwt. 12gr.} : 1\text{pwt. } 1\frac{4}{5}\text{gr.} = \text{silver.} \\ 10 : \frac{1}{2} :: \text{loz. 1pwt. 12gr.} : 1\text{pwt. } 1\frac{4}{5}\text{gr.} = \text{copper.} \end{array} \right\} \text{Ans.}$$

5. Silver 9 + copper 1 = 10; $192\text{gr.} \times 20 = 3840\text{gr.}$

$$10 : 9 :: 3840\text{gr.} : 7\text{oz. 4pwt. silver.}$$

$$10 : 1 :: 3840\text{gr.} : 16\text{pwt. copper.}$$

6. First, 1; second, $1\frac{1}{3} \times 1 = 1\frac{1}{3}$; third, $1\frac{2}{3} \times 1\frac{1}{3} = 2\frac{2}{3}$;

$$1 = \frac{2}{3}; 1\frac{1}{3} = \frac{4}{3}; 2\frac{2}{3} = \frac{8}{3}; \frac{2}{3} + \frac{4}{3} + \frac{8}{3} = 4\frac{1}{3}.$$

$$41 : 9 :: \$600 : \$131\frac{2}{3} = \text{first man receives.}$$

$$41 : 12 :: \$600 : \$175\frac{2}{3} = \text{second man receives.}$$

$$41 : 20 :: \$600 : \$292\frac{2}{3} = \text{third man receives.}$$

$$\text{Proof, } \$131\frac{2}{3} + \$175\frac{2}{3} + \$292\frac{2}{3} = \$600.$$

7. $98 + 86 + 64 = 248.$

$$\left. \begin{array}{l} 248 \text{ tons} : 98 \text{ tons} :: 93 \text{ tons} : 36\frac{3}{4} \text{ A's tons.} \\ 248 \text{ tons} : 86 \text{ tons} :: 93 \text{ tons} : 32\frac{1}{4} \text{ B's tons.} \\ 248 \text{ tons} : 64 \text{ tons} :: 93 \text{ tons} : 24 \text{ C's tons.} \end{array} \right\} \text{Ans.}$$

8. $15 + 32 = 47$ miles; $25 + 32 = 57$ miles.

$$47\text{mi.} : 15\text{mi.} :: 160\text{mi.} : 51\frac{3}{7}\text{mi. A.} \left. \vphantom{47\text{mi.} : 15\text{mi.} :: 160\text{mi.} : 51\frac{3}{7}\text{mi. A.}} \right\} \text{Ans.}$$

$$57\text{mi.} : 25\text{mi.} :: 160\text{mi.} : 70\frac{1}{5}\text{mi. B.}$$

COMPOUND PROPORTION.

(ART. 340, p. 245.)

(2.)

$$\left. \begin{array}{l} 30 \text{ days} : 20 \text{ days} \\ 9 \text{ hours} : 12 \text{ hours} \end{array} \right\} :: 117 \text{ miles} : 104 \text{ miles, Ans.}$$

$$\frac{20 \times 12 \times 117}{9 \times 30} = 104 \text{ miles, Ans.}$$

$$\begin{array}{lcl}
 24 \text{ men} : 6 \text{ men} & & \\
 8 \text{ hours} : 9 \text{ hours} & & \\
 20 \text{ feet} : 200 \text{ feet} & & \\
 6 \text{ feet} : 8 \text{ feet} & & \\
 4 \text{ feet} : 6 \text{ feet} & &
 \end{array}
 \left. \vphantom{\begin{array}{l} 24 \text{ men} \\ 8 \text{ hours} \\ 20 \text{ feet} \\ 6 \text{ feet} \\ 4 \text{ feet} \end{array}} \right\} \text{(3.)} \therefore 16 \text{ days} : 90 \text{ days, Ans.}$$

OPERATION.

$$\frac{6 \times 9 \times 200 \times 8 \times 6 \times 16}{24 \times 8 \times 20 \times 6 \times 4} = \frac{829440}{92160} = 90 \text{ days, Ans.}$$

$$\begin{array}{lcl}
 \$100 & : & \$500 \\
 12 \text{ months} : 4 \text{ months} & &
 \end{array}
 \left. \vphantom{\begin{array}{l} \$100 \\ 12 \text{ months} \end{array}} \right\} \text{(4.)} \therefore \$6 : \$10, \text{ Ans.}$$

$$\begin{array}{lcl}
 \$6 & : & \$10 \\
 4 \text{ months} : 12 \text{ months} & &
 \end{array}
 \left. \vphantom{\begin{array}{l} \$6 \\ 4 \text{ months} \end{array}} \right\} \text{(5.)} \therefore \$100 : \$500, \text{ Ans.}$$

$$\begin{array}{lcl}
 \$500 : \$100 & & \\
 \$6 : \$10 & &
 \end{array}
 \left. \vphantom{\begin{array}{l} \$500 \\ \$6 \end{array}} \right\} \text{(6.)} \therefore 12 \text{ months} : 4 \text{ months, Ans.}$$

$$\begin{array}{lcl}
 \$500 & : & \$100 \\
 4 \text{ months} : 12 \text{ months} & &
 \end{array}
 \left. \vphantom{\begin{array}{l} \$500 \\ 4 \text{ months} \end{array}} \right\} \text{(7.)} \therefore \$10 : \$6, \text{ Ans.}$$

$$\begin{array}{lcl}
 9 \text{ comp.} : 5 \text{ comp.} & & \\
 10 \text{ hours} : 11 \text{ hours} & & \\
 25 \text{ sheets} : 36 \text{ sheets} & & \\
 24 \text{ pages} : 16 \text{ pages} & & \\
 44 \text{ lines} : 50 \text{ lines} & & \\
 40 \text{ letters} : 45 \text{ letters} & &
 \end{array}
 \left. \vphantom{\begin{array}{l} 9 \text{ comp.} \\ 10 \text{ hours} \\ 25 \text{ sheets} \\ 24 \text{ pages} \\ 44 \text{ lines} \\ 40 \text{ letters} \end{array}} \right\} \text{(8.)} \therefore 16 \text{ days} : 12 \text{ days, Ans.}$$

$$\begin{array}{lcl}
 60 \text{ men} : 12 \text{ men} & & \\
 30 \text{ feet} : 300 \text{ feet} & & \\
 6 \text{ feet} : 8 \text{ feet} & & \\
 3 \text{ feet} : 6 \text{ feet} & & \\
 8 \text{ hours} : 12 \text{ hours} & &
 \end{array}
 \left. \vphantom{\begin{array}{l} 60 \text{ men} \\ 30 \text{ feet} \\ 6 \text{ feet} \\ 3 \text{ feet} \\ 8 \text{ hours} \end{array}} \right\} \text{(9.)} \therefore 15 \text{ days} : 120 \text{ days, Ans.}$$

CANCELLING.

$$\frac{12 \times \overset{10}{\cancel{300}} \times \cancel{8} \times \cancel{6} \times \overset{1}{\cancel{12}} \times \overset{1}{\cancel{15}}}{\underset{4}{\cancel{60}} \times \underset{1}{\cancel{30}} \times \underset{1}{\cancel{6}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{3}}} = 120 \text{ days, Ans.}$$

(10.)

$$\left. \begin{array}{l} 16 \text{ horses} : 32 \text{ horses} \\ 24 \text{ days} : 48 \text{ days} \end{array} \right\} :: 84 \text{ bushels} : 336 \text{ bushels, Ans.}$$

CANCELLING.

$$\frac{\overset{2}{\cancel{32}} \times \overset{2}{\cancel{48}} \times 84}{\underset{1}{\cancel{16}} \times \underset{1}{\cancel{24}}} = 336 \text{ bushels, Ans.}$$

(11.)

$$\left. \begin{array}{l} 575 \text{ pounds} : 775 \text{ pounds} \\ 150 \text{ miles} : 64 \text{ miles} \end{array} \right\} :: \$24.58 : \$14.135 + \text{Ans.}$$

(12.)

$$\left. \begin{array}{l} 7\frac{1}{4} \text{ oz.} = 7.25; 4\text{s. } 2\text{d.} = 50\text{d.}; 5\text{s. } 6\text{d.} = 66\text{d.}; 1\text{s. } 2\text{d.} = 14\text{d.} \\ 66\text{d.} : 50\text{d.} \\ 4\frac{3}{4}\text{d.} : 14\text{d.} \end{array} \right\} :: 7.25 \text{ oz.} : 16\frac{11}{32}\frac{3}{4} \text{ oz., Ans.}$$

CANCELLING.

$$\frac{\overset{2}{\cancel{50}} \times \overset{7}{\cancel{14}} \times 7.25}{\underset{33}{\cancel{66}} \times \underset{19}{\cancel{4.75}}} = 16\frac{11}{32}\frac{3}{4} \text{ oz., Ans.}$$

(13.)

$$\left. \begin{array}{l} 24 \text{ men} : 496 \text{ men} \\ 9 \text{ hours} : 11 \text{ hours} \\ 7 \text{ hard.} : 4 \text{ hard.} \\ 465 \text{ feet} : 337\frac{1}{2} \text{ feet} \\ 3\frac{2}{3} \text{ feet} : 5\frac{2}{5} \text{ feet} \\ 2\frac{1}{3} \text{ feet} : 3\frac{1}{2} \text{ feet} \end{array} \right\} :: 5\frac{1}{2} \text{ days} : 132 \text{ days, Ans.}$$

CONJOINED PROPORTION.

(ART. 341, p. 247.)

(2.)

FORM OF STATEMENT.

100A. Bradford = 120A. Haverhill;

50A. Haverhill = 65A. Methuen;

150A. Methuen.

OPERATION.

$$\begin{array}{r} 100 \times 50 \times 150 = 750000 \\ 120 \times 65 = 7800 \end{array} = 96\frac{2}{3} \text{A., Ans.}$$

(3.)

FORM OF STATEMENT.

10lb. cheese = 7lb. butter;

11lb. butter = 2bu. corn;

11bu. corn = 8bu. rye;

4bu. rye = 1 cord wood;

10 cords wood.

OPERATION.

$$\begin{array}{r} 10 \times 11 \times 11 \times 4 \times 10 = 48400 \\ 7 \times 2 \times 8 \times 1 = 112 \end{array} = 432\frac{1}{2} \text{lb., Ans.}$$

(4.)

STATEMENT.

12 men = 25 women.

5 women = 6 boys.

75 boys.

OPERATION.

$$\frac{12 \times 5 \times 75}{25 \times 6} = 30 \text{ men, Ans.}$$

11*

(5.)

STATEMENT.

6 gallons = 5 imperial gal.
 10 imp. gal. = 6 Velts.
 26 Velts = 16 Vedros.
 63 gallons.

OPERATION.

$$\frac{5 \times 6 \times 16 \times 63}{6 \times 10 \times 26} = \frac{30240}{1560} = 19\frac{5}{13} \text{ Vedros, Ans.}$$

(6.)

STATEMENT.

7 Boston = 8 Buffalo.
 10 Buffalo = 14 Chicago.
 21 Chicago = 25 Davenport.
 120 Davenport.

OPERATION.

$$\frac{7 \times 10 \times 21 \times 120}{8 \times 14 \times 25} = \frac{1764}{2800} = 630 \text{ bushels, Ans.}$$

(7.)

STATEMENT.

24s. Massachusetts = 32s. New York.
 48s. New York = 45s. Pennsylvania.
 15s. Pennsylvania = 10s. Canada.
 100s. Massachusetts.

OPERATION.

$$\frac{32 \times 45 \times 10 \times 100}{24 \times 48 \times 15} = \frac{1440000}{1728} = 83\frac{1}{3} \text{ s., Ans.}$$

MISCELLANEOUS EXAMPLES IN PROPORTION.

(PAGE 248.)

(1.)

BY ANALYSIS.

$$27 \times 7 = 189 = \text{miles A is ahead of B.}$$

36 — 27 = 9 miles that B gains each day on A. If, therefore, 9 miles are gained in one day, it will require, to gain 189 miles, $189 \div 9 = 21$ days, Ans.

FORM OF STATEMENT.

$$36 - 27 = 9\text{m.} : 189\text{m.} :: 1 \text{ day} : 21 \text{ days, Ans.}$$

(2.)

BY ANALYSIS.

2s. 3d. = 27d., price obtained for the coffee. It is evident that 27d. is $\frac{1}{3}$ of the cost; therefore, $\frac{1}{3}$ of 27d. = 20d. was the cost, Ans.

FORM OF STATEMENT.

$$135\text{d.} : 100\text{d.} :: 27\text{d.} : 20\text{d., Ans.}$$

(3.)

FORM OF STATEMENT.

$$2000 \times 12 \times 7 \times 14 = 2352000 \div 16 = 147000\text{lb. whole quantity.}$$

$$105 \times 200 = 21000\text{lb. wholly spoiled.}$$

$$147000 - 21000 = 126000\text{lb. left to subsist on.}$$

$$2000 \times 12 \times 7 = 168000 \text{ rations.}$$

$$2016000 \div 168000 = 12\text{oz. for each man per day, Ans}$$

(4.)

FORM OF STATEMENT.

$$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000\text{lb. quantity subsisted on.}$$

$$126000 \div 6 = 21000\text{lb. spoiled.}$$

$$21000 \times 7 = 147000\text{lb., the whole quantity, Ans.}$$

(5.)

FORM OF STATEMENT.

$$2000 \times 12 \times 7 \times 14 = 2352000 \div 16 = 147000\text{lb., whole weight.}$$

$$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000\text{lb. left to subsist on, Ans.}$$

(6.)

FORM OF STATEMENT.

$$\$3.00 \times 60 = \$180, \text{ price given for the Holland.}$$

$$\$4.00 \times 60 = \$240, \text{ price obtained for it.}$$

$$\$180 : \$240 :: \$240 : \$320, \text{ Ans.}$$

$$7. \ 1\frac{1}{2}\text{lb.} : 20\text{lb.} :: \$1 : \$13.33\frac{1}{3}.$$

$$15\text{lb.} : 62\text{lb.} :: \$12 : \$49.60.$$

$$\$13.33\frac{1}{3} + \$49.60 = \$62.93\frac{1}{3}, \text{ price of the tea.}$$

$$20\text{lb.} + 62\text{lb.} = 82\text{lb., whole quantity of the tea.}$$

$$3\text{lb.} : 82\text{lb.} :: \$4 : \$109.33\frac{1}{3}; \$109.33\frac{1}{3} - \$62.93\frac{1}{3} =$$

$$8. \ 2 \text{ fur. } 3\text{rd. } 3\text{yd.} = 182\frac{1}{4} \text{ feet.} \quad [\$46.40, \text{ Ans.}]$$

$$66\text{ft.} : 64\text{ft.} :: 1824\text{ft.} : 1768\frac{8}{11}\text{ft.}$$

$$1768\frac{8}{11}\text{ft.} \times 4 = 7074\frac{8}{11} \text{ feet, Ans.}$$

$$9. \ 7 \text{ cows} : 3 \text{ cows} :: 5 \text{ oxen} : 2\frac{1}{2} \text{ oxen.}$$

$$2 + 2\frac{1}{2} = 4\frac{1}{2} \text{ oxen; } 4\frac{1}{2} \text{ oxen} : 5 \text{ oxen} :: 87 \text{ days} : 105 \text{ days, Ans.}$$

$$10. \ 8\text{mo.} : 6 - 4 = 2\text{mo.} :: 360 \text{ men} : 90 \text{ men.}$$

$$360 - 90 = 270 \text{ men, Ans.}$$

$$11. \ 10\frac{1}{4} = 10.25; \ 1\frac{7}{8} = 1.875; \ 100 - 5 = 95; \ 100 : 95$$

$$:: 1.875\text{yd.} : 1.78125\text{yd.}; \ 100 : 95 :: 1.78125\text{yd.} :$$

$$1.6921875\text{yd.}; \ 1.6921875 : 1 :: 10.25\text{yd.} : 6\frac{1}{8}\frac{2}{3}\text{yd.},$$

Ans.

$$12. \ 130\text{A. } 2\text{R. } 20\text{p.} = 20900\text{p.}; \ 20900 \times 4 = 83600;$$

$$100\text{A. } 0\text{R. } 30\text{p.} = 16030\text{p.}; \ 16030 \times 5 = 80150;$$

$$83600\text{p.} : 80150\text{p.} :: \$6537.50 : \$6267.71+, \text{ Ans.}$$

$$13. \ 192 \text{ tons } 17\text{cwt. } 16\text{lb.} = 385716\text{lb.}; \ 800 + 101 + 56 + 43 = 1000.$$

$$1000 \text{ parts} : 800 \text{ parts} :: 385716\text{lb.} : 308572\frac{1}{2}\text{lb., copper.}$$

- 1000 parts : 101 parts :: 385716lb. : 38957 $\frac{79}{250}$ lb. tin.
 1000 parts : 56 parts :: 385716lb. : 21600 $\frac{125}{125}$ lb. zinc.
 1000 parts : 43 parts :: 385716lb. : 16585 $\frac{125}{250}$ lb. lead.
14. 81 w. pine : 42 w. pine :: 1 oak : $\frac{48}{81}$ oak.
 $7 : 8 :: \frac{48}{81} : \frac{48}{81}$; that is, oak is to pitch pine as 48 to 81;
 $\frac{48}{81} + \frac{48}{81} = \frac{96}{81}$; that is, $\frac{96}{81}$ cords of oak are equal to 2-
 cords of pine; therefore, $\frac{96}{81}$ oaks : 20 oak :: 2 pine : 36
 pine.
 $36 \div 2 = 18$ cords of each, Ans.
15. $63 \times \frac{4}{5} = 50\frac{2}{5}$ gallons.
 85gal. : 50 $\frac{2}{5}$:: \$116.95 : \$69.34 $\frac{2}{5}$, Ans.
16. 4 cows : 15 cows :: 3 horses : 11 $\frac{1}{4}$ horses.
 1 sheep : 7 sheep :: $\frac{1}{3}$ cow : $\frac{7}{3}$ cows.
 1 cow : $\frac{7}{3}$ cows :: $\frac{2}{3}$ horse : 1 $\frac{2}{3}$ horses.
 $10 + 11\frac{1}{4} + 1\frac{2}{3} = 23$ horses.
 Then as 23 horses : 8 horses } :: 30 days : 14 $\frac{9}{16}$ days, Ans.
 $3\frac{1}{2}$ tons : 4 $\frac{9}{10}$ tons }
17. 25 : 14 :: 1 : $\frac{14}{25}$.
 7 : 5 :: 1 : $\frac{5}{7}$.
 5 : 2 :: 1 : $\frac{2}{5}$.
 $\frac{14}{25} \times \frac{5}{7} \times \frac{2}{5} = \frac{142}{25} = \frac{4}{5}$; 2000 $\div \frac{4}{5} = 12500$ pounds of tur-
 nips, are equal to 2000 pounds of potatoes; 14 : 25 :: 35
 cts. for beets : 62 $\frac{1}{2}$ cts., the proportionate price of potatoes;
 that is, 80 cts. for potatoes is as much dearer than 35 cts.
 for beets, as 80 is more than 62 $\frac{1}{2} = 17\frac{1}{2}$ cts.; again, 5 :
 2 :: 25 cts. for carrots : 10 cts., the proportionate price
 for turnips; but 20 cts. for turnips is 10 cts. dearer than
 the proportionate price; hence, 20 cts. for turnips is 10
 cts. dearer than 25 cts. for carrots, Ans.
18. A travels 22 $\frac{1}{2}$ days at the rate of 18 miles per hour = 405
 miles. He not only travels as far as B, but the distance
 that B would travel in twice 9 days = 18 days; therefore,
 B, to travel the whole distance which A has travelled,
 would require 22 $\frac{1}{2}$ + 18 = 40 $\frac{1}{2}$ days; therefore, 405 \div
 40 $\frac{1}{2}$ = 10 miles per day, Ans.

19. By the conditions of the question, 2 men bear the expenses of the ride for 20 miles, 3 men for 52 miles, 4 men for 42 miles, and 5 men for 30 miles; therefore, each of the "two men" will pay $\frac{1}{2}$ the hire for 20 miles, $+\frac{1}{3}$ for 52 miles, $+\frac{1}{4}$ for 42 miles, $+\frac{1}{5}$ for 30 miles, = the hire for $43\frac{5}{8}$ miles.

The "two men" will pay for $43\frac{5}{8} \times 2 =$ hire for $87\frac{5}{8}$ miles.
 A will pay $\frac{1}{3}$ for 52, $+\frac{1}{4}$ for 42, $+\frac{1}{5}$ for 30 = " $33\frac{5}{8}$ miles.
 B will pay $\frac{1}{4}$ for 42, $+\frac{1}{5}$ for 30 = " $16\frac{1}{2}$ miles.
 C will pay $\frac{1}{5}$ for 30 = " 6 miles.
144

144 : $87\frac{5}{8}$:: \$ 25 : \$ $7.609\frac{19}{108}$, share of each of the "two men."

144 : $33\frac{5}{8}$:: \$ 25 : \$ $5.873\frac{21}{108}$, A's share.

144 : $16\frac{1}{2}$:: \$ 25 : \$ $2.864\frac{17}{72}$, B's share.

144 : 6 :: \$ 25 : \$ $1.041\frac{1}{3}$, C's share.

PERCENTAGE.

(ART. 346, p. 252.)

1.	.19	6.	.77 $\frac{1}{8}$
2.	.27	7.	1.06
3.	.135	8.	1.07
4.	.0175	9.	3.05
5.	.074	10.	9.998

ART. 347, p. 252.)

2.	75bu.	8.	\$ 990
3.	15cwt.	9.	48bbl.
4.	45 tons.	10.	60hhd.
5.	\$ 375	11.	\$ 0.25
6.	665chal.	12.	15.12lb.
7.	.8 miles.		

(13.)	(14.)
900	\$ 1728
.08	.15
<u>72.00</u>	<u>8640</u>
	1728
900	\$ 259.20
72	
<u>828</u>	\$ 1728
.50	<u>259.20</u>
<u>414.00</u> , Ans.	\$ 1468.80, Ans.

(15.)	
\$ 25000	\$ 25000
.40	<u>10000</u>
\$ 10000.00 = wife's share.	\$ 15000
\$ 15000	\$ 10000
.30	<u>4500</u>
\$ 4500.00 = son's share.	\$ 14500

\$ 25000
14500
 10500
60
 3)10440
 \$ 3480 = each daughter's share.

Ans. Wife, \$ 10000; son, \$ 4500; each daughter, \$ 3480.

(ART. 348, p. 253.)

2.	12½ per cent.	7.	75 per cent.
3.	10 per cent.	8.	37½ per cent.
4.	5½ per cent.	9.	50 per cent.
5.	25 per cent.	10.	20 per cent.
6.	\$ 36.00 × .25 = \$ 9.00;	11.	20 per cent.
	\$ 9.00 ÷ 150 = .06, Ans.		

12. $100 \div 140 = .71\frac{1}{2}$, the per cent. that the grammar class is of the geography class. $100 - 71\frac{1}{2} = 28\frac{1}{2}$ per cent., Ans.

13. $4 \div 32 = .12\frac{1}{2}$, Ans.

14. $22 + 3 = 25$; $25 : 22 :: 100 : 88$ per cent. of copper.
 $25 : 3 :: 100 : 12$ per cent. of nickel.

15. 25 pear-trees are $25 \times \frac{100}{250} = 10$ per cent. of the whole.
 40 per cent. + 12 per cent. + 8 per cent. + 10 per cent. =
 70 per cent.; $100 - 70 = 30$ per cent. peach-trees, Ans.

(ART. 349, p. 255.)

2.	250.	5.	$2\frac{3}{16}$.
3.	203 $\frac{1}{2}$.	6.	\$ 13.
4.	600.	7.	\$ 140.

8. $17 + 6 = 15\frac{1}{2}$ per cent. of the whole number; $\frac{23 \times 100}{15\frac{1}{2}}$
 [= 150, Ans.]

9. $.33\frac{1}{3}$ of .45 = .15; $\frac{450 \times 100}{15} = 3000$, Ans.

10. 3m. 1fur. 1rd. = 1001rd.; $\frac{1001 \times 100}{12\frac{1}{2}} = 8008$ rd. = 25m.
 [0fur. 8rd., Ans.]

11. $\frac{19 \times 100}{17\frac{1}{2}} = 110\frac{2}{3}$; $110\frac{2}{3} - 19 = 91\frac{2}{3}$, Ans.

12. $\frac{\$36.89 \times 100}{13\frac{3}{8}} = \270 ; $\frac{\$36.89 \times 100}{16\frac{3}{8}} = \221.34 ;
 [\$279.00 - \$221.34 = \$57.66, Ans.]

(ART. 350, p. 256.)

3.	120	5.	\$20.96 $\frac{3}{4}$
4.	365	6.	\$34.27 $\frac{1}{8}$

7. $\$24 \div 1.33\frac{1}{3} = \18 , Ans.

8. 2m. 6fur. 24rd. = 904rd.; $904 \div .87\frac{1}{2} = 1033\frac{1}{2}$ rd. = 3m.
 1fur. 33rd., Ans.

9. $\$123.16 \div .84 = \146.61 , Ans.

10. 6yd. $1\frac{1}{3}$ qr. $\div .25 = 25$ yd. $0\frac{4}{3}$ qr. = quantity cut off;
 25 yd. $0\frac{4}{3}$ qr. + 6yd. $1\frac{1}{3}$ qr. = 31yd. 1qr. 2na., Ans.
11. $279 \div .90 = 310$, taken; $310 - 279 = 31$, lost, Ans.
12. $100 - 86\frac{1}{2} = 13\frac{1}{2}$ per cent., which A had left.
 $100 - 55 = 45$ per cent., which B had left.
 Hence, $13\frac{1}{2} + .45 = .58\frac{1}{2}$, which they both have left of 2.00 =
- $$\$36.85\frac{1}{2}; \frac{\$36.85\frac{1}{2} \times 1.00}{.58\frac{1}{2}} = \$63;$$
- $100 : 13\frac{1}{2} :: \$63 : \$8.50\frac{1}{2}$, what A had left.
 $100 : 45 :: \$63 : \28.35 , what B had left.

MISCELLANEOUS EXAMPLES.

(PAGE 257.)

- | | | | |
|----|-------------------|-----|-----------------------|
| 1. | 117 $\frac{1}{2}$ | 7. | .07 |
| 2. | 174 $\frac{1}{5}$ | 8. | 81 $\frac{1}{4}$. |
| 3. | 351 | 9. | \$ 66. |
| 4. | 2 $\frac{1}{10}$ | 10. | 5 $\frac{3}{8}$. |
| 5. | 316 $\frac{2}{3}$ | 11. | \$ 22. |
| 6. | 40 | 12. | 117 $\frac{6}{7}$ lb. |
13. $6 \times 30 \times 12 = 2160$; $2160 \times .15 = 324$, Ans.
14. $12 : 100 :: \$0.69 : \5.75 , Ans.
15. $5400 + 6000 =$ what was excavated in the first two weeks
 $= 11400$; $40500 - 11400 = 29100$; $29100 \times .25 =$
 $7275 =$ what was excavated in the third week; 11400
 $+ 7275 = 18675$; $40500 \div 2 = 20250$; $20250 -$
 $18675 = 1575$ cubic feet, Ans.
16. $.25 \times .50 = .12\frac{1}{2}$; $\frac{.12\frac{1}{2} \times 100}{.75} = .16\frac{2}{3}$, Ans.
17. $\$0.50 \times .80 = \0.40 , the cost of a gallon; $\$0.40 \times .25$
 $= \$0.10$; $\$0.40 + \$0.10 = \$0.50$, Ans.
18. $20 \times .04 = .8$; $1 \times .05 = .05$; $20 - .8 = 19.2 =$ the
length after shrinking; $1 - .05 = .95 =$ width after
shrinking; $19.2 \times .95 = 18\frac{6}{5}$; $20 - 18\frac{6}{5} = 1\frac{1}{5}$ yd.,
Ans.

19. 15 per cent. of 1.00 = .15 = daughter's;
 10 per cent. of .85 = .085 + $\frac{1}{10}$ of an acre = son's;
 25 per cent. of .765 — $\frac{1}{10}$ = .19125 — $\frac{1}{40}$ of an acre = wife's;
 $.42625 + \frac{3}{40}$ = the shares of
 the wife, the son, and the daughter; and $100 - .42625$
 = the remainder + $\frac{3}{40}$ of an acre which is not included in
 the .42625. Hence $1.00 - .42625 = .57375 = 39 \times .5$
 = 195 + $\frac{3}{40}$ of an acre = $195\frac{3}{40}$ acres.
 $.57375 : 1.00 :: 195\frac{3}{40}$ acres : 340 acres, Ans.
20. $30,500,000 \times 1.34\frac{1}{2} = 41,022,500$, Ans.
21. Let the English = 100 per cent. ;
 $100 + 33\frac{1}{3} = 133\frac{1}{3}$ per cent. = French ;
 $133\frac{1}{3} + 8\frac{1}{3}$ of $133\frac{1}{3} = 144\frac{2}{3}$ per cent. = Turks ;
 $144\frac{2}{3} - 100 = 44\frac{2}{3}$ per cent. = 1600. Hence
 $44\frac{2}{3} : 100 :: 1600 : 3600$ = English ;
 $100 : 133\frac{1}{3} :: 3600 : 4800$ = French ;
 $100 : 144\frac{2}{3} :: 3600 : 5200$ = Turks ;
 13600 the whole number.
 $13600 : 3600 :: 100 : 26\frac{8}{17}$, percentage of English ;
 $13600 : 4800 :: 100 : 35\frac{5}{17}$, percentage of French ;
 $13600 : 5200 :: 100 : 38\frac{4}{17}$, percentage of Turks.
22. $\$7.25 \times .10 = \0.725 ; $\$7.25 - \$0.725 = \$6.525$;
 $\$7.25 + \$0.725 = \$7.975$; $\$7.975 - \$6.525 =$
 $\$1.45$; $\$6.525 : \$1.45 :: 100 : 22\frac{58}{81}$ = the percent.
 of the proceeds of the flour above the cost ; $560 \times \$1.45$
 = $\$812.00$ = profits, Ans.
23. $87500 \div 1.25 = 70000$, Ans.
24. Let the cost of the horse = 100 ; but the horse cost $62\frac{1}{2}$ per
 cent. as much as the buggy ; hence $62\frac{1}{2} : 100 :: 100 :$
 160 = the proportionate percentage of the buggy ; $100 -$
 $70 = 30$, the proportionate percentage of the harness ;
 $100 + 160 + 30 = 290$;
 $290 : 100 :: \$500 : \$172\frac{1}{2}$ = horse ;
 $290 : 160 :: \$500 : \$275\frac{2}{3}$ = buggy ;
 $290 : 30 :: \$500 : \$51\frac{3}{4}$ = harness.

INTEREST.

(ART. 354, p. 262.)

2.	\$40.50	27.	y. mo. da. 1859 9 19
3.	\$476.144		1856 6 4
4.	\$161.33		3 3 15
5.	\$888.546	\$98.25	$\times .197\frac{1}{2} = \19
6.	\$108.587		[.404, Ans.
7.	\$44.442		
8.	\$540.000	28.	y. mo. da. 1860 6 18
9.	\$1.30		1836 1 19
10.	\$1.275		24 4 29
11.	\$0.6867	\$22.763	$\times 1.464\frac{2}{3} = \$33.$
12.	\$31.85		[.344, Ans.
13.	\$116.99		
14.	\$40.10	29.	y. mo. da. 1859 9 12
15.	\$0.0758		1855 0 7
16.	\$0.9739		4 9 5
17.	\$14.607	\$175.07	$\times .285\frac{5}{8} = \$50-$
18.	\$0.227		[.04, Ans.
19.	\$31.146		
20.	\$18.674	30.	y. mo. da. 1854 0 11
21.	\$193.09		1852 11 6
22.	\$3.37 $\frac{1}{2}$		1 1 5
23.	\$26.343	\$197.285	$\times .065\frac{5}{8} = \$12-$
24.	\$0.584		[.987, Ans.
25.	\$17.438		
26.	y. mo. da. 1863 6 27	31.	\$4377.15 $\times .18 = \$787-$
	1852 0 11		.887; \$787.887 + \$43-
	11 6 16		77.15 = \$5165.037 =
			the amount, Ans.
	\$76.895 $\times .692\frac{3}{4} = \$53-$	32.	\$444.60 $\times .33 = \$146-$
	[.262, Ans.		.718, Ans.

(ART. 355, p. 265.)

(2.)

	\$ 16.75
	<u>.07</u>
Interest for 1 year	= <u>1.1725</u>
Int. for 4 mo., $\frac{1}{3}$ of a year	= <u>.3908</u>
Int. for 2 mo., $\frac{1}{2}$ of 4 mo.	= <u>.1954</u>
Int. for 1 mo., $\frac{1}{2}$ of 2 mo.	= <u>.0977</u>
Int. for 15 da., $\frac{1}{2}$ of 1 mo.	= <u>.0488</u>
Int. for 2 da., $\frac{1}{3}$ of 1 mo.	= <u>.0065</u>
Int. for 7 mo. and 17 da.	= \$ <u>0.7392</u> , Ans.

(3.)

y.	mo.	d.	
1852	11	7	
1852	3	17	
	<u>7</u>	<u>20</u>	
			\$ 11.105
			<u>.07</u>
Int. for 1 year			= <u>.77735</u>
Int. for 6 mo., or $\frac{1}{2}$ of 1 yr.			= <u>.38867</u>
Int. for 1 mo., or $\frac{1}{6}$ of 6 mo.			= <u>.06477</u>
Int. for 15 da., or $\frac{1}{2}$ of 1 mo.			= <u>.03238</u>
Int. for 5 da., or $\frac{1}{3}$ of 15 da.			= <u>.01079</u>
Int. for 7 mo. and 20 da.			= \$ <u>0.49661</u> , Ans.

4.	y.	mo.	d.
	1854	7	30
	1853	0	2
		<u>1</u>	<u>7 28</u>

$$\begin{aligned} \$12.69 \times .099\frac{2}{3} &= \$1.2647; \\ \$1.2647 \div 6 &= \$0.2107, \\ \times 7 &= \$1.475, \text{ Ans.} \end{aligned}$$

5.	\$871.111
6.	\$504.64
7.	\$0.70

8.	y.	mo.	d.
	1860	0	11
	1852	1	7
		<u>7</u>	<u>11 4</u>

$$\begin{aligned} \$17869.75 \times .475\frac{1}{3} &= \$8500. \\ .0444; \$8500.0444 \div 6 &= \\ \$1416.67406, \times 5 &= \\ \$7083.3703, \text{ Ans.} \end{aligned}$$

9.	\$21.78
----	---------

10. \$0.149

	y.	mo.	d.
11.	1863	11	15
	1861	10	11
	2	1	4

\$35.61 \times .125 $\frac{2}{3}$ = \$4.474, Ans.

12. \$10.139

	y.	mo.	d.
13.	1860	6	17
	1854	4	7
	6	2	10

\$1728.19

.371 $\frac{2}{3}$

6 \times 4 = 24) 642.31061 $\frac{2}{3}$

\$26.762,

[Ans.

14. \$397.16

.05 $\frac{1}{2}$

198580

19858

Int. for 1 yr. = 21.8438

Int. for $\frac{1}{2}$ of 1 yr. = 10.9219

Int. for 1 da., or

$\frac{1}{360}$ of 6 mo. = .0608

Ans. \$32.8265

15. \$100.25

.014 $\frac{5}{8}$

At 6 per cent. = 1.48704 $\frac{1}{8}$

At 1 per cent. = .24784

4

At 4 per cent. = .99136

\$100.25

Amount, \$101.241

12*

16. \$51.17

.04

2) \$2.0468

Int. for 6 mo. = 1.0234

Int. for 3 mo. = .5117

Int. for 15 da., or $\frac{1}{8}$ of

3 mo. = .08528

Int. for 1 da. = .00568

Int. for 13 da. = .07384

Ans. 1.6999

17. \$42.20 \times .062 $\frac{2}{3}$ = \$2.6-

445; $4\frac{1}{2} \times \$2.6445 \div 6$

= 1.983, Ans.

18. \$0.975

19. y. mo. d.

1852 6 3

1849 10 27

2 7 6

\$96.84

.07 $\frac{1}{2}$

67788

4842

Int. for 1 yr. = 7.2630

2

Int. for 2 yr. = 14.5260

Int. for 6 mo. = 3.6315

Int. for 1 mo. = .6052

Int. for 6 da., or

$\frac{1}{2}$ of 1 mo. = .1210

Ans. \$18.8837

20. \$225.925

	y.	mo.	d.
21.	1857	5	19
	1853	6	29
	3	10	20
	\$71.09,1		
	.233 $\frac{1}{2}$		
	213273		
	213273		
	142182		
	23697		
Interest at 6% per cent.*	\$16.587900		
	2		
Ans.	\$33.17,5800		

* The interest will be twice this amount at 12 per cent.

22.	\$444.163		
23.	\$7.864		
24.	\$0.886		
	y.	mo.	d.
25.	1857	1	18
	1829	4	16
	27	9	2

$$\begin{aligned} \$36.72 \times 1.665\frac{1}{2} &= \$61.151- \\ 04; \$61.15104 \times 7 \div 6 & \\ = \$71.342, \text{ Ans.} \end{aligned}$$

26.	\$8.628		
27.	\$515.60		
28.	\$3.167		
29.	\$9750 \times .08 = \\$780.00,		
	[Ans.		
30.	\$9162 \times 3 \times 1\frac{1}{2} = \\$412.29,		
	[Ans.		
31.	\$95.833		
32.	\$58.00		
33.	\$246.295		

34.	\$10000		
	.15 $\frac{1}{2}$		
	\$150000		
	3333 $\frac{1}{2}$		
Int. at 6 per cent. =	\$1533.33 $\frac{1}{2}$		
	7		
	6)10733.33 $\frac{1}{2}$		
Int. at 7 per cent. =	\$1788.88		
\$1788.88 - \$1533.33 =	\$255.55; Ans.		

(ART. 356, p. 268.)

2.	\$96.00		
Int. for 60 da.	= .96		
Int. for 30 da.	= .48		
Int. for 3 da.	= .048		
	6)528		
	.088		
	7		
Int. at 7 per cent. =	\$0.616		
	[Ans.		
3.	\$320.40		
Int. for 60 da. = $\frac{1}{10}$	of the principal = 3.204		
Int. for 3 da. = $\frac{1}{20}$ of	60 da.		
	= .1602		
	6)3.3642		
	.5607		
	5		
Int. at 5 per cent. =	\$2.8035		
	[Ans.		

4.	\$ 131.20	15.	\$ 144.50
Int. of 60 da.	= 1.3120		.024
Int. of 60 da.	= 1.3120		6)3.46800
Int. of 3 da.	= .0656		578
Ans.	\$ 2.6896		5
			73)2.890
5.	\$ 0.345		39
6.	\$ 245.65		\$ 2.851 = \$ 2.89
7.	\$ 1.681		[diminished* by $\frac{1}{3}$ of itself,
8.	\$ 0.263		[Ans.
9.	\$ 177.029 $\frac{1}{3}$		
10.	\$ 166.681 $\frac{2}{3}$	16.	\$ 20.662
11.	\$ 569.70	17.	\$ 19.65
12.	\$ 35.295	18.	\$ 80.38
13.	\$ 72.888	19.	\$ 1005.50
14.	\$ 2.223	20.	\$ 7321.90

* See note 2, p. 267.

(ART. 357, p. 269.)

2. 179£. 12s. 11d. = 179.645£. $\times .095$ = 17.0662£. = interest at 6 per cent. ; 17.0662£. $\div 6$ = 2.8443 ; 2.8443 $\times 5$ = 14.2215£. = 14£. 4s. 5 $\frac{1}{4}$ d., Ans.

(3.)

Principal = 25£.

$\frac{1}{10}$ of the principal = 2.5 int. for 20 mo.

$\frac{1}{20}$ of int. for 1yr. 8mo. = .125 int. for 1 mo.

2.625 $\div 6$ = .4375 ; .4375 $\times 5$ =

2.1875£. = 2£. 3s. 9d. = int. for 1yr. 9mo., Ans.

(4.)

Principal = 5440.5£.

$\frac{1}{3}$ of the principal = 1088.1 int. for 3yr. 4mo.

$\frac{1}{6}$ of int. for 3yr. 4mo. = 186.0125 int. for 5mo.

$\frac{2}{3}$ of int. for 5 mo. = $\left\{ \begin{array}{l} 27.2025 \\ 27.2025 \end{array} \right\}$ int. for 2mo.

1278.5175£. (Carried forward.)

(Brought forward:) $1278.5175\text{£} =$ the interest for 3yr.
11mo. $= 1278\text{£}.$ 10s. 4d., Ans.

5. $943\text{£}.$ 1s. 8d. $= 943.0833\text{£}.$; 30da. in May + 30 in June
+ 31 in July + 31 in Aug. + 30 in Sept. + 21 in Oct.
 $= 173\text{da}.$; $943.0833\text{£} \times .028\frac{1}{2} = 27.19223\text{£}.$;
 $27.19223\text{£} \times 5\frac{1}{4} \div 6 = 23.793\text{£}.$; $23.793 - .3259$
($\frac{1}{3}$ of $23.793\text{£}.$) $= 23.467 = 23\text{£}.$ 9s. 4d., Ans.

(ART. 358, p. 270.)

2. $10.08 \div .07 = \$144$, Ans.
3. $1.00 \times .08\frac{1}{4} = .0825 \times 1\frac{1}{3} = .11$; $13.20 \div .11 = \$120.00$,
Ans.
4. $40.50 \div .06 = \$675$, Ans.
5. $1.00 \times .02 \times 3 = .06$; $24.00 \div .06 = \$400$, Ans.
6. $1.00 \times .0305 = 0.0305 \div 6 = .0050\frac{5}{8}$; $.0050\frac{5}{8} \times 7\frac{1}{4} =$
 $.0368\frac{3}{4}$; $206.38\frac{1}{3} \div .0368\frac{3}{4} = \5600 , Ans.

(ART. 359, p. 270.)

2. $\$144.00 \times .01 = \1.44 ; $10.08 \div 1.44 = 7$ per cent.,
Ans.
3. $\$120 \times .01 = 1.20$; $1.20 \times 1\frac{1}{3} = 1.60$; $133.20 - 120$
 $= 13.20 \div 1.60 = 8\frac{1}{4}$ per cent., Ans.
4. $\$1.00 \times .01 = \0.01 ; $\$0.01 \times 14\frac{2}{7} = \$0.14\frac{2}{7}$; 1.00
 $\div 14\frac{2}{7} = 7$ per cent., Ans.
5. $\$1.00 \times .01 = \$0.01 \times 33\frac{1}{3} = \$.33\frac{1}{3}$; $3.00 \div .33\frac{1}{3} =$
9 per cent., Ans.
6. $\$4650 \times .01 = \46.50 ; $232.50 \div 46.50 = 5$ per cent.,
Ans.
7. $48\text{da.} = 1\frac{2}{3}\text{mo.} = \frac{1\frac{2}{3}}{12}$ of 1yr.; $\$7500 \times .01 = \75 ;
 $\$75 \times \frac{1\frac{2}{3}}{12} = \10 ; $60 \div 10 = 6$ per cent., Ans.

$$8. \$280 \times .01 = \$2.80 \times 6\frac{1}{2} = \$18.20; \$411.95 - \$280 = \$131.95; 131.95 \div 18.20 = 7\frac{1}{4} \text{ per cent., Ans.}$$

$$9. \$480 \times .01 = \$4.80 \times 1\frac{7}{8} = \$6.20; \$529.60 - \$480 = \$49.60; 49.60 \div 6.20 = 8 \text{ per cent., Ans.}$$

(ART. 360, p. 271.)

$$2. \$120 \times .08\frac{1}{4} = \$9.90; 13.20 \div 9.90 = 1\frac{1}{3} \text{ yr., Ans.}$$

$$3. \$144 \times .07 = \$10.08; 10.08 \div 10.08 = 1 \text{ yr., Ans.}$$

$$4. \$240 \times .06 = \$14.40; \$280 - \$240 = \$40; 40 \div 14.40 = 2\frac{5}{9} \text{ yr.} = 2 \text{ yr. 9mo. 10da., Ans.}$$

$$5. \$1.00 \times .05 = \$.05; 1.00 \div .05 = 20 \text{ yr., Ans.}$$

$$6. \$1.00 \times .10 = \$.10; 1.00 \div .10 = 10 \text{ yr., Ans.}$$

$$7. \$1500 \times .05 = \$75.00; \$2250 - \$1500 = \$750; 750 \div 75 = 10 \text{ yr., Ans.}$$

$$8. \$480 \times .04\frac{1}{2} = \$21.60; \$561.60 - \$480 = \$81.60; 81.60 \div 21.60 = 3\frac{7}{9} \text{ yr.} = 3 \text{ yr. 9mo. 10da., Ans.}$$

$$9. \$1728 \times .12 = \$207.36; \$3853.44 - \$1728 = \$2125.44; 2125.44 \div 207.36 = 10\frac{1}{4} \text{ yr.} = 10 \text{ yr. 3mo., Ans.}$$

$$10. \$240 \times .06 = \$14.40; \$720 - \$240 = \$480; 480 \div 14.40 = 33\frac{1}{3} \text{ yr.} = 33 \text{ yr. 4mo., Ans.}$$

$$11. \$400 \times .06 = \$24.00; 100 \div 24 = 4\frac{1}{6} \text{ yr.} = 4 \text{ yr. 2mo.};$$

1857	4	16	
	4	2	0
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$$1861 \quad 6 \quad 16 = \text{July 16, 1861, Ans.}$$

PROMISSORY NOTES.

(PAGE 275.)

(2.)

Principal on interest from June 17, 1849, . . . \$769.870

Interest from June 17, 1849, to March 1, 1850,

8mo. 14da., 32.591

Amount (carried forward), . . . \$802.461

	(Brought forward:)	\$ 802.461
First payment, March 1, 1850,		<u>75.500</u>
New principal, bearing interest from March 1, 1850,		726.961
Interest from March 1, 1850, to June 11, 1851,		
15mo. 10da.,		<u>55.733</u>
Amount,		782.694
Second payment, June 11, 1851,		<u>165.000</u>
New principal, bearing interest from June 11, 1851,		617.694
Interest from June 11, 1851, to Sept. 15, 1851,		
3mo. 4da.,		<u>9.677</u>
Amount,		627.371
Third payment, Sept. 15, 1851,		<u>161.000</u>
New principal, bearing interest from Sept. 15, 1851,		466.371
Interest from Sept. 15, 1851, to Jan. 21, 1852,		
4mo. 6da.,		<u>9.793</u>
Amount,		476.164
Fourth payment, January 21, 1852,		<u>47.250</u>
New principal, bearing interest from Jan. 21, 1852,		428.914
Interest from Jan. 21, 1852, to December 6, 1853,		
22mo. 15da.,		<u>48.252</u>
Amount,		477.166
Fifth payment, less than interest, March 5,		
1853,	12.17	
Sixth payment, more than interest, Dec. 6,		
1853,	<u>98.00</u>	
		110.170
New principal, bearing interest from Dec. 6, 1853,		366.996
Interest from Dec. 6, 1853, to July 7, 1854, 7mo. 1d.,		<u>12.906</u>
Amount (carried forward),		\$ 379.902

	(Brought forward:)	\$ 379.902
Seventh payment, July 7, 1854,		<u>169.000</u>
New principal, bearing interest from July 7, 1854,		210.902
Interest from July 7, 1854, to Sept. 25, 1855, 14mo.		
18da.,		<u>15.395</u>
Balance due Sept. 25, 1855,		\$ 226.297

(3.)

Principal on interest from April 30, 1851,		\$ 300.000
Interest from April 30, 1851, to June 27, 1852,		
13mo. 27da.,		<u>20.850</u>
Amount,		320.850

First payment, June 27, 1852,		<u>150.000</u>
New principal, bearing interest from June 27, 1852,		170.850
Interest from June 27, 1852, to December 9, 1852,		
5mo. 12da.,		<u>4.612</u>
Amount,		175.462

Second payment, Dec. 9, 1852,		<u>150.000</u>
New principal, bearing interest from Dec. 9, 1852,		25.462
Interest from Dec. 9, 1852, to Oct. 9, 1853, 10mo.,		<u>1.273</u>
Balance due Oct. 9, 1853,		\$ 26.735

(4.)

Principal on interest from Feb. 11, 1852,		\$ 54.180
Interest from Feb. 11, 1852, to July 11, 1853, 17mo.,		<u>4.605</u>
Amount,		58.785

First payment, July 11, 1853,		<u>12.250</u>
Principal bearing interest from July 11, 1853,		46.535
Interest from July 11, 1853, to Aug. 21, 1855, 25mo.		
10da.,		<u>5.894</u>
Amount (carried forward),		\$ 52.429

	(Brought forward :)	\$ 52.429
Second payment, less than interest, Aug. 15,		
1854,	2.100	
Third payment, less than interest, July 9,		
1855,	3.120	
Fourth payment, more than interest, Aug. 21,		
1855,	<u>37.180</u>	
		<u>42.400</u>
Principal bearing interest from Aug. 21, 1855, .		10.029
Interest from Aug. 21, 1855, to Dec. 17, 1855, 3mo.		
26da.,		<u>.193</u>
Balance due Dec. 17, 1855,		\$ 10.222

(6.)

Principal carrying interest from Jan. 1, 1850, .	\$ 1000.00
Interest from Jan. 1, 1850, to Sept. 28, 1850, 8mo.	
27da.,	<u>51.91</u>
Amount,	1051.91
First payment,	<u>144.00</u>
Balance for new principal,	907.91
Interest from Sept. 28, 1850, to July 17, 1851, 9mo.	
19da.,	<u>51.01</u>
Amount,	958.92
Second payment, March 1, 1851, a sum less	
than interest,	20.00
Third payment, July 17, 1851, a sum greater	
than interest,	<u>360.00</u>
	<u>380.00</u>
Balance for new principal,	578.92
Interest from July 17, 1851, to Aug. 9, 1851, 22da.,	<u>2.47</u>
Amount (carried forward),	\$ 581.39

	(Brought forward:)	\$ 581.39
Fourth payment, Aug. 9, 1851,		190.00
Balance for new principal,		391.39
Interest from Aug. 9, 1851, to Sept. 25, 1852, 1yr.		
1mo. 16da.,		30.89
Amount,		422.28
Fifth payment, Sept. 25, 1852,		170.00
Balance for new principal,		252.28
Interest from Sept. 25, 1852, to Dec. 11, 1853, 1yr.		
2mo. 16da.,		21.38
Amount,		273.66
Sixth payment, Dec. 11, 1853,		200.00
Balance for new principal,		73.66
Interest from Dec. 11, 1853, to July 4, 1855, 1yr.		
6mo. 23da.,		8.06
Amount,		81.72
Seventh payment, July 4, 1855,		75.00
Balance for new principal,		6.72
Interest from July 4, 1855, to June 1, 1857, 1yr.		
10mo. 27da.,		.89
Balance due at the time of payment,		\$ 7.61

NOTE. — Interest on the above note is computed at 7 per cent.

(ART. 372, p. 278.)

(2.)

Principal,		\$ 700.00
Interest for 9mo. 24da.,		34.30
		734.30
First payment,	160.00	
Interest for 8mo. 10da.,	6.66 $\frac{2}{3}$	
	(Carried forward:)	\$ 166.66 $\frac{2}{3}$ \$ 734.30

	(Brought forward :)	\$ 166.66 $\frac{2}{3}$	\$ 734.80
Second payment,		200.	
Interest for 5mo. 4da.,		5.13 $\frac{1}{2}$	
Third payment,		120.	
Interest for 2mo. 17da.,		1.54	
Fourth payment,		60.	
Interest for 1mo. 23da.,53	
		<u>553.87</u>	
Balance due Nov. 28, 1854,			\$ 180.43

	(3.)		
Principal,		\$ 500.00	
Interest for 12mo.,		35.00	
		<u>535.00</u>	
Payment,		200.00	
Interest for 9mo.,		10.50	
		<u>\$ 210.50</u>	
Balance due April 1, 1858,			\$ 324.50

COMPOUND INTEREST.

(ART. 376, p. 280.)

	(2.)	
\$ 500	(Brought up :)	31.80
.06		530
<u>30.00</u>		561.80
500		.06
530		33.7080
.06		561.80
<u>31.80</u> (Carried up.)		<u>\$ 595.5080, Ans.</u>

(3.)	(4.)
\$ 970	\$ 300
.06	.07
<u>58.20</u>	<u>21.00</u>
970	300
<u>1028.20</u>	<u>321</u>
.06	.07
61.6920	22.47
<u>1028.20</u>	<u>321</u>
1089.8920	343.47
.049 Int. of \$ 1 for 9mo. 24da.	.07
98090280	24.0429
43595680	343.47
53.404708	367.5129
<u>1089.8920</u>	<u>.07</u>
1143.2967	25.725903
970 First principal subtracted.	367.5129
\$ 173.296, Ans.	393.2388
	.035
	<u>19661940</u>
	11797164
	<u>13.763358</u>
	393.238
	<u>407.001</u>
	300
	<u>\$ 107.001, Ans.</u>

(5.)

\$ 316	(Brought up.) 376.361
<u>.06</u>	<u>.023</u>
18.96	1129083
<u>316</u>	<u>752722</u>
334.96	8.656303
<u>.06</u>	<u>276.261</u>
20.0976	385.017
<u>334.96</u>	<u>316</u>
355.0576	\$ 69.017, Ans.
<u>.06</u>	
21.303456	
<u>355.0576</u>	
376.361 (Carried up.)	

(ART. 377, p. 282.)

2. $\$ 1200 \times 2.104852 = 2525.8224$; $2525.8224 - 1200 = \$ 1325.822$, Ans.
3. $\$ 300 \times 1.790848 = 537.2544$; $537.2544 \times 1.0375 = 557.401$; $557.401 - 300 = \$ 257.401$, Ans.
4. $\$ 5 \times 29.457 = \$ 147.285$; $\$ 147.285 - \$ 5 = \$ 142.285$, Ans.
5. $\$ 480 \times 10.285718 = \$ 4937.144$, Ans.
6. $\$ 40 \times .310796 = \$ 12.431$, Ans.
7. The amount of \$ 1 for 50 years is \$ 18.420154; if now this amount be taken for a principal and multiplied by the amount of \$ 1 for 50 years, the result will be the amount of \$ 1 for 100 years; therefore $18.420154 \times 18.420154 = 339.302072$, the amount of \$ 1 for 100 years; and $339.302072 \times \$ 100 = \$ 33930.2072$; and $\$ 33930.2072 - \$ 100 = \$ 33830.2072$, Ans.
8. $\$ 1000 \times 6.84059 = \$ 6840.59$; $\$ 6840.59 \times 1.02 = \$ 6977.4018$; $\$ 6977.4018 - \$ 1000 = \$ 5977.4018 =$ interest at compound interest; $\$ 1000 \times 2.00 = \$ 2000 =$

interest at simple interest; \$ 5977.4018 — \$ 2000 =
\$ 3977.4018, Ans.

9. For 21y., payable semi-annually, is the same as for 42y.,
payable annually; $\$50 \times 3.460696 = \173.034 , Ans.

(10.)

Principal bearing interest from March 26, 1854,	\$ 100.000
Interest for 1y. 5mo.,	8.650
Amount of the principal to Aug. 25, 1855, . . .	<u>108.650</u>
First payment, June 11, 1854,	50.000
Compound interest from June 11, 1854, to Aug. 25, 1855, 14mo. 14da.,	3.653
Second payment, Sept. 25, 1854,	50.000
Compound interest from Sept. 25, 1854, to Aug. 25, 1855, 11mo.,	<u>2.750</u>
Amount of the endorsements,	<u>106.403</u>
Balance due Aug. 25, 1855,	<u>\$ 2.247</u>

(11.)

Principal bearing interest from Jan. 1, 1850, . . .	\$ 1000.00
Compound interest on \$1000 from Jan. 1, 1850, to April 1, 1855, 5y. 3mo.	<u>427.09</u>
Amount of principal to April 1, 1855,	<u>1427.09</u>
First payment, June 10, 1850,	70.00
Compound interest from June 10, 1850, to April 1, 1855, 4y. 9mo. 21d.,	26.94
Second payment, Sept. 25, 1851,	80.00
Compound interest from Sept. 25, 1851, to April 1, 1855, 3y. 6mo. 6da.,	21.54
Third payment, July 4, 1852,	100.00
Compound interest from July 4, 1852, to April 1, 1855, 2y. 8mo. 27da.,	20.43
Fourth payment, Nov. 11, 1853,	<u>30.00</u>

(Carried forward :) \$ 348.91 \$ 1427.09

(Brought forward:)	\$ 348.91	\$ 1427.09
Compound interest from Nov. 11, 1853, to April 1, 1855, 1y. 4mo. 20da.,	2.97	
Fifth payment, June 5, 1854,	50.00	
Compound interest from June 5, 1854, to April 1, 1855, 9mo. 26da.,	2.87	
Amount of endorsements,		404.75
Balance due April 1, 1855,		\$ 1022.34

NOTE. — The above note and endorsements are reckoned at 7 per cent.

(ART. 378, p. 284.)

2. $1.60578 \times 1.035 = 1.553255 =$ the amount of \$1 for 6y. 6mo.; $\$205.90 \div .5532 = \372.16 , Ans.
3. $1.191 \times 1.012 = 1.2052 =$ amount of \$1 for 3y. 2mo. 12da.; $\$1026.54 \div .2052 = \5000 , Ans.
4. $1.905 =$ interest of \$1 for 31y. at $3\frac{1}{2}$ per cent.; $\$857.25 \div 1.905 = \450 , Ans.

(ART. 379, p. 285.)

2. $640.405 \div 400 = 1.60101$, which for 12 years indicates by the table an interest of 4 per cent., Ans.
3. $2096.147 \div 2500 = .8384588$; and in the table this is found to be the *interest* of \$1 for 9 years at 7 per cent.
4. $200 \div 100 = 2.000000$; by the table it is seen that at 6 per cent. this will be the amount of \$1 for a time somewhere between 11 and 12 years. The amount for 11y. is 1.898299; if this, increased by $\frac{2}{100}$ of a year, be 2., then the rate is 6 per cent.; $1.898299 \times 1.0534 = 2.00000$ very nearly.
5. $\$31479.70 \div 10000 = 3.14797$; 3.0256, the amount of \$1 for 19y. at 6 per cent., is the approximation; $3.0256 \times 1.04\frac{2}{3} = 3.14797$ nearly; hence the rate is 6 per cent.

(ART. 380, p. 285.)

2. $640.405 \div 400 = 1.601012$; which is the amount of \$1 at 4 per cent. for 12 years, Ans.

3. $9021.78 \div 6000 = 1.50363$, which is the amount of \$1 at 7 per cent. for 6 years, Ans.
4. $200 \div 100 = 2.000000$; by the table the amount at 5 per cent., which approximates to 2., is the amount for 14y., which is 1.979932; $2.0000 - 1.979932 =$ that interest which the amount for 14y. (1.979932) must gain before the \$1 will be doubled; therefore, to get the time required for gaining this principal, divide (Art. 360) the given interest by the interest of 1.979932 for 1y.; that is, divide $2.000 - 1.979932$ by $2.078928 - 1.979932$; $.020068 \div .098996 = .2026y. = 2mo. 13da. \text{ nearly};$ 14y. 2mo. 13da., Ans.
5. $200 \div 100 = 2.$; by the table the nearest approximation to 2., at 6 per cent., is the amount for 11y., 1.898299; $2.012196 - 1.898299 = .113897$; $2. - 1.898299 = .101701$; $.101701 \div .113897 = .89y. = 10mo. 20+da.$; 11y. 10mo. 20+da., Ans.
6. $1781.665 \div 450 = 3.959$, which by the table is found to be the amount of \$1 at $3\frac{1}{2}$ per cent. for 40y.; but, as the principal in the problem pays $3\frac{1}{2}$ per cent. in periods of half a year, its 40 periods will be only 20y. The deposit was made when the son was 1 year old; and $20 + 1 = 21y.$, Ans.

DISCOUNT AND PRESENT WORTH.

(ART. 386, p. 287.)

2. \$1.07 = amount of \$1 for the given time; $\$802.50 \div 1.07 = \750 ; $\$802.50 - 750 = \52.50 , Ans.
3. \$1.12 = amount of \$1 for the given time; $\$117.60 \div 1.12 = \105.00 , Ans.
4. \$1.205 = amount of \$1 for 3y. 5mo.; $\$769.60 \div 1.205 = \638.672 , Ans.

5. $\$1.26875 = \text{amount of } \$1 \text{ for } 3\text{y. } 7\text{mo.}; \$678.75 \div 1.26875 = \$534.975, \text{ Ans.}$
6. $\$1.25 = \text{amount of } \$1 \text{ for the time};$
 $\$600 \div 1.25 = \$480 \text{ present worth};$
 $\$600 - \$480 = \$120 \text{ discount, Ans.}$
7. $\$1.056\frac{1}{2} = \text{amount of } \$1 \text{ for } 11\text{mo. } 11\text{da.}; \$79.87 \div 1.056\frac{1}{2} = \$75.574+.$
 $\$1.111\frac{1}{2} = \text{amount of } \$1 \text{ for } 22\text{mo. } 7\text{da.}; \$87.75 \div 1.111\frac{1}{2} = \$78.971; \$78.971 + \$75.574 = \$154.545+, \text{ Ans.}$
8. $\$1.118 = \text{amount of } \$1 \text{ for } 23\text{mo. } 18\text{da.}; \$365.87 \div 1.118 = \$327.254+.$
 $\$1.1525 = \text{amount of } \$1 \text{ for } 30\text{mo. } 15\text{da.}; \$161.15 \div 1.1525 = \$139.826+.$
 $\$1.028\frac{2}{3} = \text{amount of } \$1 \text{ for } 5\text{mo. } 22\text{da.}; \$112.50 \div 1.028\frac{2}{3} = \$109.364+.$
 $\$1.258 = \text{amount of } \$1 \text{ for } 51\text{mo. } 18\text{da.}; \$96.81 \div 1.258 = \$76.955+.$
 $\$327.254 + \$139.826 + \$109.364 + \$76.955 + = \$653.40+, \text{ Ans.}$
9. $\$67.25 \div 1.18 = \$56.99, \text{ Ans.}$
10. $\$80.095 \div 1.191 = \$67.25, \text{ Ans.}$
11. $\$110.364 \div 1.402552 = 78.687; \$110.364 - \$78.687 = \$31.677 \text{ discount, Ans.}$

(ART. 387, p. 288.)

1. $\$960 \times .05 = \$48; \$960 - \$48 = \$912, \text{ Ans.}$
2. $\$5000 \div 1.07 = \$4672.89; \$5000 - \$4672.89 = \$327.11 \text{ true discount}; \$5000 \times .07 = \$350 \text{ nominal discount}; \$350 - \$327.11 = \$22.89, \text{ Ans.}$
3. $\$4440 \times .03 = 133.20; \$4440 - \$133.20 = \$4306.80;$
 $\$4306.80 \text{ is the nominal present worth, and is the sum which must be hired at } 6 \text{ per cent. to pay the debt};$
 $\$4306.80 \times .03 = \$129.204; \$133.20 - \$129.204 = \$3.996, \text{ Ans.}$

BANKING.

(ART. 392, p. 290.)

2. $\$7800 \times (.015 + .0005 \text{ for 3 days of grace}) .0155 = \120.90 , Ans.
3. $\$1200 \times .01225 = \14.70 , Ans.
4. $\$8000 \times .0105 = \84 discount; $\$8000 - \$84 = \$7916$ proceeds, Ans.
5. $\$760 \times .0255 = \19.38 ; $\$760 - \$19.38 = \$740.62$, Ans.
6. $\$7860 \times .030\frac{1}{2} = \239.73 ; $\$7860 - \$239.73 = \$7620.27$, Ans.
7. The time from Dec. 3 to April 3 is 4mo.; 4mo. from 6mo. leave 2mo. or 60da.; 60da. + 3da. = 63da., the time the note is to run; $\$160.40 \times .0105 = \1.68 ; $\$160.40 - \$1.68 = \$158.72$, Ans.
8. 123da. from Apr. 16, 1856 = Aug. 17, 1856, the time when the note is legally due; from May 16 to Aug. 17 = 93da., the time for which the note was discounted; $\$890.50 \times .0155 = \13.80275 ; $\$13.80275 \div 6, \times 7 = \16.1032 , Ans.
9. $90 + 3\text{da.}$ from May 1 = Aug. 2, the legal time of maturity; May 1 to June 11 = 41da.; $93 - 41 = 52\text{da.}$, for which the note was discounted; $\$1340 \times .008\frac{2}{3} = \11.61 ; $\$1340 - \$11.61 = \$1328.39$ proceeds, Ans.
10. From June 19 to July 5 are 16da.; from June 19 to Aug. 19 are 61da.; $61 + 3 = 64\text{da.}$; $64 - 16 = 48$, the time the note had to run, and for which it was discounted; $\$9000 \times .032 = \288 ; $\$9000 - \$288 = \$8712$, Ans.

(ART. 393, p. 292.)

2. $\$1 \times .0180\frac{5}{8} = \$0.0180\frac{5}{8}$; $\$1.00 - \$0.0180\frac{5}{8} = \$0.9819$, the proceeds of $\$1$; $\$680 \div .9819 = \692.523 , Ans.

3. Proceeds of \$1 for 120 + 3da. = .9795; \$540.50 ÷ .9795 = \$551.81, Ans.
4. \$1938 ÷ .9695 = \$1998.968, Ans.
5. \$1681.60 ÷ .9795 = \$1665.74, Ans.
6. Present worth of \$1 for 33da. at 2 per cent. per month = .989; \$1500 ÷ .989 = \$1516.68, Ans.

(ART. 394, p. 292.)

2. The proceeds of \$1 for 30 + 3 da. = .9945; .06 ÷ .9945 = $6\frac{8}{1989}$ per cent., Ans.
3. .06 ÷ .9845 = $6\frac{186}{1989}$ per cent., Ans.
4. .02 ÷ .918 = $2\frac{22}{559}$ per cent. a month, Ans.
5. The proceeds of \$1 are .96442; .07 ÷ .96442 = $7\frac{1121}{1027333}$ per cent., Ans.
6. .06 ÷ .94 = $6\frac{1}{17}$ per cent., Ans.

(ART. 395, p. 293.)

2. The amount of \$1 for 33da. = 1.0055; .06 ÷ 1.0055 = $5\frac{844}{10055}$ per cent., Ans.
3. 2 per cent. a month = 24 per cent. per annum; .24 ÷ 1.042 = $23\frac{17}{521}$ per cent., Ans.
4. .06 ÷ 1.0155 = $5\frac{1844}{3631}$ per cent., Ans.
5. .08 ÷ 1.027333 = $7\frac{808669}{1027333}$ per cent., Ans.
6. .07 ÷ 1.03558 = $6\frac{7778}{103558}$ per cent., Ans.
7. .06 ÷ 1.06 = $5\frac{3}{13}$ per cent., Ans.
8. The amount of \$1 for 8yr. 4mo. at 5 per cent. = \$1.41 $\frac{1}{3}$; .05 ÷ 1.41 $\frac{1}{3}$ = .03 $\frac{2}{17}$, Ans.

MISCELLANEOUS EXAMPLES.

(PAGE 294.)

1. 18 of gold; 20 : 18 :: 100 : 90 per cent. of gold;
 1 of copper; 20 : 1 :: 100 : 5 per cent. of copper;
 1 of silver; 20 : 1 :: 100 : 5 per cent. of silver;
 20;

2. $\frac{1}{8}$ = amount, $\frac{1}{8}$ = principal, $\frac{1}{8}$ = discount. Therefore,
 $32 \times 16 = 512 \div 15 = 34\frac{2}{15}$ qt., Ans.
3. $\$25000 \times .04 = \1000 , Ans.
4. $206\frac{1}{4} - 192 = 14\frac{1}{4}$; $192 : 14\frac{1}{4} :: 1.00 : .07\frac{2}{3}\%$, Ans.
5. $1.50 : 1.00 :: 1.00 : .66\frac{2}{3}$; $1.00 - .66\frac{2}{3} = .33\frac{1}{3}$, Ans.
6. How long will it take \$10 to double itself, or gain \$10 interest? (Art. 360.) $10 \div .50 = 20$ yr.; $10 \div 60 = 16\frac{2}{3}$ yr.; $20 - 16\frac{2}{3} = 3\frac{1}{3}$ yr., Ans.
7. $\$500 \div 1.2155 = \411.351 , Ans.
8. $1.00 : 1.10 :: \$5 : \5.50 , the price at which the cloth was actually sold, it being 10 per cent. less than the asking price; therefore, $.90 : 1.00 :: \$5.50 : \$6.11\frac{1}{3}$, Ans.
9. $\$7860 \times .030\frac{1}{2} = \239.73 ; $\$7860 - \$239.73 = \$7620.27$, Ans.
10. $\$1500 \div 1.06 = \$1415.09\frac{2}{3}$; $\$1500 - \$1415.09\frac{2}{3} = \$84.90\frac{2}{3}$, the true discount; $\$1500 \times .06 = \90 ; $\$90 - \$84.90\frac{2}{3} = \$5.09\frac{2}{3}$, Ans.
11. $.24 \div .958 = 25\frac{2}{5}$ per cent., Ans.
12. Interest of \$1 at $1\frac{1}{2}$ per cent. a month = \$.0225; $\$36.40 \div .0225 = \$1617.77\frac{1}{3}$, Ans.
13. $.12 \div 1.011 = 11\frac{2}{3}$ per cent., Ans.
14. In the course of 693 days the bank can discount notes of 60 days 11 times, and notes of 30 days 21 times; then (Art. 393) $\$50000 \div .9945 = \50276.52 = the face of the note or notes discounted for 33 days; $\$50276.52 - \$50000 = \$276.52$ = the sum gained in each period of 33da.; $\$276.52 \times 21 = \5806.92 = the whole amount gained on 30 days' notes in 693 days; and $\$50000 \div .9895 = \50530.57 ; $\$50530.57 - \$50000 = \$530.57$; $\$530.57 \times 11 = \5836.27 = the whole sum gained on 60 days' notes in 693 days; $\$5836.27 - \$5806.92 = \$29.35$ = the amount which is gained on notes of 60 days *more* than on notes of 30 days, Ans.

15. $450 \times \$3.50 = \1575 ; $450 \times \$4.00 = \1800 ; $\$1800 \div .0305 = \54.90 ; $\$1800 - \$54.90 = \$1745.10$;
 $\$1745.10 - \$1575 = \$170.10$, Ans.
16. $6\frac{1}{2} - 6 = \frac{1}{2}$ per cent. $6\frac{1}{2} : \frac{1}{2} :: \$1.00 : \$0.076\frac{1}{3}$, the interest of \$1 for the required time at 6 per cent.; $\$0.06 : \$0.076\frac{1}{3} :: 12 \text{ months} : 15\text{m. } 11\frac{1}{3}\text{d.}$, Ans. [See solution, p. 252.]

STOCKS.

(ART. 400, p. 296.)

2. $\$24360 \times 1.35 = \32886 , Ans.
3. $\$100 \times 15 \times 1.13 = \1695 , Ans.
4. $\$100 \times 10 \times .85 = \850 , Ans.
5. $\$100 \times 30 = \3000 ; $\$3000 \times 1.08\frac{2}{3} = \3262.50 , amount; $\$3262.50 - \$3000 = \$262.50$, premium, Ans.
6. $\$50 \times 25 \times .97 = \1212.50 , Ans.
7. $\$250 \times 22 = \5500 ; $\$5500 \times .95 = \5225 , amount paid; $\$5500 - \$5225 = \$275$, Ans.
8. $\$50000 \times 1.17 = \58500 , Ans.
9. $\$19500 \times .93 = \18135 ; $\$19500 \times 1.03 = \20085 ;
 $\$20085 - \$18135 = \$1950$, Ans.

(ART. 401, p. 297.)

2. $\$6210 \div 1.035 = \6000 , Ans.
3. $\$1155 \div 11 = \105 ; $\$105 \div 1.05 = \100 , Ans.
4. $\$1230 \div 41 = \30 ; $\$30 \div .60 = \50 , Ans.
5. $\$2052 \div 1.08 = \1900 ; $\$2052 - \$1900 = \$152$, Ans.
6. $\$16245 \div .95 = \17100 ; $\$17100 - \$16245 = \$855$,
 Ans.
7. $862.50 \div 57.50 = 15 \text{ shares}$; $\$862.50 \div 1.15 = \750 ;
 $\$862.50 - \$750 = \$112.50$, Ans.
8. $\$1000 \times .88 = \880 ; $7920 \div 880 = 9 \text{ bonds}$; $\$1000 - \$880 = \$120$; $\$120 \times 9 = \1080 , Ans.

(ART. 402, p. 298.)

2. $1.00 - .25 = .75$; $.06 \div .75 = .08$, Ans.
3. $.11 \div 1.10 = 10$ per cent., Ans.
4. (402, note.) $.12 \div .8 = 1.50$; $1.50 - 1.00 = .50$, Ans.
5. $.09 \div 1.25 = .07\frac{1}{5}$; $.06 \div .75 = .08$; $.08 - .07\frac{1}{5} = \frac{1}{5}$ per cent.; therefore, it is the better investment by $\frac{1}{5}$ per cent. to buy the 6 per cent. stock.
6. $.05 \div .07 = .71\frac{3}{4}$; $1.00 - .71\frac{3}{4} = .28\frac{1}{4}$, Ans.
7. $.05 \div .80 = .06\frac{1}{4}$; $\$20000 \times .06\frac{1}{4} = \1250 ; $\$20000 \times .06 = \1200 ; $\$1250 - \$1200 = \$50$, Ans.

BROKERAGE AND COMMISSION.

(ART. 407, p. 299.)

2. $\$18768 \times .01\frac{3}{4} = \328.44 , Ans.
3. $\$112.25 \times 12 = \1347 ; $1347 \times .00\frac{1}{4} = \$3.36\frac{3}{4}$, Ans.
4. $\$12.25 \times 700 = \8575 ; $\$8575 \times .01\frac{1}{4} = \$150.06\frac{1}{4}$,
Ans.
5. $50 \div 10000 = .005 = \frac{1}{2}$ per cent., Ans.
6. $\$8.95 \times 173 \times .01\frac{7}{8} = \$29.03\frac{5}{32}$, Ans.
7. $\$107.75 \times 37 \times .00\frac{3}{8} = \$14.95\frac{3}{32}$, Ans.
8. $395\text{£}. 15\text{s}. 5\text{d}. = 94985\text{d}. ; 94985 \times .02\frac{1}{4} = 2187.1625\text{d}. = 8\text{£}. 18\text{s}. 1\frac{6}{10}\text{d}. ,$ Ans.
9. $5.46 \div 364 = .01\frac{1}{2}$, Ans.

(ART. 408, p. 301.)

2. $\$1976 \div 1.04 = \1900 ; $\$1976 - \$1900 = \$76$, Ans.
3. $\$8341.50 \div 1.005 = \8300 ; $\$8300 \div 83 = 100$ shares;
 $\$8341.50 - \$8300 = \$41.50$ brokerage, Ans.
4. $\$8960 \div 1.02 = \$8784.31\frac{1}{11}$; $\$8960 - \$8784.31\frac{1}{11} =$
 $\$175.68\frac{3}{11}$, Ans.

5. $\$5150 \div 1.03 = \5000 ; $\$5150 - \$5000 = \$150$, Ans.
6. $\$285 \div .015 = \19000 , Ans.
- 7.. $\$740.83\frac{1}{4} - \$7.50 = \$733.33\frac{1}{4}$ = the bill minus the cartage; $\$733.33\frac{1}{4} \div 1.015 = \722.50 ; $\$733.33\frac{1}{4} - \$722.50 = \$10.83\frac{1}{4}$ = commission; $250 \times 34 = 8500$ lb. of sugar; $\$722.50 \div 8500 = \$0.08\frac{1}{2}$ per pound, Ans.
8. $987\text{£. } 18\text{s. } 6\text{d.} \times .0225 = 22\text{£. } 4\text{s. } 6\frac{1}{2}\frac{3}{8}\text{d.}$; $987\text{£. } 18\text{s. } 6\text{d.} - 22\text{£. } 4\text{s. } 6\frac{1}{2}\frac{3}{8}\text{d.} = 965\text{£. } 13\text{s. } 11\frac{1}{2}\frac{1}{8}\text{d.}$; $965\text{£. } 13\text{s. } 11\frac{1}{2}\frac{1}{8}\text{d.} \times 100 = 96569\text{£. } 13\text{s. } 4\frac{1}{2}\text{d.}$; $96569\text{£. } 13\text{s. } 4\frac{1}{2}\text{d.} \div 101\frac{1}{2} = 947\text{£. } 18\text{s. } 5\frac{1}{2}\frac{1}{2}\text{d.}$; $947\text{£. } 18\text{s. } 5\frac{1}{2}\frac{1}{2}\text{d.} = 227501\frac{1}{2}\frac{1}{2}\text{d.}$; $1\text{£. } 3\text{s. } 8\text{d.} = 284\text{d.}$; $227501\frac{1}{2}\frac{1}{2}\text{d.} \div 284 = 801\frac{3}{5}\frac{7}{5}\frac{6}{5}\text{yds.}$, Ans.

ACCOUNT OF SALES.

(ART. 410, p. 303.)

(2.)

Amount of sales,	\$ 2671.40
Freight,	\$ 185.50
Commission at 3 per cent. on \$ 2671.40,						74.57	
Cartage, cooperage, &c.,	26.00	
Storage and insurance,	63.24	
Amount of charges,		\$ 349.31
Net proceeds to J. B. & Co.,		\$ 2322.09

PROFIT AND LOSS.

(ART. 412, p. 304.)

2. $\$5 \times .80 = \4 , Ans.
3. $\$8.50 \times 1.10 = \9.35 , Ans.

4. $\$0.42 \times .95 = \0.399 , Ans.
 5. $\$2500 \times 1.20 = \3000 , Ans.

(ART. 413, p. 304.)

2. $\$4 \div .80 = \5 , Ans.
 3. $\$96 \div 1.20 = \80 ; $\$80 \div 10 = \8 , Ans.
 4. $\$12.50 \div 1.17 = \10.683_{117} , Ans.
 5. $\$6.125 \div .875 = \7.00 , Ans.

(ART. 414, p. 305.)

2. $\$7 - \$6.125 = \$0.875$; $.875 \div 7 = 12\frac{1}{2}$ per cent., Ans.
 3. $\$225 - \$200 = \$25$; $25 \div 200 = 12\frac{1}{2}$ per cent., Ans.
 4. $500 \div 2500 = 20$ per cent., Ans.
 5. $\$64.864_{33}$ was given for the cloth, and $\$2.50 \times 24 = \60.00 was received for it; therefore the loss was $\$4.864_{33} - \$60.00 = \$4.864_{33}$, or $\frac{4864_{33}}{64864_{33}}$ of its value,
 $= .075 = 7\frac{1}{2}$ per cent., Ans.

(ART. 415, p. 306.)

2. $\$7 \div 1.75 = \4 prime cost; $\$4 - \$3 = \$1$, loss on
 1yd.; $1 \div 4 = .25$ loss, Ans.
 3. $\$1.25 \div .85 = \1.470_{19} par value of corn; $\$1.647_{17} - 1.470_{19} = .176_{17}$ gain per bushel if sold for $\$1.647_{17}$;
 $.176_{17} \div 1.470_{19} = .12$, Ans.
 4. $\$1.647_{17} \div 1.12 = \1.470_{19} ; $\$1.470_{19} - \$1.25 = \$0.220_{19}$ loss per bushel if sold at $\$1.25$; $.220_{19} \div 1.470_{19} = 15$ per cent., Ans.
 5. $\$75 \div .75 = \100 cost of the horse; $\$100 \times 1.30 = \130 real value of horse; $\$130 - \$75 = 55$, Ans.
 6. $\$1.25 \div .75 = \$1.66\frac{2}{3}$ cost per pound; $\$1.66\frac{2}{3} - \$1.40 = .26\frac{2}{3}$ loss per lb. if sold at $\$1.40$; $$.26\frac{2}{3} \div 1.66\frac{2}{3} = 16$ per cent. loss, Ans.

MISCELLANEOUS EXAMPLES.

(PAGE 306.)

1. $\$100 \times 1.15 = \115 ; $\$115 - \$100 = \$15$ gain on 1 share; $\$120 \div 15 = 8$ shares, Ans.
2. $\$250 \times 1.09 = \272.50 cost per share; $\$272.50 - \$25 = \$247.50 =$ what one share was sold for; $\$250 - \$247.50 = \$2.50$ loss by each share on the par value; $2.50 \div 250 = .01$ discount, Ans.
3. $\$19200 \div .96 = \20000 ; $\$20000 \times .07 = \$1400 =$ income from 7 per cent. stocks; $\$19200 \div .80 = \24000 ; $\$24000 \times .05 = \$1200 =$ income from 5 per cent. stocks; $\$1400 - \$1200 = \$200$ greater income from 7 per cent. stocks, Ans.
4. If each share of $\$250$ should pay 10 per cent., the dividend would be $\$25$; then, by proportion, $\$25 : \$15 :: \$250 : \150 , Ans.
5. $\$1.00 - \$0.20 = \$0.80$; $\$0.80 \times .015 = .012$ commission on each bill of $\$1$; $\$0.80 + \$0.012 = \$0.812 =$ the cost per cent. of the bills; $.812 \times \$200 = \162.40 the amount lost on the worthless bills; $\$162.40 + \$364 = \$526.40 =$ sum which must accrue from the bills sold at par to make a net gain of $\$364$; $\$526.40 \div .188$ (the gain on each bill of $\$1$ sold at par) $= \$2800 =$ amount of bills sold at par; $\$2800 + \200 (amount of the worthless bills) $= \$3000$, Ans.
6. $\$5640 - \$76.50 = \$5563.50 =$ proceeds of cotton *minus* the storage; $\$5563.50 - \5422.50 (sum remitted) $= \$141$ commission; $\$141 \div 5640 = .02\frac{1}{2}$ the per cent. of commission, Ans.
7. The horse was bought for 70 per cent of his real worth; and $.70 \times .75 = .525$; that is, the horse was sold for $52\frac{1}{2}$ per cent. of his original worth, which is a loss of $1.00 - .525 = 47\frac{1}{2}$ per cent.; but by the question $47\frac{1}{2}$ per cent.

- of the original value = \$55; therefore $47\frac{1}{2} : \$55 :: 52\frac{1}{2} : \$60.78\frac{1}{8}$, the sum received for the horse, Ans.
8. $\$35 \div 1.04 = \$33.65, 3\frac{1}{3}$ present worth of \$35; $\$33.65, 3\frac{1}{3} - \$30 = \$3.65, 3\frac{1}{3}$ gain, Ans.
9. 63gal. — 15gal. = 48gal.; $\$2.21, 6\frac{2}{3} \times 48 = \106.40 , price sold at; $\$112 - \$106.40 = \$5.60$ loss; $\$112 : \$5.60 :: 100 : 5$ per cent. loss, Ans.
10. 63gal. — 15gal. = 48gal. left: $\$2.21, 6\frac{2}{3} \times 48 = \106.40 ; $\$106.40 \div .95 = \112 , Ans.
11. $\$9.00 \times .90 = \8.10 , what lbbl. of the damaged flour is sold for; $\$8.10 \times 25 = \202.50 , proceeds from 25bbl. of damaged flour; $\$9.50 \times 25 = \237.50 ; $\$202.50 + \$237.50 = \$440$, the avails of the flour; $\$9 \times 50 = \450 , cost of the flour; $\$450 - \$440 = \$10$ loss, Ans.
12. $\$6135 \div 1.02\frac{1}{4} = \$6000 =$ sum paid for the flour; $\$6000 \times 1.20 = \7200 ; $\$7200 - \$6135 = \$1065$; $\$1065 - \31.63 (storage, &c.) = \$1033.37, Ans.
13. $\$102.50 \times 90 = \$9225 \div 2\frac{1}{2} = \3690 , the sum remitted; $\$3690 - \$90 = \$3600 \div 95 = 37$ T. 17cwt. 3qr. 14 $\frac{2}{3}$ lb., Ans.

PARTNERSHIP.

(ART. 417, p. 309.)

2. $\$4000 \div 10000 = 40$ per cent. gain; $\$3000 \times 40 = \1200 , Jones' part.
 $\$2000 \times .40 = \800 , Weston's part.
 $\$5000 \times .40 = \2000 , Sprague's part.
3. $\$18780 : \$6780 :: \$2000 : \722.044 , C's part.
 $\$18780 : \$12000 :: \$2000 : \1277.956 , D's part.
4. Harvey $\$2500 \times .05 = \125 , Harvey's loss.
 Blake $\$3000 \times .05 = \150 , Blake's loss.
 Horsford $\$5500 \times .05 = \275 , Horsford's loss.

5. $\$332.50 - \$120 = \$212.50$ to be divided between Elliott and Mahew; $\$850 : \$212.50 :: \$500 : \125 , Elliott's share; $\$850 : \$212.50 :: \$350 : \87.50 , Mahew's share, Griswold's gain will bear the same relation to his stock that Mahew's or Elliott's does; therefore $\$125 : \$500 :: \$120 : \480 , the value of Griswold's stock.
6. $\$5000 + \$6500 + \$7500 = \19000 , amount of stock;
 $\$19000 \times .40 = \7600 ; $\$7600 \times .90 = \6840 , net gain.
 $\$19000 : \$5000 :: \$6840 : \1800 , A's gain,
 $\$19000 : \$6500 :: \$6840 : \2340 , B's gain,
 $\$19000 : \$7500 :: \$6840 : \2700 , C's gain, } Ans.
7. $\$40000$ capital; $\$2000 + \$4500 + \$2500 + \$1500 = \$10500 =$ whole gain.
 $\$10500 : \$2000 :: \$40000 : \$7619.04\frac{1}{2}$ A's,
 $\$10500 : \$4500 :: \$40000 : \$17142.85\frac{1}{2}$ B's,
 $\$10500 : \$2500 :: \$40000 : \$9523.80\frac{2}{3}$ C's,
 $\$10500 : \$1500 :: \$40000 : \$5714.28\frac{1}{2}$ D's, } Ans.
8. $\$7500$ gains $\$1000 + \$500 = \$1500$, which is 20 per cent. on the principal; hence,
 $\$2000 \times .20 = \400 , A's gain,
 $\$3000 \times .20 = \600 , B's gain,
 $\$400 + \$600 + \$500 = \1500 , C's gain, } Ans.
9. Let $A = 1.00$; then $B = 1.50$; $C = 1.25$; $\$30000 \times .25 = \7500 whole profits; $A + B + C = 3.75$.
 $3.75 : 1.00 :: \$7500 : \2000 , A's gain,
 $3.75 : 1.50 :: \$7500 : \3000 , B's gain,
 $3.75 : 1.25 :: \$7500 : \2500 , C's gain, } Ans.
10. The fractions $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, are as the fractions $\frac{20}{60}, \frac{15}{60}, \frac{12}{60}$, or as 20, 15, 12; $20 + 15 + 12 = 47$; $\frac{20}{47} =$ Walker's, $\frac{15}{47} =$ Edwards', $\frac{12}{47} =$ Armstrong's share of the gain; but Armstrong's is divided between Walker and Edwards, and Walker's share is to Edwards' as 20 to 15; hence

Walker's share is $\frac{2}{7} + \frac{2}{8}$ of $\frac{1}{4} = \frac{1}{2}$; and Edwards' share is $\frac{1}{4} + \frac{1}{8}$ of $\frac{1}{4} = \frac{3}{8}$; then

$$\left. \begin{array}{l} \frac{1}{2} \text{ of } \$50000 = \$28571.42\frac{2}{3} \text{ Walker's,} \\ \frac{3}{8} \text{ of } \$50000 = \$21428.57\frac{1}{8} \text{ Edwards',} \end{array} \right\} \text{Ans.}$$

(ART. 418, p. 311.)

2.

BY ANALYSIS.

\$3200 for 12 months is the same as \$38400 for 1 month;
and \$4200 for 8 months is the same as \$33600 for 1 month. Therefore the whole stock may be considered
 $\$38400 + \$33600 = \$72000$. Goodwin's share of the profit will therefore be $\frac{3}{7} \frac{2}{3} \frac{2}{3} = \frac{2}{5}$ of \$240 = \$128;
Blunt's share will be $\frac{3}{7} \frac{2}{3} \frac{2}{3} = \frac{1}{5}$ of \$240 = \$112, Ans.

FORM OF STATEMENT.

$$\$3200 \times 12 = \$38400, \text{ G's product.}$$

$$\$4200 \times 8 = \$33600, \text{ B's product.}$$

$$\underline{\$72000}$$

$$\left. \begin{array}{l} \$72000 : \$38400 :: \$240 : \$128, \text{ G's gain,} \\ \$72000 : \$33600 :: \$240 : \$112, \text{ B's gain,} \end{array} \right\} \text{Ans.}$$

3.

BY ANALYSIS.

24 oxen for 8 weeks is 1 ox for 192 weeks; 18 oxen for 12 weeks is 216 oxen for 1 week; and 12 oxen for 10 weeks is 1 ox for 120 weeks. The amount of pasturing is $192 + 216 + 120 = 528$ weeks. A's share is $\frac{1}{2} \frac{2}{3} = \frac{1}{3}$ of \$26.40 = \$9.60; B's share is $\frac{2}{3} \frac{1}{2} = \frac{1}{3}$ of \$26.40 = \$10.80; C's share is $\frac{1}{2} \frac{2}{3} = \frac{1}{3}$ of \$26.40 = \$6.00, Ans.

FORM OF STATEMENT.

$$24 \text{ oxen} \times 8 = 192 \text{ oxen.}$$

$$18 \text{ oxen} \times 12 = 216 \text{ oxen.}$$

$$12 \text{ oxen} \times 10 = 120 \text{ oxen.}$$

$$\underline{528 \text{ oxen.}}$$

$$\left. \begin{array}{l} 528 : 192 :: \$26.40 : \$9.60 \text{ what A pays,} \\ 528 : 216 :: \$26.40 : \$10.80 \text{ what B pays,} \\ 528 : 120 :: \$26.40 : \$6.00 \text{ what C pays,} \end{array} \right\} \text{Ans.}$$

(4.)

$$\$300 \times 5 = \$1500$$

$$\$400 \times 8 = \$3200$$

$$\$500 \times 3 = \$1500$$

$$\underline{\$6200}$$

$$\begin{array}{l} \$6200 : \$1500 :: \$100 : \$24.19\frac{1}{3}\frac{1}{3} \text{ Barclay's,} \\ \$6200 : \$3200 :: \$100 : \$51.61\frac{3}{4} \text{ Hickman's,} \\ \$6200 : \$1500 :: \$100 : \$24.19\frac{1}{3}\frac{1}{3} \text{ Oliver's,} \end{array} \left. \vphantom{\begin{array}{l} \\ \\ \end{array}} \right\} \text{Ans.}$$

5. $\$21 + \$40 + \$24 = \85 , whole gain; and

$$\frac{2}{3}\frac{1}{5} = \text{A's for 7mo., and } \frac{2}{3}\frac{1}{5} \div 7 = \frac{2}{3}\frac{1}{35}, \text{ A's for 1mo.}$$

$$\frac{4}{5}\frac{2}{5} = \text{B's for 8mo., and } \frac{4}{5}\frac{2}{5} \div 8 = \frac{1}{5}\frac{1}{5}, \text{ B's for 1mo.}$$

$$\frac{2}{3}\frac{4}{5} = \text{C's for 12mo., and } \frac{2}{3}\frac{4}{5} \div 12 = \frac{2}{3}\frac{1}{15}, \text{ C's for 1mo.}$$

$$\frac{2}{3}\frac{1}{35} + \frac{1}{5}\frac{1}{5} + \frac{2}{3}\frac{1}{15} = \frac{1}{3}\frac{2}{5}; \text{ hence}$$

$$\left. \begin{array}{l} \frac{1}{3}\frac{2}{5} : \frac{2}{3}\frac{1}{35} :: \$1000 : \$300 = \text{A's capital,} \\ \frac{1}{3}\frac{2}{5} : \frac{1}{5}\frac{1}{5} :: \$1000 : \$500 = \text{B's capital,} \\ \frac{1}{3}\frac{2}{5} : \frac{2}{3}\frac{1}{15} :: \$1000 : \$200 = \text{C's capital,} \end{array} \right\} \text{Ans.}$$

6. Let 100 = gain of each; $100 + 100 = 200$, gain of both.

$$\frac{1}{2}\frac{2}{3} = \text{White's gain in 12mo., and } \frac{1}{2}\frac{2}{3} \div 12 = \frac{1}{4}\frac{1}{3}, \text{ White's gain in 1mo.}$$

$$\frac{1}{2}\frac{2}{3} = \text{D's gain in 8mo., and } \frac{1}{2}\frac{2}{3} \div 8 = \frac{1}{4}\frac{1}{3}, \text{ D's gain in 1mo.}$$

$$\frac{1}{4}\frac{1}{3} + \frac{1}{4}\frac{1}{3} = \frac{1}{2}\frac{1}{3}, \text{ gain of both in 1mo., or in any equal time.}$$

$$\left. \begin{array}{l} \frac{1}{4}\frac{1}{3} : \frac{1}{4}\frac{1}{3} :: \$6300 : \$2520, \text{ White's capital,} \\ \frac{1}{4}\frac{1}{3} : \frac{1}{4}\frac{1}{3} :: \$6300 : \$3780, \text{ Daniel's capital,} \end{array} \right\} \text{Ans.}$$

(7.)

$$\$4000 \times 4 = 16000$$

$$\underline{500}$$

$$4500 \times 12 = 54000$$

$$\underline{1000}$$

$$3500 \times 4 = 14000$$

$$\text{A's product} = \underline{84000}$$

$$\$3000 \times 10 = 30000$$

$$\underline{1500}$$

$$1500 \times 4 = 6000$$

$$\underline{3000}$$

$$4500 \times 6 = 27000$$

$$\text{B's product} = \underline{63000}$$

$$\begin{array}{r} \$2000 \times 6 = 12000 \\ \hline 2000 \end{array}$$

$$\begin{array}{r} 4000 \times 8 = 32000 \\ \hline 2000 \end{array}$$

$$\begin{array}{r} 6000 \times 2 = 12000 \\ \hline 1500 \end{array}$$

$$\begin{array}{r} 4500 \times 4 = 18000 \\ \hline \end{array}$$

$$\text{C's product} = 74000$$

$$\left. \begin{array}{l} \$221000 : \$84000 :: \$4420 : \$1680, \text{A's gain,} \\ \$221000 : \$63000 :: \$4420 : \$1260, \text{B's gain,} \\ \$221000 : \$74000 :: \$4420 : \$1480, \text{C's gain,} \end{array} \right\} \text{Ans.}$$

$$\begin{array}{r} \$12000 \times 6 = 72000 \\ \hline 2500 \end{array}$$

$$\begin{array}{r} 9500 \times 3 = 28500 \\ \hline 1000 \end{array}$$

$$\begin{array}{r} 8500 \times 3 = 25500 \\ \hline \end{array}$$

$$\text{G's product} = 126000$$

$$\begin{array}{r} \$18000 \times 6 = 108000 \\ \hline 2500 \end{array}$$

$$\begin{array}{r} 15500 \times 3 = 46500 \\ \hline 6000 \end{array}$$

$$\begin{array}{r} 9500 \times 3 = 28500 \\ \hline \end{array}$$

$$\text{T's product} = 183000$$

$$126000 + 183000 = 309000, \text{sum of products.}$$

$$\$8500 + 9500 = 18000; 18000 - 15000 = \$3000, \text{whole loss.}$$

$$\begin{array}{l} \$3000 \times \frac{126000}{309000} = \$1223.30_{\frac{10}{103}}, \text{G's loss;} \\ \$3000 \times \frac{183000}{309000} \\ = \$1776.69_{\frac{93}{103}}, \text{T's loss.} \end{array}$$

$$\begin{array}{l} \$8500 - \$1223.30_{\frac{10}{103}} = 7276.69_{\frac{93}{103}}, \text{G's share;} \\ \$2500 - \\ \$1776.69_{\frac{93}{103}} = \$7723.30_{\frac{10}{103}}, \text{T's share.} \end{array}$$

$$\begin{array}{r} \$4000 \times 6 = 24000 \\ \hline 2000 \end{array}$$

$$\begin{array}{r} 2000 \times 18 = 36000 \\ \hline \end{array}$$

$$\text{J's product} = 60000$$

$$\begin{array}{r} \$2500 \times 10 = 25000 \\ \hline 2000 \end{array}$$

$$\begin{array}{r} 4500 \times 14 = 63000 \\ \hline \end{array}$$

$$\text{B's product} = 88000$$

$$\begin{array}{r} \$3500 \times 8 = 28000 \\ \hline 1500 \end{array}$$

$$\begin{array}{r} 2000 \times 16 = 32000 \\ \hline \end{array}$$

$$\text{E's product} = 60000$$

$$\text{J's product } 60000$$

$$\text{E's " } 60000$$

$$\text{B's " } 88000$$

$$\hline 208000$$

(9.)

$$\left. \begin{array}{l} 208000 : 60000 :: \$1041.80 : \$300.51\frac{1}{3}, J's, \\ 208000 : 60000 :: \$1041.80 : \$300.51\frac{1}{3}, E's, \\ 208000 : 88000 :: \$1041.80 : \$440.76\frac{2}{3}, B's, \end{array} \right\} \text{Ans.}$$

10. This problem may be solved by analysis, thus: \$10000 gain \$22.50 in 12mo., and gain $2\frac{1}{2}\%$ = \$187.50 in 1mo., and $\frac{187.50}{10000} = .01875$, the gain of \$1 in 1mo.; and $\$2800 \times .01875 = \52.50 = Parkman's gain in 1mo.; and $420 \div 52.50 = 8$ mo., the time Parkman's money was in trade; and $\$3600 \times .01875 = \67.50 , Delano's gain in 1mo.; hence, $405 \div 67.50 = 6$, the number of months Delano's money was in trade. To get the rate per cent., divide the gain for 1y. by the capital, $2250 \div 10000 = .22\frac{1}{2}$ per cent., Ans.
The problem may be solved by Art. 360. Obtaining the rate per cent. as before, divide the gain by the interest of the principal for one year.

$$\begin{array}{l} \$2800 \times .22\frac{1}{2} = \$630; 420 \div 630 = \frac{2}{3}y. = 8\text{mo.}, P's \text{ time.} \\ \$3600 \times .22\frac{1}{2} = \$810; 405 \div 810 = \frac{1}{2}y. = 6\text{mo.}, D's \text{ time.} \end{array}$$

BANKRUPTCY.

(ART. 420, p. 313.)

2. $\$52384 - \$1584 = \$50800$; $50800 \div 63500 = .80$;
 $\$8361.55 \times .80 = \6689.24 , amount received by Dayton, Ans.
3. $\$600 + \$760 + \$840 + \$800 = \$3000$; $\$2275 \div 3000$
 $= \$0.75\frac{5}{8}$, the amount paid on \$1;
 $\$600 \times .75\frac{5}{8} = \455.00 , A's dividend,
 $\$760 \times .75\frac{5}{8} = \$576.33\frac{1}{3}$, B's " } Ans.
 $\$840 \times .75\frac{5}{8} = \637.00 , C's "
 $\$800 \times .75\frac{5}{8} = \$606.66\frac{2}{3}$, D's "
4. $\$400 + \$300 + \$1000 = \1700 ; $600 \div 1700 = .35\frac{3}{17}$;
 $\$400 \times .35\frac{3}{17} = \$141.17\frac{1}{17}$, A's share, }
 $\$300 \times .35\frac{3}{17} = \$105.88\frac{4}{17}$, B's " } Ans.
 $\$1000 \times .35\frac{3}{17} = \$352.94\frac{2}{17}$, C's "

5. $\$180000 + \$40000 + \$12875 = \232875 , and this amount contains both the sum distributed to the creditors and the $3\frac{1}{2}$ per cent. charges; therefore, $\$232875 \div 1.03\frac{1}{2} = \225000 = the amount distributed to the creditors; and $225000 \div 300000 = .75$, the per cent. paid; $\$1360.60 \times .75 = \1020.45 , amount received by Henderson, Ans.

TAXES.

(ART. 425, p. 316.)

2. $\$1310 \times .005 = \$ 6.55$, amount on F's property.
 $\$1.50 \times 7 = \10.50 , Forster's poll tax.
 $\$17.05$, Forster's tax, Ans.
3. $\$415.35 \times .005 = \2.07675 , Ans.
4. $\$3506300 = \text{———}$, taxable property;
 $\$14018.90 - \$3500 = \$10518.90$, to be assessed on property;
 $\$10518.90 \div 3506300 = .003 = \text{tax on } \1 ;
 $\$29010 \times .003 = \$87.03 = \text{A's property tax}$;
 $\$87.03 + \$2 = \$89.03 = \text{amount of A's tax, Ans.}$

(ART. 426, p. 316.)

2. $\$10450 \div .95 = \11000 , Ans.
3. $\$9700 \div .97 = \$10000 = \text{assessment}$; $\$10000 - \$9700 = \$300 = \text{cost of collection, Ans.}$
4. $\$756000 = \text{———}$, taxable property;
 $\$18000 \div .97 = \$18556.70 = \text{amount of tax to be raised}$;
 $\$1.50 \times 450 = \$675 = \text{amount assessed on the polls}$;
 $\$18556.70 - \$675 = \$17881.70 = \text{amount to be assessed on the property}$;
 $\$17881.70 \div 756000 = \0.0236 (nearly), tax on each dollar.

$\$15600 \times .0236 = \368.16 , A's property tax ;
 $\$1.50 \times 3 = \4.50 , A's poll tax ;
 $\$368.16 + \$4.50 = \$372.66$, amount of A's tax, Ans.

(Art. 427, p. 317.)

2. $\$500 + \$150 + \$30 = \680 , amount of expenditures;
 $\$680 - \$350 = \$330$, amount of rate money ;
 $\$330 \div 5500 = \0.06 , rate per day ;
 $60 + 60 + 30 = 150$ days' attendance for A ;
 $150 \times 0.06 = \$9$, A's bill, Ans.
3. $\$150 + \$18.50 = \$168.50$, amount of expenditures ;
 $\$168.50 - \$63.50 = \$105$, amount of rate money ;
 $\$105 \div 3000 = \0.035 , rate per day for each pupil ;
 $121 \times 0.035 = \$4.235$, A's rate bill, }
 $173 \times 0.035 = \$6.055$, B's rate bill, } Ans.

GENERAL AVERAGE.

(Art. 432, p. 320.)

(2.)

CONTRIBUTORY INTERESTS.

Vessel,	\$ 31500
Cargo,	6235
Freight less $\frac{1}{2}$,	2080
Entire contrib. int.,	\$ 39815

LOSS FOR GENERAL BENEFIT.

Thrown overboard,	\$ 6500
Repairs less $\frac{1}{3}$,	700
Cost of detention,	142
Entire loss,	\$ 7342

$\$7342 \div 39815 = .18\frac{1}{2}$ (nearly), the loss per cent. ;
 $\$31500 \times .18\frac{1}{2} = \5827.50 , am't payable by vessel ;
 $2080 \times .18\frac{1}{2} = 384.80$, am't payable by freight ;
 $2145 \times .18\frac{1}{2} = 396.825$, am't pay'e by Manning & Bro. ;
 $1460 \times .18\frac{1}{2} = 270.10$, am't pay'e by Anderson & Fiske ;
 $960 \times .18\frac{1}{2} = 177.60$, am't payable by Smidt & Huber ;
 $670 \times .18\frac{1}{2} = 123.95$, am't pay'e by Greenwood & Co. ;
 $1000 \times .18\frac{1}{2} = 185.00$, am't payable by A., R., & H.

EQUATION OF PAYMENTS.

(ART. 435, p. 322.)

3. \$200 to be paid in 3 months is the same as \$100 in 6 months; and \$300 in 5 months is the same as \$100 in 15 months; and \$500 in 10 months is the same as \$100 in 50 months. If, then, \$100 is to be paid in $6 + 15 + 50 = 71$ months, \$1000 should be paid in $\frac{1}{10}$ of 71 months $= \frac{71}{10} = 7\text{mo. } 3\text{da.}$, Ans.

OPERATION.

$$\begin{array}{r}
 \$ 200 \times 3 = 600 \\
 \$ 300 \times 5 = 1500 \\
 \$ 500 \times 10 = 5000 \\
 \hline
 \$1000 \quad 1000 \overline{)7100} (7\frac{1}{10} \text{ months, Ans.} \\
 \underline{7000} \\
 100
 \end{array}$$

$$\begin{array}{r}
 4. \quad \$2000 \times \\
 \quad 500 \times 12 = 6000 \\
 \quad 4000 \times 24 = 96000 \\
 \hline
 6500 \quad 102000
 \end{array}$$

$$102000 \div 6500 = 15\frac{2}{3}\text{mo.} = 15\text{mo. } 20\frac{2}{3}\text{da.}, \text{ Ans.}$$

5. \$50 to be paid in 2 months is the same as \$100 in 1 month; \$150 in 8 months is the same as \$100 in 12 months. The equated time, then, will be as if \$100 were to be paid in 1 month, \$100 in 5 months, and \$100 in 12 months; that is, \$100 in $1 + 5 + 12 = 18$ months; consequently \$300 in $\frac{1}{3}$ of 18 months $= 6$ months, Ans.

OPERATION.

$$\begin{array}{r}
 \$ 50 \times 2 = 100 \\
 \$100 \times 5 = 500 \\
 \$150 \times 8 = 1200 \\
 \hline
 \$300 \quad 300 \overline{)1800} (6 \text{ months, Ans.} \\
 \underline{1800}
 \end{array}$$

6. The first bill due is that for \$ 600, on May 1.

Due May 1, \$ 600

“ July 7, $370 \times 67 = 24790$

“ July 15, $560 \times 75 = 42000$

“ Oct. 20, $420 \times 172 = 72240$

1950

139030

$139030 \div 1950 = 71$ days, nearly; May 1 + 71 days =
July 11, Ans.

7. The first bill due is that for \$ 675.25, on June 1, 1855.

Due June 1, \$ 675

“ Nov. 4, $376 \times 156 = 58656$

“ “ 25, $822 \times 177 = 145494$

1856.

Due June 1, $961 \times 366 = 351726$

“ Apr. 1, $145 \times 305 = 44225$

“ Aug. 10, $811 \times 436 = 353596$

“ “ 12, $568 \times 438 = 248784$

“ “ 15, $370 \times 441 = 163170$

4728

1365651

$1365651 \div 4728 = 289$ days; June 1, 1855 + 289 days
= March 16, 1856, Ans.

(ART. 436, p. 324.)

2. As $\frac{1}{2}$ of \$ 144 is paid 7 months before it is due, it is evident the remainder, \$ 72, is payable in 14 months; but \$ 48 of this sum is paid in 4 months, which is 10 months before it is due. There will then remain \$ 24 unpaid; and the question will be, How long may \$ 24 be kept to pay the interest of \$ 72 for 10 months? \$ 72 for 10 months is the same as \$ 1 for 720 months; and \$ 1 for 720 months is the same as \$ 24 for $\frac{1}{24}$ of 720 months = $720 \div 24 = 30$ months. To the 30 months we add the 4 months, and we have the whole equated time; thus, $30 + 4 = 34$ months = 2y. 10mo., Ans.

OPERATION.

$$\begin{array}{r} 144 \times 7 = 1008 \\ \hline \end{array}$$

72

$$\begin{array}{r} 48 \times 4 = 192 \\ \hline \end{array}$$

$$\begin{array}{r} 120 \quad 24)816(34\text{mo.} = 2\text{y. } 10\text{mo., Ans.} \\ \hline 24 \end{array}$$

72

96

96

3. The payments in this problem were not due before Nov. 23, 1856; the days are reckoned from the time each payment was made to the end of 8 months, Nov. 23.

\$ 8000

$$\text{Paid April 5, } 1200 \times 232 = 278400$$

$$\text{" July 4, } 1500 \times 142 = 213000$$

$$\text{" Sept. 25, } 1800 \times 59 = 106200$$

$$\text{" Oct. 1, } 1000 \times 53 = 53000$$

$$\text{" Nov. 20, } 500 \times 3 = 1500$$

6000

652100

Balance unpaid, \$ 2000

$$652100 \div 2000 = 326 \text{ days; Nov. 23, 1856} + 326 \text{ days} = \\ \text{[Oct. 15, 1857, Ans.]}$$

4. $\$133\frac{1}{3}$ in 2 months is the same as \$1 in $266\frac{2}{3}$ months; and $\$266\frac{2}{3}$ in 3 months is the same as \$1 in 800 months; \$400 in 6 months is the same as \$1 in 2400 months; and $\$1$ in $266\frac{2}{3} + 800 + 2400 = 3466\frac{2}{3}$ months is the same as \$800 for $\frac{1}{800}$ of $3466\frac{2}{3}$ months = $3466\frac{2}{3} \div 800 = 4\frac{1}{3}$ months. And if $\frac{1}{2}$ of \$800 be paid down, the remainder, \$400, may be kept twice $4\frac{1}{3}$ months = $8\frac{2}{3}$ months, Ans.

OPERATION.

$$\begin{array}{r}
 \$133\frac{1}{3} \times 2 = 266\frac{2}{3} \\
 266\frac{2}{3} \times 3 = 800 \\
 400 \times 6 = 2400 \\
 \hline
 800 \quad 400)3466\frac{2}{3} (8 \text{ months.} \\
 400 \quad \underline{3200} \\
 400 \quad \quad 266\frac{2}{3} \\
 \quad \quad \quad 30 \\
 \quad \quad \quad \hline
 400)8000 (20 \text{ days.} \\
 800 \\
 \hline
 0
 \end{array}$$

5. The payments made in this bill were not due before March 25, 1857, 6 months from Sept. 25, 1856.

$$\begin{array}{r}
 1856. \quad \quad \quad 3051 \\
 \text{Paid Oct. 4, } 4,476 \times 172 = 81872 \\
 \text{" Nov. 12, } 375 \times 133 = 49875 \\
 \text{" Dec. 5, } 800 \times 110 = 88000 \\
 1857. \\
 \text{" Jan. 1, } 200 \times 83 = 16600 \\
 \hline
 1851 \quad \quad 236347
 \end{array}$$

Balance unpaid, \$1200

$236347 \div 1200 = 197$ days, nearly; March 25, 1857 + 197
[days = Oct. 8, 1857, Ans.]

AVERAGING OF ACCOUNTS.

(ART. 438, p. 327.)

2. The first item in this account was due July 4, 1855.

Days reckoned from July 4, 1855.

$$\begin{array}{r}
 \text{July 4, } \$376 \\
 \text{Aug. 20, } 816 \times 47 = 38352 \\
 \text{Aug. 29, } 178 \times 56 = 9968 \\
 \text{Sept. 25, } 387 \times 83 = 32121 \\
 \text{Dec. 5, } 419 \times 154 = 64526 \\
 \hline
 \$2176 \quad \quad 144967
 \end{array}$$

Days reckoned from July 4, 1855.

$$\begin{array}{r}
 \text{Aug. 10, } \$316 \times 37 = 11692 \\
 \text{Sept. 1, } 675 \times 59 = 39825 \\
 \text{Sept. 25, } 512 \times 83 = 42496 \\
 \text{Nov. 20, } 162 \times 139 = 22518 \\
 \text{Dec. 1, } 100 \times 150 = 15000 \\
 \hline
 \$1765 \quad \quad 131531
 \end{array}$$

\$2176 - \$1765 = \$411, difference of items; 144967 - 131531 = 13436, difference of products; $13436 \div 411 = 32 +$ days; July 4, 1855 + 33 days = Aug. 6, 1855, Ans.

3. The first item due in this account is the cash payment of \$700, April 1; from this date the time is reckoned.

Due 1856.

Sept. 1, \$721 \times 153 = 110818

Sept. 20, 815 \times 172 = 140180

Oct. 11, 588 \times 193 = 113484

Oct. 30, 800 \times 212 = 68600

Dec. 15, 625 \times 258 = 161250

1857.

Jan. 18, 560 \times 292 = 163520

* Feb. 28, 685 \times 333 = 228105

Mar. 25, 865 \times 358 = 130670

\$4659

1111122

Due 1856.

April 1, \$700

Nov. 30, 570 \times 243 = 138510

July 20, 500 \times 110 = 55000

Sept. 25, 100 \times 177 = 17700

1857.

Mar. 30, 750 \times 363 = 272250

April 30, 330 \times 394 = 130020

May 20, 500 \times 414 = 207000

\$3450

820480

1111122 - 820480 = 290642, balance of products;

\$4659 - \$3450 = \$1209, balance of the items;

$290642 \div 1209 = 240$ days; 240 days forward from April 1,

1856 = November 27, 1856, Ans.

* See note 2, p. 350, National Arithmetic.

4. In this problem the time is reckoned from the first date in the account to the date of each item, it being due from date.

1856. (Time from March 1.)

Mar. 1, \$86.25

April 7, 18.15 \times 37 = 671.55

June 15, 48.26 \times 106 = 5115.56

July 21, 91.20 \times 142 = 12950.40

Aug. 1, 30.00 \times 153 = 4590.00

\$223.86

23327.51

1856. (Time from April 1.)

April 1, \$48.25

May 20, 90.10 \times 49 = 4414.90

June 17, 12.50 \times 77 = 962.50

July 4, 20.00 \times 94 = 1880.00

July 10, 25.00 \times 100 = 2500.00

\$195.85

9757.40

$23327.51 \div 223.86 = 104$ days; March 1 + 104 days = June 13;

$9757.40 \div 195.85 = 50$ days, nearly; April 1 + 50 days =

May 21;

15*

June 13 is the average time of the debit items becoming due; and May 21 is the average time of the credit items becoming due. From May 21 to June 13 = 23 days = difference between the average dates; then the amount of the smaller side, $195.85 \times 23 = 4504.55$; $4504.55 \div 28.01 (= 223.86 - 195.85, \text{ balance of the account}) = 161 \text{ days nearly}$; hence, June 13 (the average date of the larger size) + 161 days = Nov. 21, 1856, Ans.

$\$223.86 - \$195.85 = \$28.01$, balance due Nov. 21; $\$28.01 \div 1.01$ (the amount of \$1 from Sept. 21, 1856, to Nov. 21, 1856) = $\$27.73$, the cash value of balance, Sept. 1, 1856, Ans.

5. In the adjustment of this account, the time is reckoned for the debit items from the first date; after ascertaining the average time of payment, the 8 months of credit are added.

1856.			1857.		
May 1,	\$ 800		Jan. 1,	\$ 500	
July 7,	$760 \times 67 =$	50920	*Oct. 18,	$482 \times 290 =$	139780
Sept. 11,	$417 \times 133 =$	55461	Mar. 19,	$750 \times 77 =$	57750
Nov. 25,	$288 \times 208 =$	59904	April 1,	$210 \times 90 =$	18900
Dec. 20,	$571 \times 233 =$	133043	May 25,	$100 \times 144 =$	14400
	<u>\$ 2336</u>	299328		<u>\$ 2042</u>	230830

$299328 \div 2336 = 128 \text{ days}$; May 1 + 128 days = Sept. 6, 1856.

$230830 \div 2042 = 113 \text{ days}$; Jan. 1 + 113 days = April 24, 1857.

Sept. 6, 1856, is the average date of the debits; and Sept. 6, 1856 + 8 mo. = May 6, 1857, the average time of the debits becoming due; April 24 is the average time of the credits becoming due. The difference between April 24 and May 6 is 12 days; 2042 (amount of smaller side) $\times 12 = 24504$; $24504 \div 294$ (balance of the account)

* Due Oct. 18.

= 83 days; May 6 (average time of the larger side) + 83 days = July 28, 1857, Ans.

Also, what will be the value of the balance on Nov. 28, 1857? \$294 is the balance; from July 28 to Nov. 28 are 4 months; the amount of \$294 for 4 months = $\$294 \times 1.02 = \299.88 , Ans.

6. In this problem the first item of debits becoming due is that of \$560, on June 1; and the first item of credits becoming due is that of \$100, cash paid June 20.

Due June 1, \$560.

846	$\times 71$	=	60066
728	$\times 111$	=	80808
400	$\times 59$	=	23600
560	$\times 345$	=	193200
600	$\times 200$	=	120000
500	$\times 343$	=	171500

\$4194 649174

Due June 20, \$100.

820	$\times 119$	=	97580
350	$\times 79$	=	27650
800	$\times 56$	=	44800
630	$\times 271$	=	170730
400	$\times 130$	=	52000
750	$\times 254$	=	190500

\$3850 583260

$649174 \div 4194 = 155$ days; June 1 + 155 days = Nov. 3, 1856.

$583260 \div 3850 = 151$ days; June 20 + 151 days = Nov. 18, 1856.

Nov. 18 — Nov. 3 = 15 days, the difference between the average date of the debits and credits becoming due; 3850 (the amount of the smaller side) $\times 15 = 57750$; $57750 \div 344$ (the balance of the account) = 168 days. Since the amount of the larger side becomes due first, Nov. 3, the 168 days must be counted backward from that date; therefore, Nov. 3, 1856 — 168 days = May 19, 1856, Ans.

The time from this date to the time of settlement, 12mo. 5da.; therefore, $\$344 \times 1.060\frac{1}{2} = \364.93 , the value of the balance, May 24, 1857, Ans.

(ART. 439, p. 331.)

ALFRED HICKS in account with KEEN & LEE.

1857.	1857.
Due July 1, \$ 156 × 158 = 23868	Due Feb. 1, \$ 120 × 303 = 36360
" Feb. 3, 100 × 301 = 30100	" July 20, 420 × 134 = 56280
" July 20, 817 × 134 = 42478	" Nov. 1, 300 × 30 = 9000
" " 30, 162 × 124 = 20088	" " 1, 50 × 30 = 1500
" May 15, 100 × 200 = 20000	
1858.	1858.
" Jan. 10, $\frac{100}{\$ 835}$ × 40 = $\frac{4000^*}{218}$	" Feb. 20, $\frac{213}{\$ 890}$ × 81 = $\frac{17258^*}{6)120398}$
	100 20.065
	\$ 990
218	
28.422	
\$ 1048	

\$ 1048 — \$ 990 = \$ 58 = balance of items.

\$ 23.422 — \$ 20.065 = \$ 3.36 = balance of interest.

\$ 58 + \$ 3.36 = \$ 61.36, balance due Dec. 1, 1857.

* These products becoming due after settlement, are transferred each to its opposite side ; but the items themselves must be added in their own places.

4. In finding the balance of this account, the time is reckoned from the date of each item, since it is due from date.

\$ 200 × 264 = 52800	\$ 300 × 248 = 74400
300 × 198 = 59400	350 × 222 = 77700
250 × 181 = 45250	400 × 205 = 82000
600 × 175 = 105000	320 × 144 = 46080
400 × 125 = 50000	600 × 109 = 65400
500 × 103 = 51500	100 × 95 = 9500
100 × 69 = 6900	200 × 52 = 10400
120 × 14 = 1680	150 × 25 = 3750
\$ 2470 372530	\$ 2420 369230

372530 — 369230 = 3300 ; $3300 \div 6000 = \$ 0.55$, balance of interest at 6 per cent. ; $\$ 0.55 \div 6 = \$ 0.09\frac{1}{6}$;
 $\$ 0.09\frac{1}{6} \times 7 = \$ 0.64$, balance of interest at 7 per cent. ;

Items of debit, \$2470	Balance of items, \$50
Items of credit, 2420	Balance of interest, 0.64
Balance of items, \$50;	Bal. due Mar. 25, 1857, \$50.64, Ans.

ACCOUNTS OF STORAGE.

(ART. 441, p. 333.)

2. 1857.	chests.	da.	prod.
May 16, Rec.	4560	× 14	= 63840
" 30, Deliv.	564		
	Bal. 3996	× 2	= 7992
June 1, Deliv.	904		
	Bal. 3092	× 38	= 117496
July 9, Deliv.	1000		
	Bal. 2092	× 25	= 52300
Aug. 3, Deliv.	1500		
	Bal. 592	× 13	= 7696
" 16, Deliv.	592		
		30)	249324

Chests chargeable for 1mo., 83108

 $8310.8 \times \$0.03 = \$249.32, 4$, cost of storage, Ans.

3. 1857.	Bales.	da.	prod.
Jan. 1, Rec.	2310	× 15	= 34650
" 16, "	120		
	2430	× 16	= 38880
Feb. 1, Rec.	300		
	2730	× 11	= 30030
Feb. 12, Deliv.	1000		
	1730	× 17	= 29410
Mar. 1, Deliv.	600		
	1130	× 33	= 37290
Apr. 3, Deliv.	400	(carried forward).	

Apr. 3, Deliv. $\frac{400}{730} \times 7 = 5110$ (brought forward).

Apr. 10, Deliv. $\frac{312}{418} \times 21 = 8778$

May 1, In store, 418ba. $30)184148$

Bales chargeable for 1 month, 6138

$6138 \times \$0.05 = \306.90 , cost of storage up to May 1.

MISCELLANEOUS EXAMPLES.

(PAGE 333.)

1. The ratio of \$1120, A's gain, to \$3500, A's stock = $\frac{1120}{3500} = \frac{8}{25}$.

The ratio of \$880, B's gain, to \$2200, B's stock = $\frac{880}{2200} = \frac{4}{10} = \frac{2}{5}$.

The ratio of \$1200, C's gain, to \$2500, C's stock = $\frac{1200}{2500} = \frac{12}{25}$.

$\frac{12}{25} - \frac{8}{25} = \frac{4}{25}$ = the gain of 2mo., B's time more than A's;
 $\frac{4}{25} : \frac{2}{5} :: 2\text{mo.} : 8\text{mo.}$, A's time; $8 + 2 = 10\text{mo.}$, B's time;

$\frac{4}{25} : \frac{12}{25} :: 2\text{mo.} : 12\text{mo.}$, C's time, Ans.

2. $\$540 + \$660 = \$1200$; $\$540 \div 1200 = 45\text{cts. on } \1 ;
 $\$15000 \times .45 = \6750 , paid by the bankrupt; then,
 $\$15000 - \$6750 = \$8250$, Ans.

3. $\$64.50 - \$1.50 = \$63$, A's property tax; $\$50.50 - \$1.50 = \$49$, B's property tax; $\$63 \div .007 = \9000 , A's property; $\$49 \div .007 = \7000 , B's property, Ans.

4. $\$20 + \$30 + \$150 = \200 , total expenses; $\$200 - \$50 = \$150$, raised by tax; $\$150 \div .03 = 5000$, days of attendance, Ans.

5. Kimball's whole loss was made up *minus* a 5 per cent. tax on his share of the cargo; therefore, $\$7000 \times .05 = \350 , Ans.

6. \$50 for 4mo. is \$1 for 200mo.; and \$100 for 8mo. is \$1 for 800mo. If, then, \$1 is to be paid in $200 + 800 = 1000$ months, \$150 should be paid in $\frac{1}{150}$ of 1000mo. $= \frac{1000}{150} = 6\frac{2}{3}$ mo. It appears, then, that \$150 is paid $6\frac{2}{3}$ mo. before it is due. The question now is, In how much *less* than 10mo. should the \$250 be paid? As we have before shown that \$1 is to be paid in 1000mo., \$250 should be paid in $\frac{1}{250}$ of 1000mo. $= 1000 \div 250 = 4$ mo. We find, then, that the time in which B is to pay A is 4mo. *less* than it otherwise would have been, on account of A's making présent payment. We therefore subtract 4mo. from 10, and find the true answer; thus $10 - 4 = 6$ mo.. Ans.

OPERATION.

$$\begin{array}{r} 50 \times 4 = 200 \\ 100 \times 8 = 800 \\ \hline 150 \end{array} \quad \begin{array}{r} 150 \overline{)1000} (6\frac{2}{3} \text{ mo.} \\ \underline{1000} \end{array}$$

$$150 \times 6\frac{2}{3} = 1000 \div 250 = 4\text{mo.}$$

$$10 - 4 = 6\text{mo., Ans.}$$

(7.)

$$\begin{array}{rcl} 3 & = & \frac{1}{12} \\ 4 & = & \frac{1}{12} \times 6 = \frac{1}{2} \\ 6 & = & \frac{1}{12} \times 8 = \frac{1}{3} \\ 8 & = & \frac{1}{12} \times 12 = 1 \end{array} \qquad \begin{array}{rcl} \frac{1}{2} + \frac{1}{3} + \frac{1}{2} + \frac{1}{3} & = & 1\frac{1}{2} \\ \frac{1}{2} + \frac{1}{3} + \frac{1}{3} & = & \frac{7}{6} \\ \frac{7}{6} \div \frac{1}{12} & = & 5\frac{1}{3}\text{mo.} = \\ & & 5\text{mo. } 25\text{da., Ans.} \end{array}$$

8. \$2000 + his services are worth \$800 per year ; but \$4000 + his services are worth \$1100 per year ; therefore, \$2000 are worth \$300 per year ; hence his services without the \$2000 are worth \$800 - \$300 = \$500, Ans.
9. $\begin{array}{r} \$100 \times 4 = 400 \\ \$100 \times 8 = 800 \\ \$200 \times 12 = 2400 \\ \hline \$400 \qquad 3600 \end{array}$ $3600 \div 400 = 9\text{mo. for } \$400 ;$
 $200 : 400 :: 9 : 18\text{mo. for } \$200 ;$
that is, \$200 running 18mo. is
equal to \$400 running 9mo. If
now 18 be divided into two parts,

9. $\$100 \times 4 = 400$
 $\$100 \times 8 = 800$
 $\underline{\$200 \times 12} = \underline{2400}$
 $\frac{\$400}{3600}$

$3600 \div 400 = 9\text{mo. for } \$400;$
 $200 : 400 :: 9 : 18\text{mo. for } \$200;$
 that is, \$200 running 18mo. is
 equal to \$400 running 9mo. If
 now 18 be divided into two parts,

one of which shall be twice as great as the other, we have 12 and 6; hence, if one note of \$200 run 12mo., and the other note of \$200 run 6mo., it will be equivalent to \$200 in 18mo., or to \$400 in 9mo.; therefore, in 6mo. and 12mo., Ans.

10. The question involved in this problem is, In what time will the interest of \$360 be \$3.78; $\$360 \times .06 = \21.60 , interest of \$360 for 1y.; then, $3.78 \div 21.60$ will be the time in years (Art. 360) = $\frac{378}{2160} = \frac{7}{40}y. = 2.1mo. = 63da.,$ Ans.

INSURANCE.

(ART. 445, p. 335.)

2. $\$896 \times .12 = \$107.52,$ Ans.
3. $\$17289 \times .01\frac{1}{4} = 216.11\frac{1}{4},$ Ans.
4. $\$35000 \times \frac{3}{4} \times .03\frac{1}{4} = \$82.50;$ and $\$75000 \times \frac{3}{4} \times .02\frac{1}{4} = \$1250;$ $\$1250 + \$82.50 = \$1932.50,$ the amount of premium, Ans.
5. The loss was what the insurance cost; therefore, $\$8675 \times .04\frac{1}{8} = \$179.15\frac{5}{8},$ the premium; $\$179.15\frac{5}{8} + \$1,$ the cost of the policy = $\$180.15\frac{5}{8},$ Ans.
6. $690.90 \div 9870 = .07,$ Ans.
7. $.04\frac{1}{4} + .03\frac{1}{8} + .04\frac{3}{8} + .05 + .05\frac{1}{2} = .23\frac{3}{8};$ $\$47600 \times .23\frac{3}{8} = \$11007.50,$ Ans.

(ART. 446, p. 336.)

2. $\$26250 \div .87\frac{1}{2} = \$30000,$ Ans.
3. $\$6370 \div .98 = \$6500,$ Ans.
4. In this problem the rate per cent. and interest are given, by which to find the principal; then (Art. 358), $\$50 \div .01\frac{1}{4} = \$4000,$ Ans.
5. As the policy is 10 per cent. of \$600, the property must be 90 per cent. of \$600; therefore $\$600 \times .90 = \$540,$ Ans.

6. $1.00 - .03 = .97$; $\$1000 \div .97 = \$1030.927+$.
 $1.00 - .05 = .95$; $\$1030.927 \div .95 = \$1085.187+$.
 $1.00 - .06 = .94$; $\$1085.187 \div .94 = \$1154.454+$.
 $1.00 - .07 = .93$; $\$1154.454 \div .93 = \$1241.348+$
 Ans.

LIFE INSURANCE.

(ART. 454, p. 340.)

2. $\$8000 \times .0114 = \91.20 , Ans.
 3. $\$78000 \times .0491 = \3829.80 , Ans.
 4. $\$2000 \times .0088 = \17.60 , Ans.
 5. $\$12000 \times .0373 = \447.60 , the annual premium; the insured died in the third year, so that this premium was paid for three years; therefore, $\$447.60 \times 3 = \1342.80 , and $\$12000 - \$1342.80 = \$10657.20$, Ans.
 6. $80y. - 50. = 30y.$, for which the annual premium was paid; $\$5000 \times .046 = \230 ; $\$230 \times 30 = \6900 , the amount of premiums paid; therefore $\$6900 - \$5000 = \$1900$, Ans.
 7. The number of years the premium is paid is 4;
 $\$4000 \times \frac{1}{20} = \208 , the annual premium;
 The interest of \$1 for 4y., and 3y., and 2y., and 1y., is respectively $.24 + .18 + .12 + .06 = .60$; and $\$208 \times .60 = \124.80 , the amount of interest; $\$208 \times 4 = \832 , the amount of premium; $\$124.80 + \$832 = \$956.80$; then, $\$4000 - \$956.80 = \$3043.20$, Ans.
 8. $\$10000 \times .02 = \200 , the annual premium.
 The last annual premium paid is not charged with interest; to shorten the work, we take the amount of \$1 at compound interest from the table, for the periods of 11y., and 10y., and 9y., etc.; these added together give \$15.869-947, which is the sum of the amounts of an annual premium of \$1 for 11y.; a premium of \$200 will be 200

times this = $\$15.869947 \times 200 = \3173.9894 ; to this add the premium for the 12th year, which is not chargeable with interest; $\$3173.9894 + \$200 = \$3373.9894$; and $\$10000 - \$3373.9894 = \$6626.01$,
Ans.

NOTE. — The work of this problem might be much shortened by referring to the table of Annuities later in the book.

CUSTOM-HOUSE BUSINESS.

(ART. 460, p. 343.)

2. $\$4350 \times .08 = \348 , Ans.
3. $\$7890 \times .15 = \1183.50 ; $\$1183.50 \times .19 = \224.865 , Ans.
4. $986 \times 4.84 \times .24 = \1145.3376 , Ans.
5. $112.50 \div 750 = 15$ per cent., Ans.
6. $60 \times .95 \times 70 = 3990$ fr., value of champagne;
 $36 \times .95 \times 35 = 1197$ fr., value of port;
 $50 \times (31 - 1) \times 4 = 6000$ fr., value of sherry;
 $\underline{11187}$
 $11187 \times .30 = 3356.1$ fr. duties; $3356.1 \times .18\frac{6}{10} =$
 $\$624.2346$, Ans.
7. $\$53.76 \div .08 = \672 ; $60 \times (112 - 12) = 6000$ lb.;
 $\$672 \div 6000 = 11\frac{1}{2}$ cts., Ans.
8. $40 \times 63 \times .98 = 2469.6$ gal. molasses charged with duty;
 $24 \times 400 \times .90 = 8640$ lb. sugar charged with duty;
 $2469.6 \times 3 \times 10 \times .24 = \177.81
 $8640 \times 1 \times 5 \times .24 = 103.68$
 $260 \times 2 \times .08 = 41.60$
 $410 \times 7 \times .30 = 861.00$
 $\underline{\$1184.09}$, Ans.
9. $270 \times \$50 = \13500 ; $\$16740 - \$13500 = \$3240$;
 $3240 \div 13500 = .24$, Ans.

10. Whole invoice \$20560

$$3000 \times .04 = \$120.00$$

$$4200 \times .08 = 336.00$$

$$2100 \times .15 = 315.00$$

$$6000$$

$$\underline{\$15300}$$

$$\text{Bal. of invoice } \$5260 \times .30 = \underline{1578.00}$$

$$\$2349.00, \text{ Ans.}$$

$$11. 10 \times 40 \times \frac{5}{10} \times .24 = 24. \quad \text{£.}$$

$$200 \times \frac{4}{10} \times .19 = 7.6 \quad \text{£.}$$

$$100 \times \frac{2\frac{1}{2}}{20} \times .15 = 1.875\text{£.}$$

$$60 \times .04 = 2.4 \quad \text{£.}$$

$$\underline{35.875\text{£.}} \times 4.84 = \$173.635,$$

[Ans.]

12. $32 \times 32 = 1024\text{yds.}$; $\$122.88 \div .24 = \512 , invoice value of the cloth; $\$512 \div 1024 = \0.50 per yard; $\$512 + \122.88 , the cost of duties, + $\$40.96$, other charges, = $\$675.84$; $\$675.84 \div 1024 = \0.66 , cost per yard, Ans.

COINS AND CURRENCIES.

(ART. 470, p. 348.)

3. $46\text{£. } 16\text{s. } 6\text{d.} = 46.825\text{£.}$; $46.825 \div \frac{2}{3} = \$117.06\frac{1}{4}$,
Ans.

4. $1032 \times \frac{3}{8} = 387\text{£.}$, Ans.

5. $515.70 \div 4 = 128.925\text{£.}$; $128.925\text{£.} = 128\text{£. } 18\text{s. } 6\text{d.}$,
Ans.

6. $160.50 \times .18\frac{6}{10} = \29.853 , Ans.

7. $728.41 \div 4.84 = 150\frac{24\frac{1}{4}}{4}\text{£.} = 150\text{£. } 9\text{s. } 11\frac{6\frac{1}{2}}{12}\text{d.}$, Ans.

8. $12\text{£. } 12\text{s.} = 12.6\text{£.}$; $12.6 \div \frac{7}{10} = \54 , Ans.

9. $128\text{£. } 18\text{s. } 6\text{d.} = 128.925\text{£.}; 128.925 \times 4 = \515.70 ,
Ans.
10. $740.45 \times 0.75 = \$555.33\frac{3}{4}$, Ans.
11. $46\text{£. } 16\text{s. } 6\text{d.} = 46.825\text{£.}; 46.825 \div \frac{2}{3} = \$117.66\frac{1}{4}$;
 $117.66\frac{1}{4} \times \frac{3}{5} = 35.118\frac{3}{4}\text{£.} = 35\text{£. } 2\text{s. } 4\frac{1}{2}\text{d.}$, Ans.
12. $151 \times 1.12 = \$169.12$; $169.12 \div .10 = 1691.2$ reals
plate, Ans.
13. $1000 \times 1.06 = \$1060$; $1060 \div 18\frac{9}{10} = 5698\frac{8}{9}$ francs,
Ans.
14. $30 \times 30 \times 15 = 13500\text{s. sterling} = 675\text{£.}; 675 \times 4.84$
 $= \$3267$; $3267 \div 4 = 816.75\text{£.} = 816\text{£. } 15\text{s.}$ Canada
currency, Ans.
15. $410 \times .80 = \$328$; $328 \div .69 = 475\frac{2}{3}$ thalers, Ans.

EXCHANGE.

(ART. 493, p. 354.)

2. $\$1950.50 \times 1.00\frac{1}{4} = \1955.87 , Ans.
3. $\$2000 \times 1.005 = \2010 , Ans.
4. $\$1744.40 \div .98 = \1780 , face of draft, Ans.
5. $\$600 \times .99 = \594 ; $\$594 - \6.30 (the interest of \$600
for 63 days) $= \$587.70$, Ans.
6. $\$9256.40 \times 1.00\frac{3}{4} = 9291.11$; $\$9291.11 - \50.91 (the
interest of \$9256.40 for 33 days) $= \$9240.20$, Ans.
7. Find the value of \$1 at $\frac{3}{4}$ per cent. premium, and interest
off for 33 days at 6 per cent.; $\$1 \times 1.00\frac{3}{4} = \1.00375 ;
 $\$1.00375 - \$0.0055 = \$0.99825$, the value of \$1 by
the conditions of the question; hence, as \$0.99825 is to
\$1, so is \$9240.20 to that sum of which \$9240.20 is
the value or yield; therefore, $\$9240.20 \div .99825 =$
 $\$9256.39\frac{1}{2}$, Ans.
8. The yield of \$1, by the conditions of the question, is \$0.982;
then, $\$1190.184 \div .982 = \1212 , Ans.

(ART. 499, p. 357.)

3. $3676 \div 5\frac{20}{100} = \$706.92\frac{4}{13}$, Ans.
4. $3000\frac{1}{8} \times 0.35 = \$1050.21\frac{1}{8}$, Ans.
5. $81727.75 \div 4.867 = 16792.223\text{£} = 16792\text{£} \text{ 4s. } 5\frac{1}{2}\text{d.}$,
Ans.
6. $79000 \times .186 = \$14694$, the par value of 79000 francs;
 $\$14694 - \$14400.12 = \$293.88$; $293.88 \div 14694 =$
.02, Ans.
7. $5763.75 \times 0.74 = \$4265.175$, Ans.
8. $550.66 \div 1.10 = 500\frac{6}{10}$ millreas = 500 millreas 600 reas,
Ans.
9. $10000 \div .68 = 14705\frac{8}{9}$ thalers = 14705 thalers 26
groschen $5\frac{1}{3}$ pfennings, Ans.
10. $400\frac{1}{3} \times 1.06 = \424.265 , Ans.
11. 300 thalers 20 groschen 0 pfennings = $300.6\frac{1}{3}$ thalers;
 $300.6\frac{1}{3} \times .69 = \$207.47\frac{1}{12}$, Ans.
12. $5694 \div .40 = 14235$ rupees, Ans.
13. $615.60 \div .80 = 769.5$ ducats = 769 ducats 5 carlini,
Ans.
14. $7300 \div 1500 = \$4.86\frac{2}{3}$; $4.86\frac{2}{3} - 4.44\frac{1}{3} = .42\frac{2}{3}$; $.42\frac{2}{3} \div$
 $4.44\frac{1}{3} = .09\frac{1}{3}$, Ans.
15. $868\text{£} \text{ 17s. } 6\text{d.} = 868.875\text{£}$; $868.875 \times 23.60 = 20505.45$
francs = 20505 francs 45 centimes, Ans.
16. $2640 \div .18 = 14666.66\frac{2}{3}$ lire = 14666 lire $66\frac{2}{3}$ centesimi,
Ans.
17. $17280 \div .186 = 92903\frac{7}{11}$ francs; $92903\frac{7}{11} - 88128 =$
 $4775\frac{7}{11}$; $4775\frac{7}{11} \div 88128 = 5\frac{1}{4}\frac{2}{11}$ per cent., Ans.
18. English money at $8\frac{1}{2}$ per cent. premium = $\frac{4}{9} \times 1.085 =$
 $4\frac{2}{3}$ = $\$4.82\frac{2}{3}$; $17000 \times 4.82\frac{2}{3} = \81977.777 , Ans.

ARBITRATION OF EXCHANGES.

(ART. 504, p. 361.)

(2.)

1m. = 5.95fr.

5.20fr. = \$1

\$3500 = -- m.

$$\frac{1 \times 5.20 \times 3500}{1 \times 5.95} = 3058.823\frac{9}{7} \text{ millreas} = 3058 \text{ millreas}$$

823 $\frac{9}{7}$ reas, Ans.

(3.)

EXCHANGE DIRECT.

\$4 $\frac{3}{4}$ = 1£.

2000£. = \$9777.777

EXCHANGE THROUGH PARIS.

\$1 = 5.20fr.

24fr. = 1£.

2000£. = \$9230.769

EXCHANGE THROUGH HAMBURG.

\$0.35 = 1 mar. banc.

13 $\frac{3}{4}$ m. b. = 1£.

2000£. = \$9625.

Exchange through Paris is better than exchange direct by \$547.008; and better than exchange through Hamburg by \$394.231.

4. \$5000.00

12.50, $\frac{1}{4}$ per cent. discount added;

\$5012.50, exchange value of \$5000 at New Orleans, to be paid in New York;

\$5012.50

25.0625, $\frac{1}{2}$ per cent. of \$5012.50 subtracted;

\$4987.4375, exchange value of \$5012.50 at St. Louis, or New Orleans. The exchange being made through New Orleans, \$4987.4375 in St. Louis will pay a debt of \$5000 in New York; $5000 \times 1.015 = 5075$ = the amount that must be paid by direct exchange; therefore, $5075 - 4987.4375 = 87.56\frac{1}{4}$, Ans.

5. $\$4.84\frac{1}{2} = 1\text{£}.$

$1\text{£} = 1\frac{2}{7}\text{ thalers}.$

$9760\text{th.} = \$6698.25 +; \$6698.25 \times 1.005 = \$6731.74 +$
[Ans.]

6. $\$0.34 = 1\text{ mark banco}.$

$2\frac{1}{2}\text{ m. b.} = 1\text{ ruble}.$

$\text{---} = \$650.$

$\frac{650}{.34 \times 2\frac{1}{2}} = 764\frac{1}{7}\text{ rubles} = 764\text{ rubles } 70\frac{1}{7}\text{ kopecks},$
[Ans.]

7. By the question, $\$1$ on Boston $= \$1.00\frac{1}{4}$ on Philadelphia, by direct exchange; and $\$1$ on Boston $= \$1$ on Chicago, and $\$1$ on Chicago $= \$0.98$ on Philadelphia; consequently, $\$1$ on Boston $= \$0.98$ on Philadelphia, by circuitous exchange; therefore, exchange on Boston through Chicago is better than direct exchange by the difference between $\$1.00\frac{1}{4}$ and $\$0.98 = 2\frac{1}{4}\text{ per cent.},$ Ans.

8. $8\text{ guilders} = 1\text{ Amsterdam ell},$
 $1\text{ ell} = \frac{3}{8}\text{ yd.},$ and
 $8\text{ guilders} = \$3.20,$ and
 $\$3.20 = \frac{3}{8}\text{ yd.},$ or $\$4.26\frac{2}{7} = 1\text{ yd.}$ } Amsterdam.

$3\frac{1}{2}\text{ thalers} = 1\text{ Berlin ell},$
 $1\text{ ell} = \frac{25\frac{1}{2}}{36} = \frac{1}{2}\frac{1}{4}\text{ yd.},$ and
 $3\frac{1}{2}\text{ thalers} = 3\frac{1}{2} \times .66 = \$2.31,$
 $\$2.31 = \frac{1}{2}\frac{1}{4}\text{ yd.},$ or $\$3.26\frac{2}{7} = 1\text{ yd.}$ } Berlin.

$1\text{£} = \$4.86\frac{2}{3};$ $\frac{1}{8}\text{ of } \$4.86\frac{2}{3} = \$3.65,$ } England.
 $\$3.65 = 1\text{ yd.},$ bought in England.

Therefore, Berlin is better for the purchase than Amsterdam or England, by the difference between $\$3.26\frac{2}{7}$ and $\$4.26\frac{2}{3},$ or $\$3.65$ respectively.

(9.)

$$\$4.80 = 1\text{£.}$$

$$1\text{£.} = 25\frac{1}{4} \text{ francs.}$$

$$7000 \text{ francs} = \$ \text{ --- }$$

$$\frac{4.80 \times 7000}{25\frac{1}{4}} = \$1330.69, \text{ Ans.}$$

(10.)

$$1\text{£.} = 13\frac{1}{3} \text{ guilders.}$$

$$1\text{g.} = 2\frac{1}{2} \text{ francs.}$$

$$\text{--- francs} = 1000\text{£.}$$

$$\frac{13\frac{1}{3} \times 2\frac{1}{2} \times 1000}{1} = 29333\frac{1}{3} \text{ francs; } 29333\frac{1}{3} - 146\frac{2}{3} (\frac{1}{2} \text{ per cent. commission}) = 4186\frac{2}{3} \text{ whole gain; } 4186\frac{2}{3} \div 25000 (1000\text{£.} \times 25 = 25000 \text{ francs}) = 16\frac{2}{3}\% \text{ gain per cent., Ans.}$$

ALLIGATION.

(ART. 506, p. 363.)

(2.)

$$\$1.00 \times 19 = \$19.00$$

$$.66 \times 40 = 26.40$$

$$.50 \times 11 = 5.50$$

$$\frac{70}{\$50.90}$$

$$\$0.727\frac{1}{2}, \text{ Ans.}$$

(3.)

$$22 \times 3 = 66$$

$$20 \times 3 = 60$$

$$6) \overline{126}$$

$$21, \text{ Ans.}$$

(4.)

$$70 \times 20 = \$14.00$$

$$60 \times 15 = 9.00$$

$$40 \times 80 = 32.00$$

$$\frac{115}{55.00}$$

$$\$0.47\frac{1}{2}\frac{1}{2}, \text{ Ans.}$$

(ART. 509, p. 365.)

(2.)

$$80 \left\{ \begin{array}{l} \text{1bu. at 45, gain 35} \\ \text{1bu. at 75, gain 5} \\ \hline \text{1bu. at 100, loss 20} \\ \text{1bu. at 100, loss 20} \end{array} \right\} = \begin{array}{l} 40 \text{ gain.} \\ \\ 20 \text{ loss} \\ 20 \text{ loss} \end{array} \left. \vphantom{\begin{array}{l} \text{1bu. at 45, gain 35} \\ \text{1bu. at 75, gain 5} \\ \hline \text{1bu. at 100, loss 20} \\ \text{1bu. at 100, loss 20} \end{array}} \right\} = 40 \text{ loss.}$$

1bu. of barley, 1bu. of rye, and 2 of wheat, Ans.

(3.)

$$20 \left\{ \begin{array}{l} \text{1 of 24, loss 4} \\ \text{1 of 23, loss 3} \\ \hline \text{1 of 19, gain 1} \\ \text{1 of 15, gain 5} \\ \hline \text{1 of 19, gain 1} \end{array} \right\} = \begin{array}{l} 7 \text{ loss.} \\ \\ 6 \text{ gain.} \\ \\ 1 \text{ gain} \end{array} \left. \vphantom{\begin{array}{l} \text{1 of 24, loss 4} \\ \text{1 of 23, loss 3} \\ \hline \text{1 of 19, gain 1} \\ \text{1 of 15, gain 5} \\ \hline \text{1 of 19, gain 1} \end{array}} \right\} = 7 \text{ gain.}$$

1 part of 24, 1 of 23, 1 of 15, and 2 of 19, Ans.

(4.)

$$75 \left\{ \begin{array}{l} \text{1 gal. at 0, gain 75} \\ \text{1 gal. at 60, gain 15} \\ \hline \text{1 gal. at 80, loss 5} \\ \text{1 gal. at 120, loss 45} \\ \hline \text{8 gal. at 80, loss (8 \times 5) 40} \end{array} \right\} = \begin{array}{l} 90 \text{ gain.} \\ \\ \\ \\ 90 \text{ loss.} \end{array}$$

1 gal. of water, 1 of 60 cts., 1 of 120 cts., and 9 of 80 cts, Ans.

(ART. 510, p. 366.)

(2.)

$$100 \left\{ \begin{array}{l} \text{20 at 0, 2000 gain} \\ \hline \text{1 at 175, 75 loss} \\ \text{1 at 125, 25 loss} \\ \hline \text{19 at 175, 1425 loss} \\ \text{19 at 125, 475 loss} \end{array} \right\} = \begin{array}{l} 2000 \text{ gain.} \\ \\ \\ 2000 \text{ loss.} \end{array}$$

20 gallons of each price, Ans.

$$\begin{array}{r}
 \text{(3.)} \\
 190 \left\{ \begin{array}{l} 1 \text{ at } 180, \text{ gain } 10 \\ 10 \text{ at } 170, \text{ gain } 200 \\ 4 \text{ at } 220, \text{ loss } 120 \\ 1 \text{ at } 200, \text{ loss } 10 \\ \hline 8 \text{ at } 200, \text{ loss } 80 \end{array} \right\} = 210 \text{ gain.} \\
 \left\{ \begin{array}{l} 1 \text{ at } 200, \text{ loss } 10 \\ \hline 8 \text{ at } 200, \text{ loss } 80 \end{array} \right\} = 210 \text{ loss.}
 \end{array}$$

9 bushels at \$2.00; 1 bushel at \$1.80, Ans.

$$\begin{array}{r}
 \text{(4.)} \\
 \left. \begin{array}{l} 1 \text{ at } 8, \text{ gain } 4\frac{1}{2} \\ 3 \text{ at } 9\frac{1}{4}, \text{ gain } 9\frac{3}{4} \\ 4 \text{ at } 10\frac{1}{2}, \text{ gain } 8 \end{array} \right\} = 22\frac{1}{4} \text{ gain.} \\
 \left. \begin{array}{l} 1 \text{ at } 13, \text{ loss } \frac{1}{2} \\ 6 \text{ at } 13\frac{1}{2}, \text{ loss } 6 \\ 1 \text{ at } 14, \text{ loss } 1\frac{1}{2} \\ \hline 4\frac{1}{2} \text{ at } 13, \text{ loss } 2\frac{1}{4} \\ 8 \text{ at } 14, \text{ loss } 12 \end{array} \right\} = 22\frac{1}{4} \text{ loss.}
 \end{array}$$

Ans. 11lb. at 8 cts., 5 $\frac{1}{2}$ lb. at 13 cts., and 9lb. at 14 cts.

$$\begin{array}{r}
 \text{(5.)} \\
 50 \left\{ \begin{array}{l} 10 \text{ of } 58, \text{ loss } 80 \\ 1 \text{ of } 45, \text{ gain } 5 \\ \hline 15 \text{ of } 45, \text{ gain } 75 \end{array} \right\} = 80 \text{ loss.} \\
 \left\{ \begin{array}{l} 1 \text{ of } 45, \text{ gain } 5 \\ \hline 15 \text{ of } 45, \text{ gain } 75 \end{array} \right\} = 80 \text{ gain.}
 \end{array}$$

Ans. 16 bushels of barley.

(ART. 511, p. 367.)

$$\begin{array}{r}
 \text{(2.)} \\
 15 \left\{ \begin{array}{l} 1 \text{ at } 8, \text{ gain } 7 \\ 1 \text{ at } 10, \text{ gain } 5 \\ 1 \text{ at } 12, \text{ gain } 3 \\ 1 \text{ at } 20, \text{ loss } 5 \\ \hline 2 \text{ at } 20, \text{ loss } 10 \end{array} \right\} = 15 \text{ gain.} \\
 \left\{ \begin{array}{l} 1 \text{ at } 20, \text{ loss } 5 \\ \hline 2 \text{ at } 20, \text{ loss } 10 \end{array} \right\} = 15 \text{ loss.}
 \end{array}$$

$1 + 1 + 1 + 3 = 6$; sugar at 8, 10, and 12 cents, will be each 1 part in 6; and sugar at 20 cents will be 3 parts in 6; $\frac{1}{6}$ of 200 = 33 $\frac{1}{3}$ lb. of 8, 10, and 12 cents; $\frac{3}{6}$ of 200 = 100lb. of 20 cents, Ans.

(3.)

$$1.90 \left\{ \begin{array}{l} 10\text{bu. at } 1.70, \text{ gain } 2.00 \\ 4\text{bu. at } 2.20, \text{ loss } 1.20 \\ \hline 8\text{bu. at } 2.00, \text{ loss } 0.80 \end{array} \right\} = 2.00 \text{ gain.}$$

$10 + 4 + 8 = 22\text{bu.}$; $48 - 22 = 26\text{bu.}$ Then, as the gain on 1bu. at \$1.80 equals the loss on 1bu. at \$2.00, $\frac{1}{2}$ of 26 = 13bu. = the quantity at \$1.80, and $13 + 8 = 21\text{bu.}$ = the quantity at \$2.00, Ans.

(4.)

5lb. = 60oz.; 5oz. at 22 carats give a loss equal the gain on the 5oz. at 18 carats; and $5 + 5 = 10\text{oz.}$; $60 - 10 = 50\text{oz.}$ Then,

$$20 \left\{ \begin{array}{l} 1\text{oz. at } 15, \text{ gain } 5 \\ 1\text{oz. at } 17, \text{ gain } 3 \\ \hline 4\text{oz. at } 22, \text{ loss } 8 \end{array} \right\} = 8 \text{ gain.}$$

$1 + 1 + 4 = 6$; $\frac{1}{4}$ of 50 = $8\frac{1}{2}\text{oz.}$ = the quantity at 15 carats, and at 17 carats; and $\frac{1}{4}$ of 50 = $33\frac{1}{2}\text{oz.}$; $33\frac{1}{2} + 5 = 38\frac{1}{2}\text{oz.}$ = the quantity at 22 carats, Ans.

(5.)

As 20 animals were bought for \$20, the average price of them was \$1.00.

$$100 \left\{ \begin{array}{l} 2 \text{ sheep at } 400, \text{ loss } 600 \\ 1 \text{ sheep at } 400, \text{ loss } 300 \\ 1 \text{ lamb at } 50, \text{ gain } 50 \\ 1 \text{ kid at } 25, \text{ gain } 75 \\ \hline 1 \text{ kid at } 25, \text{ gain } 75 \\ 14 \text{ lambs at } 50, \text{ gain } 700 \end{array} \right\} = 900 \text{ loss.}$$

Ans. 3 sheep, 15 lambs, 2 kids.

MISCELLANEOUS EXAMPLES.

(PAGE 368.)

(1.)

$$97\frac{1}{2} \left\{ \begin{array}{l} 9 \text{ at } 80, \text{ gain } 157\frac{1}{2} \\ 1 \text{ at } 120, \text{ loss } 22\frac{1}{2} \\ \hline 6 \text{ at } 120, \text{ loss } 135 \end{array} \right\} = 157\frac{1}{2} \text{ loss.}$$

$6 + 1 = 7$ men at \$1.20, Ans.

2. 5000 rix dollars 12 skillings = $5000\frac{1}{4}$ rix dollars; $5000\frac{1}{4} \times 1.06 = \5300.265 , Ans.
3. If Sanford remits the amount, he buys a bill at Liverpool on New Orleans, and pays 10 per cent. premium; but if Lassale at New Orleans draws on Liverpool for the amount, he receives the amount at only 8 per cent. premium; therefore the difference is the difference between 1£. at 10 per cent. premium ($\$4.88\frac{3}{4}$) and 1£. at 8 per cent. premium ($\$4.80$) = $\$4.88\frac{3}{4} - \$4.80 = \$0.08\frac{3}{4}$, and this difference multiplied by $1500\frac{1}{4}$; then $\$0.08\frac{3}{4} \times 1500\frac{1}{4} = \133.40 , Ans.
4. $\$1.56$ is a gain of 20 per cent.; therefore $\$156 \div 1.20 = \1.30 , the actual value of the mixture; hence,

$$130 \left\{ \begin{array}{l} 17 \text{ at } 126, \text{ gain } 68. \\ 7 \text{ at } \text{---} \text{ loss } 68. \end{array} \right.$$

That is, 7 gallons cost so much that the loss, when sold at 130, was 68; then $68 \div 7 = 9\frac{1}{2}$ cts. loss per gallon; and $\$1.30 + \$0.09\frac{1}{2} = \$1.39\frac{1}{2}$, Ans.

5. 100 ounces 20 tari 10 graini = $100.68\frac{1}{2}$ ounces; $100.68\frac{1}{2} \times 2.40 = \$241.64 \div .16 = 1510.25$ lire = 1510 lire 25 centesimi, Ans.

6. 20 U. S. gal. = 1 eimer of Sweden.
 3 eimers of S. = 4 eimers of Trieste.
 24 eimers of T. = 9 ahms Danish.
 33 ahms D. = 5 carri of Naples.
 12 carri of N. = — U. S. gal.

$$\frac{20 \times 3 \times 24 \times 33 \times 12}{1 \times 4 \times 9 \times 5} = 3168 \text{ gal.}; 3168 \times \$0.80$$

(value of 1 ducat) = $\$2534.40$, cost of 12 carri of wine.
 Again,

- 20 U. S. gal. = 1 eimer of Sweden.
 3 eimers of S. = 4 eimers of Trieste.
 170 eimers of T. = — U. S. gal.

$$\frac{20 \times 3 \times 170}{1 \times 4} = 2550 \text{ gal.}; 2550 \times \$0.84\frac{1}{2} \text{ (the value of}$$

1 florin 45 kreutzers in U.S. money) = \$2164.31 $\frac{1}{4}$, cost of 170 eimers of Trieste; hence, 12 carri of wine will cost \$2534.40 - \$2164.31 $\frac{1}{4}$ = \$370.08 $\frac{3}{4}$ more than 170 eimers of Trieste of wine, Ans.

7. 60s. = 3£.; 3 × 4.80 = \$14.40; \$14.40 ÷ 8.25 (bu. in 1qr.) = \$1.74 $\frac{6}{11}$, the limited cost of a bushel delivered in Liverpool; this sum *less* the freight will be the limited cost of a bushel in Baltimore; 12d. = $\frac{1}{20}$ of 1£., of \$4.80 = \$0.24; then, \$1.74 $\frac{6}{11}$ - \$0.24 = \$1.50 $\frac{6}{11}$, Ans.

8. 87 ÷ 1.16 = 75 cts., the actual value of the mixture. Since there is a difference of 12 cts. per pound in the two ingredients of the mixture, 11 pounds cancels eleven parts of the difference, and the 5 pounds cancels five parts of the difference; therefore, $\frac{1}{6}$ of 12 (the amount cancelled by the 11 pounds) must be deducted from 75 in finding the value of the 5 pounds per pound; 75 - $\frac{1}{6}$ of 12 = 66 $\frac{3}{4}$ cts., the value of each pound of the 5 pounds; 75 + $\frac{5}{6}$ of 12 = 78 $\frac{3}{4}$ cts., the value per pound of the 11 pounds, Ans.

INVOLUTION.

(ART. 515, p. 370.)

1.	5.	5.
	512.	6.
2.	1024.	7.
3.	27.	8.
4.	504.	9.

512.

15625.

1612800.

282475249.

.000004100625.

(ART. 516, p. 370.)		9.	96889010407
2.	2097152.	10.	2176782336.
3.	40353607.	11.	205891132094649+.
4.	60466176.	12.	363.691179+.
5.	281950621875.		
6.	$\frac{64}{729}$.	13.	$\left\{ \begin{array}{l} 157\frac{233}{1024} \\ 1.800943+ \\ 116\frac{133}{248} \end{array} \right.$
7.	22071204.		
8.	36372961.		

EVOLUTION.

EXTRACTION OF THE SQUARE ROOT.

(ART. 525, p. 376.)		12.	7 $\frac{1}{2}$.
5.	1856.	13.	9 $\frac{1}{2}$.
6.	999.	14.	19 $\frac{1}{19}$.
7.	$\frac{7}{12}$.	15.	1.4.
8.	15.3.	16.	81.
9.	6 $\frac{1}{2}$.	17.	27.
10.	.027.	18.	64.
11.	4.16.	19.	3105671.
(ART. 526, p. 378.)		4.	1.77482393+.
2.	2.5298+.	5.	19.3132079+.
3.	1.41421+.	6.	2.98831055+.

EXTRACTION OF THE CUBE ROOT.

(ART. 529, p. 383.)			
3.	426.	9.	478.
4.	$\frac{5}{7}$.	10.	11.
5.	4.39.	11.	3 $\frac{2}{11}$.
6.	379.	12.	327.
7.	392.	13.	4968.
8.	.899.	14.	7583.

(ART. 530, p. 383.)

1.	1.2599 +.	3.	1.442249 +.
2.	2.2239 +.	4.	2.08008382301904 +.

EXTRACTION OF ANY ROOT.

(ART. 532, p. 384.)

(2.)	(3.)	(4.)
*3)998001	*2)262144	*3)43046721
3)332667	2)131072	3)14348907
*3)110889	2)65536	3)4782969
3)36963	*2)32768	3)1594323
*3)12321	2)16384	*3)531441
3)4107	2)8192	3)177147
*37)1369	*2)4096	3)59049
37)37	2)2048	3)19683
1	2)1024	*3)6561
$3 \times 3 \times 3 \times 37 = 999,$	*2)512	3)2187
[Ans.	2)256	3)729
	2)128	3)243
	*2)64	*3)81
	2)32	3)27
	2)16	3)9
	*2)8	3)3
	2)4	1
	2)2	$3 \times 3 \times 3 \times 3 = 81,$
	1	[Ans.
$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64, \text{Ans.}$		

(5.)

$$\begin{array}{r}
 *3)14348907 \\
 \underline{3)4782969} \\
 3)1594323 \\
 \underline{3)581441} \\
 3)177147 \\
 *3)59049 \\
 \underline{3)19683} \\
 3)6561 \\
 \underline{3)2187} \\
 3)729 \\
 *3)243 \\
 \underline{3)81} \\
 3)27 \\
 \underline{3)9} \\
 3)3
 \end{array}$$

(6.)

$$\begin{array}{r}
 *5)11390625 \\
 \underline{5)2278125} \\
 5)455625 \\
 \underline{5)91125} \\
 5)18225 \\
 \underline{5)3645} \\
 *3)729 \\
 \underline{3)243} \\
 3)81 \\
 \underline{3)27} \\
 3)9 \\
 \underline{3)3} \\
 1
 \end{array}$$

$$5 \times 3 = 15, \text{ Ans.}$$

$$1 \quad 3 \times 3 \times 3 = 27, \text{ Ans.}$$

HORNER'S METHOD OF EXTRACTING ROOTS.

(Art. 533, p. 387.)

(3.)

$$\begin{array}{r}
 0 \quad 0 \quad 41678648568(3467, \text{ Ans.}) \\
 3 \quad 9 \quad 27 \\
 \underline{8 \quad 9 \quad 14678} \\
 3 \quad 18 \quad 12304 \\
 \underline{6 \quad 2700 \quad 2369648} \\
 3 \quad 376 \quad 2117736 \\
 \underline{90 \quad 3076 \quad 251912568} \\
 4 \quad 392 \quad 251912568 \\
 \underline{94 \quad 346800} \\
 4 \quad 6156 \\
 \underline{98 \quad 352956} \\
 4 \quad 6192 \quad (\text{Brought up.}) \\
 1020 \quad 35914800 \quad 1082 \\
 6 \quad 72709 \quad 6 \\
 \underline{1026 \quad 35987509 \quad 10880} \\
 6 \quad 7 \\
 1082 (\text{Carried up.}) \quad 10887
 \end{array}$$

(4.)

$$\begin{array}{r}
 0 \quad 0 \quad 48614208(352, \\
 8 \quad 9 \quad 27 \quad [\text{Ans.}] \\
 \underline{8 \quad 9 \quad 16614} \\
 8 \quad 18 \quad 15875 \\
 \underline{6 \quad 2700 \quad 739208} \\
 8 \quad 475 \quad 739208 \\
 \underline{90 \quad 3175} \\
 5 \quad 500 \\
 \underline{95 \quad 367500} \\
 5 \quad 2104 \\
 100 \quad 369604 \\
 5 \\
 \underline{1050} \\
 2 \\
 1052
 \end{array}$$

(5.)

0	0	1.05(1.016397, Ans.
1	1	1
1	1	<u>50000</u>
1	2	<u>30301</u>
2	30000	<u>19699</u>
1	301	<u>18468</u>
300	30301	<u>1231</u>
1	302	<u>929</u>
301	30608	<u>302</u>
1	18	<u>279</u>
302	3078	<u>23</u>
1	18	<u>22</u>
303	3096	<u>1</u>
	...	

(6.)

				184528125(45, Ans.
4	16	64	256	<u>1024</u>
4	16	64	256	<u>82128125</u>
4	32	192	1024	<u>82128125</u>
8	48	256	12800000	
4	48	384	3625625	
12	96	640000	16425625	
4	64	85125		
16	16000	725125		
4	1025			
200	17025			
5				
205				

KEY TO

(7.)

0	0	0	100(8.162278, Ana.
8	9	27	81
8	9	27	190000
8	18	81	118521
6	27	108000	7647900
8	27	5521	7860080
9	5400	118521	287820
8	121	5648	252672
120	5521	1191640	35148
1	122	85040	25294
121	5648	1226680	9854
1	123	35484	8853
122	5766	1262164	1001
1	74	120	1012
123	5840	126336	
1	74	120	
124	5914	126456	
	74	1	
	5988	12647	

(8.)

0	0	0	0	6561(5.7995466,
5	25	125	625	8125 [Ana.
5	25	125	625	348600000
5	50	875	2500	289192057
10	75	500	31250000	54407948
5	75	750	10063151	49025772
15	150	1250000	41818151	5882171
5	100	187593	11466854	5078080
20	25000	1487593	52780006	809141
5	1799	200529	169308	282755
250	26799	1638122	5447308	26886
7	1848	213898	171986	22624
257	28647	1851930	5619244	3762
7	1897	292	1745	3394
264	30544	18812	563670	368
7	1946	292	1745	339
271	32490	19104	565415	29
7		292	10	
278		19896	56557	
7			10	
285			56561	

APPLICATION OF POWERS AND ROOTS.

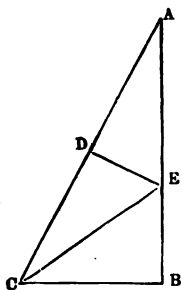
(PAGE 392.)

1. $\sqrt{141376} = 376$, Ans.
2. $1760 \times 9 = 15840$; $\sqrt{15840} = 125.857 +$ feet, Ans.
3. $144 \times 144 = 20736$; $64 \times 64 = 4096$; $20736 - 4096 = 16640$; $\sqrt{16640} = 128.99 +$ feet, Ans.
4. $20 \times 20 = 400$; $16 \times 16 = 256$; $12 \times 12 = 144$; $400 + 256 + 144 = 800$; $\sqrt{800} = 28.28 +$ feet, Ans.
5. $40 \times 40 = 1600$; $1600 \div 4 = 400$; $\sqrt{400} = 20$ rods, Ans.
6. $60 : 90 :: 66^2 = 4356 : 6534$; $\sqrt{6534} = 80.83 +$ rods, Ans.
7. $2h. : 3h. :: (\frac{3}{4})^2 = \frac{9}{16} = .5625 : .84375$; $\sqrt{.84375} = .918 +$ inch, Ans.
8. $2^2 = 4 : 1.5^2 = 2.25 :: 50m. : 28m. 7\frac{1}{2}sec.$, Ans.
9. $4 \times 4 \times 3 = 48 : 6 \times 6 \times 2 = 72 :: 4h. : 6h.$, Ans.
10. $40 \times 90 = 3600$; $\sqrt{3600} = 60$, Ans.
11. $31\frac{1}{4} \times 20 = 625$; $\sqrt{625} = 25lb.$, Ans.
12. $2 : 3 :: 2400 : 3600$; $\sqrt{3600} = 60$ trees in length. $3 : 2 :: 2400 : 1600$; $\sqrt{1600} = 40$ trees in breadth. $60 - 1 = 59 \times 7 = 413$; $40 - 1 = 39 \times 7 = 273$; $413 \times 273 = 112749$ square yards, Ans.
13. $50 \div 2 = 25$; $25^2 = 625$; $625 - 600 = 25$; $\sqrt{25} = 5$; $25 + 5 = 30$ years the older; $25 - 5 = 20$ years the younger, Ans.
14. $128 \times 128 = 16384$; $72 \times 72 = 5184 + 16384 = 21568$; $\sqrt{21568} = 146.86 +$ miles, Ans.
15. $100 \times 100 = 10000$; $70 - 5 = 65$; $65 \times 65 = 4225$; $10000 - 4225 = 5775$; $\sqrt{5775} = 75.993420 +$; $80 \times 80 = 6400$; $50 - 5 = 45$; $45 \times 45 = 2025$; $6400 - 2025 = 4375$; $\sqrt{4375} = 66.143782 +$; 75.993420

+ 66.143782 = 142.137202 \times 142.137202 = 20202-
 .984192388804; $70 - 50 = 20$; $20 \times 20 = 400$; 20202-
 .984192388804 + 400 = 20602.984192388804; $\sqrt{20602-}$
 .984192388804 = 143.537306 + feet, Ans.

16. $\sqrt{16} = 4$; $44 + 4 = 48$; $48 \div 2 = 24$, the larger num-
 ber; $44 - 4 = 40$; $40 \div 2 = 20$, the smaller, Ans.

17. Let A B represent the height of the tree, E the top of the
 stump, C the point on which the top of the
 tree will fall. As the tree will rest on the
 stump, it is evident that A E will be equal
 to C E. By drawing D E at right angles to
 A C, it is evident that A D will be equal to
 D C; that is, the line A C is bisected in D;
 therefore we have two similar triangles, A B
 C and A D E. This is evident from the fact
 that each of these triangles has one right
 angle, and that the angle A is common
 to both triangles. Therefore, as A B is to A C, so is A D to
 A E. If, then, we take A E from A B, the remainder, E B, will
 be the answer required.



FIRST OPERATION.

A B = 80; B C = 40; by Art. 535, A C = $\sqrt{A B^2 + B C^2} = \sqrt{80^2 + 40^2} = \sqrt{8000}$; and A D = $\frac{1}{2}$ A C = $\sqrt{40^2 + 20^2} = \sqrt{2000}$. Then, by Art. 539, A B : A C :: A D : A E, and A B² : A C² :: A D² : A E²; hence, 6400 : 8000 :: 2000 : 2500, and $\sqrt{2500} = 50$. Therefore, 80ft. - 50ft. = 30ft., Ans.

BY ALGEBRA.

Let x represent B E, $80 - x = C E$, and $40 = C B$. Then
 $80 - x - x^2 = 40^2$; $6400 - 16x + x^2 - x^2 = 1600$. And
 $160x = 6400 - 1600 = 4800$.
 $x = 30$ feet = B E, Ans.

SECOND OPERATION.

80 feet evidently equals the length of the part broken off and the stump together. The distance from the bottom of the tree to the point on which the top may fall, 40 feet, represents the base of a right-angled triangle, of which the part of the tree broken off is the hypotenuse, and the part left standing on the stump is the perpendicular. Then, according to ART. 553, $40^2 = 1600 \div 80 = 20$; $80 - 20 = 60$; $60 \div 2 = 30$ feet, Ans.

$$18. \quad 400 \times 400 = 160000; \quad 160 \times 160 = 25600; \quad 160000 - 25600 = 134400; \quad \sqrt{134400} = 366.6 - 50 = 316.6, \\ \text{Ans.}$$

$$19. \quad 70 \times 70 = 4900; \quad 40 \times 40 = 1600; \quad 30 \times 30 = 900; \\ 4900 - 1600 = 3300; \quad \sqrt{3300} = 57.445 +; \quad 4900 - 900 = 4000; \quad \sqrt{4000} = 63.245 + 57.445 = 120.69 + \\ \text{feet, Ans.}$$

$$20. \quad r_1^2 \times r_1^2 = r_2^2 : 1^2 \text{ in.} : : 450 \text{ lb.} : 45000 \text{ lb., Ans.}$$

21. 10 acres = 435600 square feet; $\sqrt{435600} = 660$ feet = each side of the square plat; $660 \div 6 = 110$ = the number of vines in each row, and also the number of rows, in the square order; $110 \times 110 = 12100$ = the number of vines, in the square order.

In the quincunx order, the vines are disposed in the form of equilateral triangles; the distance between the rows, therefore, must equal the altitude of such triangles, having 6 feet for each of their equal sides. Hence, $6^2 - 3^2 = 27$; $\sqrt{27} = 5.196 +$ feet = the distance of the rows from each other; $660 \div 5.196 = 127$ = the number of rows in the field; and the number of vines will be the same as in the square order. Therefore, $127 \times 110 = 13970$ = the number of vines in the quincunx order; and $13970 - 12100 = 1870$ = the number more of vines in the quincunx than in the square order, Ans.

In the square order no vine need be set *nearer* than 3 feet of the edge of the plat, and in the quincunx order nearer than $1\frac{1}{2}$ feet of the edge of the plat, and answer the conditions of the question.

$$22. \quad 40 \times 40 = 1600; \quad 4 \times 4 = 16; \quad 1600 - 16 = 1584;$$

$1584 \div 4 = 396$; $396 \times 3 = 1188$; $1188 + 16 = 1204$;
 $\sqrt{1204} = 34.698$; $40 - 34.698 = 5.302$; $5.302 \div 2 =$
 2.651in. , the part A will grind off. $1204 - 396 = 808$;
 $\sqrt{808} = 28.425$; $34.698 - 28.425 = 6.273$; $6.273 \div 2 =$
 $3.136+\text{in.}$, the part B will grind off. $808 - 396 = 412$;
 $\sqrt{412} = 20.297$; $28.425 - 20.297 = 8.128$; $8.128 \div 2 =$
 4.064in. , part C grinds off. $412 - 396 = 16$; $\sqrt{16} = 4$;
 $20.297 - 4 = 16.297$; $16.297 \div 2 = 8.148+\text{in.}$, part D will
 grind off.

Ans. A, 2.651in.; B, 3.136+in.; C, 4.064in.; D, 8.148+in.

23. $49\frac{3}{4}$ feet $= 49.625$ feet; $49.625 \times 144 = 7146$ square
 inches; $1.5 \times 1.5 \times 2 = 4.5$; $7146 - 4.5 = 7141.5$; 7141.5
 $\div 6 = 1190.25$; $\sqrt{1190.25} = 34.5$; $34.5 + 1.5 = 36$ inches,
 Ans.

To understand the operation of this question, we will take six small square pieces of board, each of the same dimensions, and with these we will construct a cubic box; but, in so doing, we shall find that we need two small cubes, each of which is of the thickness of the board or plank used.

Now, if the box, after being completed, was a cube whose sides measured 36 inches each, and if the board was $1\frac{1}{2}$ inches thick, it would require 7146 square inches of the board to make the box, as the pupil can readily perceive.

But our box was made of six square pieces of board and two small cubes, each measuring, in the present case, $1\frac{1}{2}$ inches square. If we deduct the contents of these two squares, $= 1.5 \times 1.5 \times 2 = 4.5$ square inches, from the superficial contents of the board, $= 7146$ inches, we have $7146 - 4.5 = 7141.5$ square inches remaining; and if we divide these inches by 6, we have the superficial contents of one of the square boards of which we make our box. Thus, $7141.5 \div 6 = 1190.25$. The square root of this number, $\sqrt{1190.25} = 34.5$, will be the width of each board. To this number we must add the thickness of the board, 1.5 inches, and we have the answer, $34.5 + 1.5 = 36$ inches.

24. $22\frac{3}{4}$ feet $= 22\frac{3}{4} \times 144 = 3242$ square inches; $2.5 \times$
 $2.5 = 6.25$ inches; $3242 - 6.25 = 3235.75$; $3235.75 \div 7 =$

462.25; $\sqrt{462.25} = 21.5$; $21.5 + 2.5 = 24$ inches, the width of the box; $24 \div 2 = 12$ inches, the height; $24 \times 2 = 48$ inches, the length.

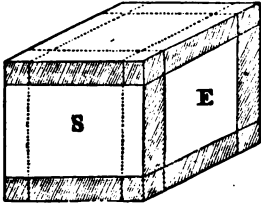
From each of these numbers we subtract 5 inches, the thickness of the two sides of the box; $12 - 5 = 7$; $24 - 5 = 19$; $48 - 5 = 43$. The inside dimensions of the box will therefore be 7, 19, and 43 inches; and its contents will therefore be $7 \times 19 \times 43 = 5719$ cubic inches, Ans.

To understand the above operation, we will construct a box that shall be twice as wide as its height, and twice as long as its width. With the materials used to construct this box the pupil will find that he can construct a cubical box whose sides will measure half the length of the former box. And, if it be constructed as the box in question 44, there will be material remaining sufficient to make one of the sides, after deducting the square of the thickness of the plank. Thus, if our box was 12 inches high, 24 inches wide, and 48 inches long, and was made of a plank $2\frac{1}{2}$ inches thick, it would require one whose superficial contents were 3242 inches. From this sum we deduct the square of the thickness of the plank, $2.5 \times 2.5 = 6.25$ inches; $3242 - 6.25 = 3235.75$ inches. This sum, as we have before shown, is sufficient not only to construct the cubical box, but there will be sufficient remaining, wanting the square of the thickness of the plank, to make another, similar to one of the six squares of which we made the box. This will be evident, if the box be cut into two parts, and one of them placed on the other. Therefore, we divide 3235.75 by 7, because there are materials sufficient for 7 squares, and the quotient is 462.25. The square root of this number is the length of one of the squares of which the box is made. $\sqrt{462.25} = 21.5$ inches. To this number we add the thickness of the plank, and we have the width of the required box. $21.5 + 2.5 = 24$ inches, width of the box; $24 \div 2 = 12$ inches, height; and $24 \times 2 = 48$ inches, length, Ans.

We have copied, by permission, from the *Massachusetts Teacher*, the following illustrations, with diagrams, of the 23d and 24th problems.

(23.)

FIG. 1.



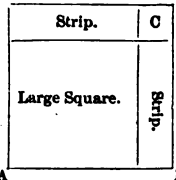
Suppose the 6 pieces of which the box is made to be so arranged (Fig. 1) that the edges of the top, bottom, and sides, will appear at the ends, as in the shaded parts about E; and that the edges of the top and bottom will appear at the sides, as in the shaded parts above and below S.

The external area of each face of the box may be considered as divided into, 1st, a large square, as at E; 2d, 4 strips, one on each side of the large square, having each a width equal to the thickness of the board ($1\frac{1}{2}$ in.), and the same length as one side of the large square; and 3d, 4 small squares, each having, for the length of a side, the thickness of the board. In the 6 faces of the box, then, there are 6 large squares + 24 strips + 24 small squares.

But the shaded parts, though forming part of the external area of the sides and ends, do *not* form part of the side and end *pieces* of the box,* and are not to be reckoned as consuming any part of the surface of the board. If, then, from the large squares, strips, and small squares, that make up the whole external area of the box, we deduct 12 strips (2 for each side, and 4 for each end), and 16 small squares (4 for each side, and 4 for each end), we have left 6 large squares + 12 strips + 8 small squares = the area of the board = $49\frac{1}{2}$ sq. ft. = 7146 sq. in.

From this deduct the area of the 8 small squares ($(1\frac{1}{2}$ in.)² \times 8 = 18 sq. in.) and we have 6 large squares + 12 strips = 7146 sq. in. - 18 sq. in. = 7128 sq. in. Divide this by 6, and we find one large square + 2 strips = 1188 sq. in. This large square and the 2 strips may be arranged as in Fig. 2, which will be an exact square, if we fill the corner C ($1\frac{1}{2}$ in.)²

FIG. 2.

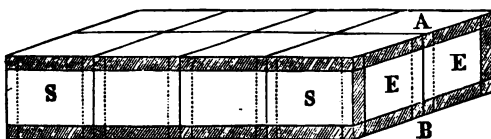


* Notice the difference between the side and side *piece*, and the end and end *piece*, of the box.

$= 2\frac{1}{4}$ sq. in. Then 1188 sq. in. $+ 2\frac{1}{4}$ sq. in. $=$ the area of the square (Fig. 2) completed $= 47\frac{61}{4}$ sq. in. Then, $\sqrt{47\frac{61}{4}}$ sq. in. $= 6\frac{9}{2}$ in. $=$ length of one side of the square, or the line A B. This, it is evident (see Fig. 1), is $1\frac{1}{2}$ in. less than the length of a side of the box; therefore, $6\frac{9}{2}$ in. $+ 1\frac{1}{2}$ in. $= 36$ in., the length of one side.

(24.)

FIG. 3.



Let, in Fig. 3, the shaded parts about E E represent the edges of the top, bottom, and sides, at the end of the box; and the shaded parts above and below S S, the edges of the top and bottom, at the side. These shaded parts represent the thickness of the plank, and form part of the external area of the side and end of the box, but no part of the side and end *pieces*.

If we draw the line A B vertically across the middle of the end of the box, the whole end is divided into 2 equal squares; but, taking away the shaded portions, it is evident that the end *piece* is not divided by this line into squares, since we have diminished each square of the end in one direction by twice the thickness of the plank, and in the other by only once its thickness.

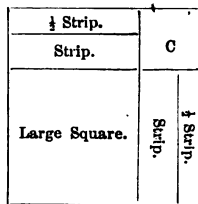
To get two equal squares in the end *piece*, let there be marked off on each side of A B, by dotted lines, two strips, each as wide as the plank is thick. We may now suppose the *end* of the box (not the end *piece*) to be divided into, 1st, 2 large squares, E and E; 2d, 8 strips, each equal in length to the side of a *large square*, and in width to the thickness of the plank ($2\frac{1}{2}$ in.); and 3d, 8 small squares, each having its side equal in length to the thickness of the plank. Since the box is twice as wide as it is high, and twice as long as wide, the area of an end is doubled in a side, and quadrupled in the top or bottom. Then an end, a side, and the top or bottom, contain 7 times as many large squares, strips, and small squares, as an end; that is, 14 large squares $+ 56$

strips + 56 small squares; and both ends, both sides, and the top and bottom, or the whole external area of the box, = 28 large squares + 112 strips + 112 small squares.

But these exceed the area of the 6 *pieces* of which the box is made, by the strips and small squares in the edges of the top and bottom at the sides, and of the top, bottom, and sides, at the ends. Referring to Fig. 3, and noticing that the strips marked off by dotted lines are part of the pieces in which they are found, and not to be deducted, we see that we must deduct, for each side, 8 strips and 16 small squares; and for an end, 6 strips and 8 small squares; or 28 strips and 48 small squares for both sides and both ends. Subtracting these from the large squares, strips, and small squares of the whole external area of the box, we have 28 large squares + 84 strips + 64 small squares = the area of the 6 pieces = $22\frac{7}{2}$ sq. feet. = 3242sq. in. From this deduct the 64 small squares ($(2\frac{1}{2}\text{in.})^2 \times 64 = 400\text{sq. in.}$), and we have 28 large squares + 84 strips = 3242sq. in. — 400sq. in. = 2842 sq. in. Divide by 28, and we find 1 large square + 3 strips = $2\frac{3}{4}$ sq. in. = $2\frac{3}{4}$ sq. in.

This large square and the three strips may be arranged as in Fig. 4; namely, a strip and a half on each of two adjacent sides of the *large square*. This will be an exact square if we fill out the corner C, which contains a square each side of which is one and a half times the width of a *strip* ($2\frac{1}{2}\text{in.} \times 1\frac{1}{2} = 1\frac{5}{4}\text{in.}$). Adding this small square ($(1\frac{5}{4}\text{in.})^2 = 2\frac{25}{16}\text{sq. in.}$), we have $2\frac{3}{4}$ sq. in. + $2\frac{25}{16}\text{sq. in.} = 1\frac{13}{8}\text{sq. in.} = \text{area of the square Fig. 4, completed. Then } \sqrt{1\frac{13}{8}\text{sq. in.}} = 1\frac{3}{4}\text{in.} = \text{length of a side of the square, or the line D F.}$

FIG. 4.



Comparing this with Fig 3, we see it wants half the thickness of the plank ($2\frac{1}{2}\text{in.} \div 2 = 1\frac{1}{4}\text{in.}$) of being the external height of the box. Then, $1\frac{3}{4}\text{in.} + 1\frac{1}{4}\text{in.} = 1\frac{5}{4}\text{in.} = 1\frac{5}{4}\text{in.} = 12\text{in.} = \text{external height of the box; } 12\text{in.} \times 2 = 24\text{in.} = \text{external width of the box; and } 24\text{in.} \times 2 = 48\text{in.} = \text{external length of the box.}$

Subtract from each of these dimensions twice the thickness of the plank, and we have 12in. — 5in. = 7in. = internal height of the box; 24in. — 5in. = 19in. = internal width of the box; 48in. — 5in. = 43in. = internal length of the box, and the product of the internal dimensions = 5719 cubic in. = the contents of the box.

$$25. 3 \times 3 \times 3 = 27 : 6 \times 6 \times 6 = 216 :: 4\text{lb.} : 32\text{lb.},$$

Ans.

$$26. 1 \times 1 \times 1 = 1 : 3.5 \times 3.5 \times 3.5 = 42.875 :: \$120$$

: \$5145, Ans.

$$27. 5\text{ft. } 10\text{in.} = 70\text{in.}; 10\text{ft. } 4\frac{1}{2}\text{in.} = 124.6\text{in.}; 70 \times 70 \times 70$$

$$= 343000; 124.6 \times 124.6 \times 124.6 = 1934434.936;$$

$$343000 : 1934434.936 :: 180\text{lb.} : 1015.1\text{+lb.}, \text{ Ans.}$$

$$28. 2\text{lb.} : 2000\text{lb.} :: 4 \times 4 \times 4 = 64\text{in.} : 64000;$$

$$\sqrt[3]{64000} = 40\text{in.} = 3\text{ft. } 4\text{in. high};$$

$$2\text{lb.} : 2000\text{lb.} :: 3 \times 3 \times 3 = 27\text{in.} : 27000\text{in.};$$

$$\sqrt[3]{27000} = 30\text{in.} = 2\text{ft. } 6\text{in. wide};$$

$$2\text{lb.} : 2000\text{lb.} :: \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} : \frac{1920}{64};$$

$$\sqrt[3]{\frac{1920}{64}} = \frac{1}{4} = 2\frac{1}{2}\text{in. thick, Ans.}$$

$$29. 12096 \div 56 = 216; \sqrt[3]{216} = 6; 56 \times 6 = 336;$$

$$336 \times 6 = 2016; 336 \text{ and } 2016, \text{ mean proportions};$$

$$\text{Then, } 56 : 336 :: 2016 : 12096, \text{ Ans}$$

$$30. 5 \times 5 \times 5 = 125\text{ft.} : 20 \times 20 \times 20 = 8000\text{ft.} :: 1\text{cwt.}$$

: 64cwt., Ans.

$$31. 6 \times 6 \times 6 = 216\text{ft.} : 10 \times 10 \times 10 = 1000\text{ft.} :: 1\text{da.}$$

: 4.629+da., Ans.

$$32. 6 \times 6 \times 6 = 216\text{ft.} : 8 \times 8 \times 8 = 512\text{ft.} :: 600\text{lb.}$$

: 1422.2+lb., Ans.

$$33. 5 \times 5 \times 5 = 125; 125 \div 4 = 31.25; 125 - 31.25 =$$

$$93.75; \sqrt[3]{93.75} = 4.542+; 5 - 4.542 = .45\text{+in.,}$$

the first woman's share. $93.75 - 31.25 = 62.50;$

$$\sqrt[3]{62.50} = 3.968; 4.542 - 3.968 = .57\text{+in., share}$$

of the second woman. $62.50 - 31.25 = 31.25; \sqrt[3]{31.25}$

$$= 3.149; 3.968 - 3.149 = .82\text{+in., third woman's}$$

share. $3.149\text{+in., fourth woman's share.}$

34. If Wells have 100 per cent., Rowe will have 108 per cent., and Northend 110 per cent. But Pierce has 10 per cent. more than Rowe; therefore he will have $108 \times 1.10 = 118.8$ per cent. $118.8 + 108 + 100 + 110 = 436.8$; 5 tons = 100cwt.

Then $436.8 : 118.8 :: 100\text{cwt.} : 27\frac{54}{273}\text{cwt.}$ for Pierce.

And $436.8 : 108 :: 100\text{cwt.} : 24\frac{198}{273}\text{cwt.}$ for Rowe.

And $436.8 : 100 :: 100\text{cwt.} : 22\frac{243}{273}\text{cwt.}$ for Wells.

And $436.8 : 110 :: 100\text{cwt.} : 25\frac{50}{273}\text{cwt.}$ for Northend.

To find the feet in height that each must take, we adopt the following rule, and say, As the relative value of all their shares is to the relative value of each share, so is the cube of the height of the pyramid or stack to the cube of the height of each man's part of the stack. But we are to compute from the top of the stack each time, and then subtract as in the following process :

$16 \times 16 \times 16 = 4096\text{ft.}$, cube of the height of the stack.
 $436.8 : 118.8 :: 4096 : 1114.02197$; $\sqrt[3]{1114.02197} = 10.366 + \text{ft.}$ in height for Pierce.

$436.8 : 118.8 + 108 = 226.8 :: 4096 : 2126.76923076$;
 $\sqrt[3]{2126.76923076} = 12.859 +$ from the top of the stack. From this number we subtract the height of Pierce's stack, $12.859 - 10.366 = 2.493\text{ft.}$ for the height of Rowe's stack.

$436.8 : 118.8 + 108 + 100 = 326.8 :: 4096 : 3064.49816849$;
 $\sqrt[3]{3064.49816849} = 14.525$ from the top of the stack. From this we subtract the height of the other two stacks; $14.525 - 12.859 = 1.666\text{ft.}$ for the height of Wells' stack.

If from the height of the stack, 16ft., we subtract the last root, we have the remaining height of the stack for Northend; thus, $16 - 14.525 + = 1.474\text{ft.}$, height of Northend's stack.

NOTE. — The decimals in the answer to the above question will vary according to the degree of accuracy required.

ARITHMETICAL PROGRESSION.

(ART. 557, p. 396.)

$$2. \frac{39 - 3}{19 - 1} = 2y., \text{ Ans.}$$

$$3. \frac{45 - 5}{11 - 1} = 4m., \text{ Ans.}$$

(ART. 558, p. 397.)

$$2. \frac{51 - 7}{4} + 1 = 12da., \text{ Ans.}$$

$$3. \frac{103 - 1}{2} + 1 = 52w., \text{ Ans.}$$

(ART. 559, p. 398.)

$$2. \frac{100 + 4}{2} \times 17 = 884, \text{ Ans.}$$

3. $320 \times 30 = 9600 = \text{rods in } 30m.;$ but there will be one more stone in this distance than there are rods, because there will be a stone at each end of the 30 miles. The man must travel 2 rods to bring the first stone to the basket, and 60 miles and 2 rods to bring the last stone; wherefore the following formula:

$$\frac{19202 + 2 \times 9601}{2} = 92188802rd. = 288090m. 2rd., \text{ Ans.}$$

(ART. 560, p. 398.)

$$2. \frac{45 + 3 \times 45 - 3 + 2}{2 \times 2} = 528. \text{ sum of the series, Ans.}$$

$$3. \frac{618 \times 6 \times 618 - 6 + 12}{2 \times 12} = \$162.24, \text{ Ans.}$$

(ART. 561, p. 399.)

$$2. \frac{100010000 \times 2}{10000} - 20000 = 2, \text{ Ans.}$$

$$3. \frac{528 \times 2}{22} - 3 = \$45; \frac{45 - 3}{22 - 1} = \$2 \text{ difference, Ans.}$$

(ART. 562, p. 400.)

$$2. \frac{49 - 4}{6 - 1} = 9, \text{ common difference.}$$

As there are 6 terms, the third and fourth are required;
 $4 + 9 + 9 = 22$, the third term; $22 + 9 = 31$, the
 fourth term, Ans.

$$3. \frac{30 - 20}{7 - 1} = 1\frac{2}{3}, \text{ the common difference.}$$

$20 + 1\frac{2}{3} = 21\frac{2}{3}$; $21\frac{2}{3} + 1\frac{2}{3} = 23\frac{1}{3}$; $23\frac{1}{3} + 1\frac{2}{3} = 25$;
 $25 + 1\frac{2}{3} = 26\frac{2}{3}$; $26\frac{2}{3} + 1\frac{2}{3} = 28\frac{1}{3}$. $21\frac{2}{3}$, $23\frac{1}{3}$, 25,
 $26\frac{2}{3}$, and $28\frac{1}{3}$, Ans.

GEOMETRICAL PROGRESSION.

(ART. 565, p. 401.)

$$2. 5 \times 3^{7-1} = 3645, \text{ seventh term, Ans.}$$

$$3. \frac{72}{3^{9-1}} = \frac{8}{27}, \text{ first term, Ans.}$$

$$4. \frac{885735}{3^{12-1}} = \frac{885735}{177147} = 5, \text{ twelfth term;}$$

$5 \times 3 = 15$, eleventh term; $15 \times 3 = 45$, tenth term,
 [Ans.]

$$5. 5 \div (\frac{1}{3})^{7-1} = 5 \div \frac{1}{729} = 3645, \text{ first term, Ans.}$$

$$6. 50 \times 1.06^{5-1} = 63.123848, \text{ last term, Ans.}$$

$$7. 2 \times 2^{30-1} = \$10737418.24, \text{ Ans.}$$

$$8. \$160 \times 1.06^{7-1} = \$226.96305796096, \text{ Ans.}$$

9. $\$300 \times 1.05^{5-1} = \$443.23, 6+,$ Ans.
 10. $\$100 \times 1.06^{31-1} = \$574.34911729132501162641063323-10802645846357252196069357387776,$ Ans.

(ART. 566, p. 402.)

5. $\frac{1.06^4 - 1}{1.06 - 1} \times 50 = 218.7308,$ sum of the series, Ans.
 6. $\frac{2^{40} - 1}{2 - 1} \times 10 = \$109951162777.50,$ Ans.
 7. $\frac{2}{3} \div (1 - \frac{1}{2}) = \frac{2}{3} \div \frac{1}{2} = \frac{4}{3} = 1\frac{1}{3},$ Ans.
 8. By examining this question, we find there have been 21 deposits. The amount of the last deposit is \$10.60, the \$10 being on interest only one year. The last but one is \$11.236. The last but two is \$11.91016. The last but three is \$12.6247696, and so on. Thus we have a regular geometrical series, the ratio of which is 1.06, the first term \$10.60, the number of terms 21, to find the sum of all the series.

$$\frac{1.06^{21} - 1}{1.06 - 1} \times 10.60 = \$423.922 +, \text{ Ans.}$$

9. $.008 = \frac{8}{1000}; .000497133 = \frac{497133}{1000000000},$ the first term;
 $\frac{1000000}{1000000},$ the ratio.
 $1 - \frac{1000000}{1000000} = \frac{999999}{1000000}; \frac{497133}{1000000000} \div \frac{999999}{1000000} = \frac{497133000000}{999999000000} = \frac{497133}{999999}; \frac{497133}{999999} + \frac{8}{1000} = \frac{885735}{999999} = \frac{885735}{999999},$ Ans.
 10. $10 \div (1 - \frac{9}{10}) = 10 \div \frac{1}{10} = 100$ miles, Ans.

(ART. 567, p. 403.)

2. $512 \div 1 = 512; \sqrt[2]{512} = 2$ ratio, Ans.
 3. $\frac{1328600 - 5}{1328600 - 885735} = 3$ ratio, Ans.
 4. $\sqrt[12-1]{\frac{2048}{1}} = 2$ ratio. $\frac{2048 \times 2 - 1}{1} = \4095 debt,
 [Ans.]

(ART. 568, p. 404.)

2. $128 \div \frac{1}{2} = 256$; $\sqrt[4]{256} = 4$ ratio; $\frac{1}{2} \times 4 = 2$; $2 \times 4 = 8$; $8 \times 4 = 32$. 2, 8, and 32, Ans.
3. $2187 \div 3 = 729$; $\sqrt[3]{729} = 9$; $3 \times 3 = 9$; $9 \times 3 = 27$; $27 \times 3 = 81$; $81 \times 3 = 243$; $243 \times 3 = 729$. 9, 27, 81, 243, and 729, Ans.

(ART. 569, p. 405.)

2. $20480 \div 5 = 4096 = 4^6$; $6 + 1 = 7$, Ans.
3. $2048 \div 1 = 2048 = 2^{11}$; $11 + 1 = 12$ months, Ans.

ANNUITIES.

(ART. 574, p. 408.)

2. $\frac{1.06^5 - 1}{1.06 - 1} \times \$500 = \$2818.546$, Ans.
3. $\frac{1.05^9 - 1}{1.05 - 1} \times \$80 = \$882.125$, Ans.
4. $\frac{1.035^{15} - 1}{1.035 - 1} \times \$1000 = \$19295.68$, Ans.
5. $\frac{1.035^6 - 1}{1.035 - 1} \times \$30 = \$196.50$, Ans.
6. $\frac{1.015^{22} - 1}{1.015 - 1} \times \$150 = \$3875.63$, Ans.

(ART. 575, p. 409.)

2. $\$6.801692 \times 100 = \680.169 , Ans.
3. $\$5.786373 \times 200 = \1157.274 , Ans.
4. $\$13.331709 \times 500 = \6665.854 , Ans.
5. $\$7.360087 \times 500 = \3680.043 , Ans.
6. $\$12.561102 \times 80 = \1004.888 , Ans.

(ART. 576, p. 409.)

2. $\$963 \div .06 = \16050 , Ans.
3. $\$6335 \div .07 = \90500 , Ans.
4. $\$1200 \div .05 = \24000 , Ans.

(ART. 577, p. 410.)

2. $\$11.469421 - \$6.801692 = \$4.667729$; $\$4.667729 \times 350 = \1633.705 , Ans.
3. $\$13.590326 - \$6.732745 = \$6.857581$; $\$6.857581 \times 70 = \480.03 , Ans.
4. $\$240 \div .06 = \4000 , the present worth of $\$240$ in perpetuity.
 $\$16.617546$, the present value of an annuity of $\$1$, due 100 years hence.
 $\$16.617546 \times 240 = \3988.211 ;
 $\$4000 - \$3988.211 = \$11.789$, Ans.
or,
 $\$1.06^{100} = \$339.302084+$.
 $\$4000 \div 339.302084 = \11.789 , Ans.

(ART. 578, p. 411.)

2. $\$3680.04 \div 7.360087 = \500 , Ans.
3. $\$882.12,5 \div 11.026564 = \80 . Ans.
4. $\$279 \div 6.975319 = \40 , Ans.

PERMUTATIONS AND COMBINATIONS.

(ART. 581, p. 412.)

2. $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ changes, Ans.
3. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 3628800$ days, Ans.
4. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$ changes. 365d. 5h. 49m. = 525949 min-

utes; $479001600 \div 10 = 47900160$ minutes; $47900160 \div 525949 = 91\text{y. } 38801\text{m.} = 91\text{y. } 26\text{d. } 22\text{h. } 41\text{m.}$, Ans.

5. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20 \times 21 \times 22 \times 23 \times 24 \times 25 \times 26 = 40329146112660563-5584000000$ changes, Ans.

(ART. 582, p. 412.)

2. $6 \times 5 \times 4 \times 3 = 360$ changes, Ans.
3. $26 \times 25 \times 24 \times 23 \times 22 \times 21 = 165765600$, Ans.

(ART. 583, p. 413.)

2. $\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} = \frac{720}{6} = 120$, Ans.
3. $\frac{100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10} = \frac{628156509555294720}{3628800} = \173103094564.40 , Ans.

ANALYSIS BY POSITION.

(ART. 587, p. 414.)

(2.)	(3.)
Supposed number, 120	Supposed number, 36
$\frac{1}{3}$ of 120 = <u>40</u>	$\frac{1}{2}$ of 36 = 18
$\frac{1}{4}$ of 120 = <u>30</u>	$\frac{1}{3}$ of 36 = 12
<u>70</u>	$\frac{1}{4}$ of 36 = <u>9</u>
Result found, 50	Result found, 75
50 : 60 :: 120 : 144, Ans.	75 : 125 :: 36 : 60, Ans.
(4.)	(5.)
C's assumed age, 20	Assumed sum, \$400
B's age = <u>60</u>	<u>1.60</u>
A's age = <u>120</u>	Amount for 10 yr., \$640.00
<u>200</u>	640 : 560 :: 400 : 350, Ans.
200 : 140 :: 20 : 14, C's age;	
14 \times 3 = 42, B's age; 42 \times 2 = 84, A's age, Ans.	

(ART. 588, p. 415.)

2. First suppose each laid out \$500; then

$\begin{array}{r} 4)\$500 \\ \underline{125} \\ 625 = \text{A's money.} \end{array}$	$\begin{array}{r} \$500 \\ \underline{225} \\ 275 = \text{B's money.} \end{array}$
--	--

Then, by the question, $2 \times 275 = \$550$ should be A's money, but A's money is \$625; therefore $\$625 - \$550 = \$75$ is the first error.

Again, we suppose each laid out \$800; then

$\begin{array}{r} 4)\$800 \\ \underline{200} \\ 1000 = \text{A's money.} \end{array}$	$\begin{array}{r} \$800 \\ \underline{275} \\ 575 = \text{B's money.} \end{array}$
---	--

Then, by the question, $2 \times \$575 = \1150 should be A's money, but A's money is \$1000; therefore $\$1150 - \$1000 = \$150$ +, second error; then, by the rule, $75 + 150 : 800 - 500 :: 75 : 100$; and $100 + 500 = 600$, the sum of dollars each invested, Ans.

3. We first suppose the age of the youngest to be 10. Then, by the question, the age of the next older will be 14, and the next 18, and the oldest will be 22. But, by the supposition, the age of the oldest was twice the age of the youngest, that is, $2 \times 10 = 20$; but the age of the oldest is 22, therefore the first error is $22 - 20 = 2$ — too small.

Again, we suppose the age of the youngest to be 16. Then the age of the next older will be 20, and the age of the next 24, and the age of the oldest will be 28. But, by the supposition, the age of the oldest was twice the age of the youngest, that is, $2 \times 16 = 32$; but the age of the oldest is 28; therefore, the second error is $32 - 28 = 4$ + too large; then, by the rule, $2 + 4 : 16 - 10 :: 4 : 4$; $16 - 4 = 12$, the age of the youngest; $12 \times 2 = 24$, age of the oldest; $24 - 4 = 20$, age of the second; $20 - 4 = 16$, age of the third son, Ans.

4. We suppose the value of the first horse to be \$20. Then $20 + 50 = \$70$ will be twice the value of the second horse; that is, the value of the second horse will be \$35. Then, by the question, $35 + 50 = \$85$ will be three times the value of the first horse; that is, the first horse will be worth \$28 $\frac{1}{3}$. But, by the supposition, he should be worth but \$20; therefore $28\frac{1}{3} - 20 = \$8\frac{1}{3}$ — is the first error. Again, we suppose the value of the first horse to be \$32. Then $32 + 50 = \$82$ will be twice the value of the second horse; that is, the second horse will be worth \$41. Then, by the question, $41 + 50 = \$91$ will be three times the value of the first horse; that is, the first horse will be worth \$30 $\frac{1}{3}$. But, by the supposition, he should be worth \$32; therefore, $32 - 30\frac{1}{3} = 1\frac{2}{3}$, the second error; then, $8\frac{1}{3} + 1\frac{2}{3} : 32 - 20 :: 8\frac{1}{3} : 10$; and $10 + 20 = \$30$, value of the first horse; $30 + 50 = \$80$, value of the second horse, Ans.
5. If we suppose the time to be 3 o'clock, it will be 9 hours to midnight, and $\frac{2}{3}$ of 3 hours will be equal to $\frac{2}{3}$ of 9 hours; but $\frac{2}{3}$ of 3 hours is $\frac{2}{3} \times 3 = 2$ hours, and $\frac{2}{3}$ of 9 hours is $\frac{2}{3} \times 9 = 6$ hours; therefore the first error will be $6 - 2 = 4$. Again, we suppose the time to be 4 o'clock. Then the time to midnight will be 8 hours; therefore, by the question, $\frac{2}{3}$ of 4 hours will be equal to $\frac{2}{3}$ of 8 hours, but $\frac{2}{3}$ of 4 hours is $\frac{2}{3} \times 4 = 2\frac{2}{3}$ hours; and $\frac{2}{3}$ of 8 hours is $\frac{2}{3} \times 8 = 5\frac{1}{3}$ hours; therefore, the second error will be $5\frac{1}{3} - 2\frac{2}{3} = 3$. Then, by the rule, $4 - 3 :: 3 : 1$; $3 + 1 = 4$ hours, the time after noon = 12 minutes past 3, Ans.
6. We first suppose their income to be \$360 each. Then, as A saves $\frac{1}{12}$ of his income, he will spend $\$360 \div 12 = \30 ; $\$360 - \$30 = \$330$ annually. As B spends \$100 per annum more than A, he will spend $\$330 + \$100 = \$430$ each year; and in 10 years he will spend

$10 \times \$430 = \4300 . As his income, by the supposition, is only \$360 per annum, it would be in ten years $10 \times \$360 = \3600 . His debt would therefore be $\$4300 - 3600 = \700 . But by the question it was only \$600; therefore the first error will be $\$700 - \$600 = \$100$ — too small.

Again, we suppose their income to be \$300 annually. Then as A saves $\frac{1}{12}$ of his, he will spend $\$300 \div 12 = \25 ; $\$300 - \$25 = \$275$ annually; and B, by the question, will spend $\$275 + \$100 = \$375$, and in 10 years he will spend $10 \times \$375 = \3750 . B's income by the question will be only $10 \times \$300 = \3000 ; therefore his debt will be $\$3750 - \$3000 = \$750$. But, by the question, it was only \$600, therefore the second error will be $\$750 - \$600 = \$150$ — too small; then (the errors being both made by results too small, the difference is taken), $150 - 100 : 360 - 300 :: 100 : 120$; $120 + 360 = \$480$, Ans.

7. It appears, by the tenor of the question, that 90 must be so divided into two parts that, if the larger part be multiplied by 60, and the smaller part by 80, the difference of the products shall be 3300. We therefore suppose the larger part to be 60, and the smaller part 30, and proceed according to the rule, thus: $60 \times 60 = 3600$; $30 \times 80 = 2400$. The difference between these products is $3600 - 2400 = 1200$, which is $3300 - 1200 = 2100$ less than it should be. Again, we divide 90 into the two parts 80 and 10, and multiply them by 60 and 80 as before, and find the result as follows: $80 \times 60 = 4800$; $10 \times 80 = 800$; $4800 - 800 = 4000$, which is $4000 - 3300 = 700$ more than it should be. Hence we have the statement, $700 + 2100 : 80 - 60 :: 700 : 5$; $80 - 5 = 75$, the days of labor; and $90 - 75 = 15$, days of idleness, Ans.

8. First suppose 48 the weight of the fish; as the body weighs

as much as the head and tail, its weight will be half of the whole fish; half of 48 = 24; but $24 + 15 + (15 + \frac{1}{2} \text{ of } 24) = 58$; therefore, $58 - 48 = 10$, first error.

Again, suppose the weight of the fish to be 60; but $30 + 15 + (15 + \frac{1}{3} \text{ of } 30) = 65$ for the weight of the fish by the conditions of the question; therefore $65 - 60 = 5$, second error. Then, $10 - 5 : 60 - 48 :: 10 : 25$; therefore, $25 + 48 = 72$, weight of the fish, Ans.

9. Assume 300 pounds to be growing on each acre (any other number would answer as well); then $3\frac{1}{3}$ acres will contain 1000 pounds. We now suppose the weekly increase to be 9 pounds; then $3\frac{1}{3}$ acres will, in four weeks, produce 120 pounds.

$$3\frac{1}{3} \times 300 = 1000$$

$$9 \times 3\frac{1}{3} \times 4 = 120$$

Amount of $3\frac{1}{3}$ acres in 4 weeks = 1120lbs.

Having found that 12 oxen will eat 1120 pounds of grass in 4 weeks, we wish to know how many pounds 21 oxen would eat in 9 weeks, which, by the following process, we find to be 4410 pounds. Thus,

$$\begin{array}{l} 12 : 21 \\ 4 : 9 \end{array} \left. \vphantom{\begin{array}{l} 12 : 21 \\ 4 : 9 \end{array}} \right\} :: 1120 : 4410\text{lbs.}$$

But we find the amount of the grass of 10 acres in 9 weeks to be 3810 pounds. Thus,

$$300 \times 10 = 3000$$

$$9 \times 10 \times 9 = 810$$

3810lbs.

But, by the supposition, 21 oxen in 9 weeks would eat 4410lbs.

The first error will therefore be

-600

We next suppose the weekly increase to be 18 pounds per acre; therefore the amount of the grass of $3\frac{1}{3}$ acres in 4 weeks will be 1240 pounds. Thus,

$$300 \times 3\frac{1}{3} = 1000$$

$$18 \times 3\frac{1}{3} \times 4 = 240$$

1240lbs.

And the amount of 10 acres in 9 weeks would be 4620lbs.

Thus,

$$\begin{array}{r} 10 \times 300 = 3000 \\ 10 \times 18 \times 9 = 1620 \\ \hline 4620\text{lbs.} \end{array}$$

But, by the last supposition, we find that 21 oxen in 9 weeks would eat 4882½ pounds. Thus,

$$\left. \begin{array}{l} 12 : 21 \\ 4 : 9 \end{array} \right\} :: 1240 : 4882\frac{1}{2}\text{lbs.}$$

By subtracting 4620 from 4882½ pounds, we find 4882½
the second error, to be —262½.

$$\begin{array}{r} 4620 \\ -262\frac{1}{2} \end{array}$$

Hence, the statement, $600 - 262\frac{1}{2} : 18 - 9 :: 600 : 16$;
and $16 \div 9 = 25$ pounds, weekly increase.

Having assumed 300 pounds to the acre, and found the weekly increase to be 25 pounds to the acre, which is 8½ per cent., we now proceed to find the amount of the produce of 24 acres for 18 weeks. Thus,

$$\begin{array}{r} 24 \times 300 = 7200 \\ 25 \times 18 \times 24 = 10800 \\ \hline \text{Amount of 24 acres for 18 weeks} = 18000\text{lbs.} \end{array}$$

$$\begin{array}{r} 3\frac{1}{3} \times 300 = 1000 \\ 3\frac{1}{3} \times 4 \times 25 = 333\frac{1}{3} \end{array}$$

Actual amount of 3½ acres for 4 weeks = 1333½lbs.

The question now is, If 12 oxen eat 1333½ pounds of grass in 4 weeks, how many oxen will eat 18000 pounds in 18 weeks? Then,

$$\left. \begin{array}{l} 1333\frac{1}{2}\text{lbs.} : 18000\text{lbs.} \\ 18\text{ weeks} : 4\text{ weeks} \end{array} \right\} :: 12 : 36\text{ oxen, Ans.}$$

10. By trial the required number is found to be between 26 and 27, which numbers may be assumed successively. Then, by extracting the square root of 26, trebling the root, and taking the result from 26, we get 10.703; 11 —

10.703 = .297, first error; proceeding the same way with 27, the result is 11.412; $11.412 - 11 = .412$, second error; then, by the rule, $.297 + .412 : 27 - 26 :: .297 : .42$; and $.42 + 26 = 26.42$, the first approximation. Assume now 26.42 for the number; then, $26.42 - 3$ times its square root = 10.999883269; $11 - 10.999883269 = .000116731$, first error; next assume 26.421; $26.421 - 3$ times its square root = 11.000591445; $11.000591445 - 11 = .000591445$, second error. Then, $.000116731 + .000591445 : 26.421 - 26.42 :: .000116731 : .0001648$; and $26.42 + .0001648 = 26.4201648$,
 Ans.

SCALES OF NOTATION.

(ART. 592, p. 418.)

(2.)	(3.)
2)37	3)1000000
2)18 1	3)333333 1
2)9 0	3)111111 0
2)4 1	3)37037 0
2)2 0	3)12345 2
1 0	3)4115 0
Ans. 100101.	3)1371 2
	3)457 0
	3)152 1
	3)50 2
	3)16 2
	3)5 1
	1 2

Ans. 1212210202001, in the ternary scale; and

$$\begin{array}{r}
 9 \overline{)1000000} \\
 9 \overline{)111111} \quad 1 \\
 9 \overline{)12345} \quad 6 \\
 9 \overline{)1371} \quad 6 \\
 9 \overline{)152} \quad 3 \\
 9 \overline{)16} \quad 8 \\
 1 \quad 7
 \end{array}$$

In the nonary scale, 1783661, Ans.

$$\begin{array}{r}
 (4.) \\
 12 \overline{)476897} \\
 12 \overline{)39741} \quad 5 \\
 12 \overline{)3311} \quad 9 \\
 12 \overline{)275} \quad e \\
 12 \overline{)22} \quad e \\
 1 \quad t \quad \text{Ans. } 1tee95.
 \end{array}$$

(ART. 593, p. 419.)

$$\begin{array}{r}
 (2.) \\
 234 \\
 \underline{5} \\
 13 \\
 \underline{5} \\
 69, \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (3.) \\
 21122 \\
 \underline{8} \\
 7 \\
 \underline{3} \\
 22 \\
 \underline{3} \\
 68 \\
 \underline{3} \\
 206, \text{ Ans.}
 \end{array}$$

222

KEY TO

(4.)
 100101
 $\begin{array}{r} 2 \\ 2 \\ 2 \\ 4 \\ 2 \\ 9 \\ 2 \\ 18 \\ 2 \\ \hline \end{array}$
 37, Ans.

(5.)
 13579
 $\begin{array}{r} 12 \\ 15 \\ 12 \\ 185 \\ 12 \\ 2227 \\ 12 \\ 26733 \end{array}$

11)26733
 $\begin{array}{r} 11)2430 \quad 3 \\ 11)220 \quad t \\ 11)20 \quad 0 \\ \quad 1 \quad 9 \\ \hline \end{array}$
 Ans. 190t3.

(6.)
 123454321
 $\begin{array}{r} 6 \\ 8 \\ 6 \\ 51 \\ 6 \\ 310 \\ 6 \\ 1865 \\ 6 \\ 67167 \\ 6 \\ 403004 \\ 6 \\ 2418025 \end{array}$

12)2418025
 $\begin{array}{r} 12)201502 \quad 1 \\ 12)16791 \quad t \\ 12)1399 \quad 3 \\ 12)116 \quad 7 \\ \quad 9 \quad 8 \\ \hline \end{array}$
 Ans. 9873t1.

(ART. 594, p. 420.)

<p>(1.)</p> $ \begin{array}{r} 45324502 \\ 25405534 \\ \hline 115134440, \text{ the sum, } \\ 15514524, \text{ the diff., } \end{array} \left. \vphantom{\begin{array}{r} 45324502 \\ 25405534 \end{array}} \right\} \text{Ans.} $	<p>(2.)</p> $ \begin{array}{r} 2483 \\ 589 \\ \hline 11985 \\ 18502 \\ \hline 11184 \\ 131225, \text{ Ans.} \end{array} $
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<p>(3.)</p> $ \begin{array}{r} 589)1184323(2483, \text{ Ans.} \\ \underline{e56} \\ 22t3 \\ \underline{1te0} \\ 3e32 \\ \underline{39t0} \\ 1523 \\ \underline{1523} \end{array} $	<p>(4.)</p> $ \begin{array}{r} 1112244i(2405, \text{ Ans.} \\ \underline{4} \\ 44)312 \\ \underline{304} \\ 5205)42441 \\ \underline{42441} \end{array} $
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DUODECIMALS.

(ART. 598, p. 421.)

<p>(1.)</p> $ \begin{array}{r} 121\text{ft. } 3' \ 9'' \\ 105\text{ft. } 11' \ 8'' \\ 80\text{ft. } 0' \ 6'' \\ 15\text{ft. } 10' \ 0'' \ 4''' \\ \hline \text{Ans. } 323\text{ft. } 1' \ 11'' \ 4''' \end{array} $	<p>(2.)</p> $ \begin{array}{r} 462\text{ft. } 4' \ 9'' \\ 307\text{ft. } 9' \ 1'' \\ \hline \text{Ans. } 154\text{ft. } 7' \ 8'' \end{array} $
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3. $92\text{ft. } 0' \ 6'' - 21\text{ft. } 9' \ 10'' = 70\text{ft. } 2' \ 8''$; and $70\text{ft. } 2' \ 8'' + 19\text{ft. } 10' \ 8'' \ 6''' = 90\text{ft. } 0' \ 11'' \ 6'''$, Ans.

(ART. 600, p. 422.)

2. $48\text{ft. } 6' \times 24\text{ft. } 3' = 1176\text{sq. ft. } 1' \ 6''$, Ans.

3. $20\text{ft.} + 14\text{ft. } 6' = 34\text{ft. } 6', \times 2 = 69\text{ft.} \times 10\text{ft. } 4' = 713\text{ft.};$
 $3\text{ft. } 2' \times 6\text{ft.} \times 2 = 38\text{ft.}; 4\text{ft. } 4' \times 4\text{ft.} = 17\text{ft. } 4', +$
 $38\text{ft.} = 55\text{ft. } 4'; 713\text{ft.} - 55\text{ft. } 4' = 657\text{ft. } 8'; 657\text{ft. } 8' \div 9 = 73\frac{2}{3}\text{ square yards, Ans.}$
4. $53\text{ft. } 6' \times 10\text{ft. } 3' \times 2\text{ft.} = 1096\text{ft. } 9', \text{ Ans.}$
5. $6\text{ft. } 8' + 5\text{ft. } 9' + 4\text{ft. } 6' + 3\text{ft. } 10' = 20\text{ft. } 9', \times 3\text{ft. } 5' \times 4 = 283\text{sq. ft. } 7', \text{ Ans.}$
6. $97\text{ft. } 9' \times 3\text{ft. } 6' = 342\text{ft. } 1' 6'', \div 4 = 85\frac{5}{8}\text{ cord ft., } \div 8 = 10\frac{1}{2}\frac{1}{4}\text{ cords, Ans.}$
7. $100\text{ft.} \times 6\text{ft. } 11' = 691\text{ft. } 8' \div 4 = 172\frac{1}{2}\text{ cord ft. } \div 8 = 21\frac{1}{8}\frac{3}{8}\text{ cords, Ans.}$

(ART. 601, p. 423.)

2. $834\text{sq. ft. } 3' \div 17\text{ft. } 9' = 47\text{ft., Ans.}$
3. $18\text{ft. } 9' \times 3 = 56\text{ft. } 3'; 84\text{ft. } 4' 6'' \div 56\text{ft. } 3' = 1\text{ft. } 6', \text{ Ans.}$

NOTE. — The thickness of the plank, 3', should not be regarded as 3' in working the problem, but simply as 3 times the thickness of board measure.

4. $792\text{ft. } 6' 9'' 2''' \div 12\text{ft. } 7' 8'' = 62\text{ft. } 8' 6'', \text{ Ans.}$

MISCELLANEOUS EXAMPLES.

(PAGE 424.)

1. As \$52.50 is the average of salary, $\$52.50 - \$20 = \$32.50$, average of increase; $\$32.50 \times 2 = \65 , whole increase; $\$65 + 20 = \85 , salary received last month = last term; then, (Art. 558) $85^{\frac{5}{5}20} + 1 = 14$ months, Ans.
2. $20\text{ft.} + 16\text{ft. } 6' = 36\text{ft. } 6' \times 2 = 73\text{ft.} \times 9\text{ft. } 6' = 693\text{ft. } 6'; 20\text{ft.} \times 16\text{ft. } 6' = 330\text{ft.}; 330\text{ft.} + 693\text{ft. } 6' = 1023\text{ft. } 6' \times 3 = 3070\text{ft. } 6' \div 9 = 341\text{yd. } 1\text{ft. } 6'; 341\text{yd. } 1\text{ft. } 6' - 90\text{yd.} = 251\text{yd. } 1\text{ft. } 6', \text{ Ans.}$
3. (Art. 565.) $(1\frac{1}{2})^{10} = \frac{5^{9049}}{1624}; \frac{5^{9049}}{1624} \times 1024 = \59049 , the share of the eldest, Ans.
4. (Art. 583.) $\frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}$
 $\{ = \$1847.56, \text{ Ans.}$

5. $340\text{ft.} \div 3\text{ft. } 9' = 90\text{ft. } 8' = 30\text{yd. } 0\text{ft. } 8', \text{ Ans.}$
6. By Table (p. 406) the amount of \$1 for 6 years is \$6.97-5319; therefore, $\$700 \times 6.975319 = \$4882.72, \text{ Ans.}$
7. (1.) We first suppose the time to be $60\frac{1}{2}$ seconds after 12 o'clock. The hour-hand then will have passed $\frac{60.5}{43200}$ of the distance from 12 to 12 again, and the second-hand will have passed once round and $\frac{1}{120}$ of another time. The difference between these two numbers is $\frac{1}{120} - \frac{60.5}{43200} = \frac{3595}{86400}$. The minute-hand will have passed $\frac{60.5}{3600}$ of the distance from 12, and the difference between this number and $\frac{1}{120}$ is $\frac{60.5}{3600} - \frac{1}{120} = \frac{722}{3600}$. We now find the difference between this last number and $\frac{3595}{86400}$; $\frac{722}{3600} - \frac{3595}{86400} = \frac{3425}{86400}$, first error, too small.

We next suppose the time to be 61 seconds after 12 o'clock. The hour-hand then will have passed $\frac{61}{43200}$ of the distance from 12 to 12, and the second-hand will have passed once round and $\frac{1}{80}$ of another time. The difference of these two numbers is $\frac{1}{80} - \frac{61}{43200} = \frac{529}{86400}$. We now find the difference between the second-hand and minute-hand. The minute-hand has moved in 61 seconds $\frac{61}{3600}$ of the distance from 12 to 12; the difference between $\frac{61}{3600} - \frac{1}{80} = \frac{127}{7200}$. We next find the difference between this last number and $\frac{529}{86400}$; $\frac{127}{7200} - \frac{529}{86400} = \frac{1294}{86400} = \frac{1294}{86400} +$, second error. As the denominators of our errors are the same number, we may reject them in the operation; for when fractions have a common denominator their values are as their numerators. $133 + 1294 : 61 - 60.5 :: 133 : \frac{665}{14400}$; and $\frac{665}{14400} + 60.5 = 60\frac{89}{1440}$ seconds, Ans.

- (2.) Suppose the time to be $61\frac{1}{2}$ seconds after 12 o'clock. Then the hour-hand will have moved $\frac{61.5}{43200}$ of the distance from 12 o'clock to 12 again, and the minute-hand will have moved $\frac{61.5}{3600}$ of this distance, and the second-hand will have moved once round and $\frac{1}{80}$ again. The difference between $\frac{61.5}{3600}$ and $\frac{1}{80}$ is $\frac{775}{7200}$. The difference between $\frac{775}{7200}$ and $\frac{61.5}{43200}$ is $\frac{3425}{86400}$. We then find the dif-

ference between $\frac{576}{43200}$ and $\frac{3420}{43200} = \frac{334.5}{43200}$ —, the first error.

We then suppose the time to be 62 seconds after 12 o'clock. The hour-hand will then have moved from 12 o'clock $\frac{62}{43200}$ of the distance to 12 again, and the minute-hand have moved $\frac{362}{3600}$ of the distance, and the second-hand will have gone once round and $\frac{2}{90}$ of the distance again. The difference between $\frac{362}{3600}$ and $\frac{62}{43200}$ is $\frac{682}{43200}$. The difference between $\frac{2}{90}$ and $\frac{362}{3600}$ is $\frac{82}{43200}$. The difference between $\frac{682}{43200}$ and $\frac{82}{43200}$ is $\frac{1}{43200} +$, second error. $334.5 + 14 : 62 - 61.5 :: 14 : \frac{14}{637}$; and $62 - \frac{14}{637} = 61\frac{687}{637}$ seconds, Ans.

- (3.) We will first suppose the time to be 59 seconds after 12 o'clock; the hour-hand will then have advanced $\frac{59}{43200}$ of the distance from 12 o'clock to 12 again, and the second-hand will be within $\frac{1}{90}$ of the distance to 12; therefore, the whole distance between the hour-hand and the second-hand will be $\frac{1}{90} + \frac{59}{43200} = \frac{779}{43200}$; and the distance between the hour-hand and minute-hand will be $\frac{59}{3600} - \frac{59}{43200} = \frac{649}{43200}$, and the difference between $\frac{779}{43200}$ and $\frac{649}{43200} = \frac{139}{43200}$ —, the first error.

Again, we will suppose the time to be 58 minutes after 12 o'clock. The distance then between the second-hand and hour-hand will be $\frac{2}{90} + \frac{58}{43200} = \frac{1498}{43200}$. The distance between the hour-hand and minute-hand will be $\frac{58}{3600} - \frac{58}{43200} = \frac{638}{43200}$, and $\frac{1498}{43200} - \frac{638}{43200} = \frac{860}{43200}$ — will be the second error. $860 - 130 : 59 - 58 :: 130 : \frac{13}{43}$; and $59 + \frac{13}{43} = 59\frac{13}{43}$ seconds, Ans.

MENSURATION.

(ART. 613, p. 426.)

1. $18 \times \frac{1}{2} = 108$, Ans.
2. $15.6 + 9.2 + 10.4 = 35.2$ feet; $35.2 \div 2 = 17.6$ feet;
 $17.6 - 15.6 = 2.00$; $17.6 - 9.2 = 8.4$; $17.6 - 10.4$

$$= 7.2; 17.6 \times 2 \times 8.4 \times 7.2 = 2128.896; \sqrt{2128.896} = 46.139+ \text{ feet, Ans.}$$

3. $40 \times \frac{15}{2} = 300, \text{ Ans.}$

4. $336 \div 3 = 112$, each side of the field; $112 \div 2 = 56$, half the base; $112^2 - 56^2 = 9408$; $\sqrt{9408} = 97$, the perpendicular height, very nearly; $112 \times \frac{97}{2} = 5432$ rods = 33 acres 152 rods, Ans.

(ART. 621, p. 427.)

1. $15 \times 2 = 30, \text{ Ans.}$

2. $128 \times 48 = 6144, \text{ Ans.}$

3. $12 \times 8 = 96, \text{ Ans.}$

4. $358 \times 194 = 69452, \text{ Ans.}$

5. $693 \times 693 = 480249\text{ft.}$; $480249 \div 272\frac{1}{2} = 1764$ rods; $1764 \div 160 = 11$ acres 4 rods, Ans.

6. $40 \times 40 = 1600$; $20 \times 20 \times 2 = 800$; $1600 - 800 = 800, \text{ Ans.}$

7. $\sqrt{3600} = 60$ yards; $3600 \div 2 = 1800$ yards; $\sqrt{1800} = 42.427+$ yards; $60 \text{ yards} - 42.427 \text{ yards} = 17.573$ yards; $17.573 \div 2 = 8.78+$ yards, Ans.

(ART. 622, p. 428.)

1. $15\frac{1}{2} \times 20 = 1080\text{sq. ft., Ans.}$

2. $186\frac{1}{2} \times 986 = 620687$ links; $620687 \div 625$ (the links in a square rod) = 993 rods 3 yards = 6 acres 33 rods 3 yards, Ans.

(ART. 623, p. 428.)

1. Drawing the diagonal divides the garden into two triangles, with sides 328, 598, 298, and 598, 456, 572.

$$328 \quad 612 - 328 = 284, \text{ rem.}$$

$$598 \quad 612 - 598 = 14, \text{ rem.}$$

$$298 \quad 612 - 298 = 314, \text{ rem.}$$

$$\begin{array}{r} 2)1224 \\ \hline \end{array}$$

$$612, \text{ half sum.}$$

$$598 \quad 813 - 598 = 215, \text{ rem.}$$

$$456 \quad 813 - 456 = 357, \text{ rem.}$$

$$572 \quad 813 - 572 = 241, \text{ rem.}$$

$$\hline 2)1626$$

813, half sum.

$$612 \times 284 \times 14 \times 314 = 764059968; \sqrt{764059968} = 27641.63; 813 \times 215 \times 357 \times 241 = 15038837415; \sqrt{15038837415} = 122632.97; 27641.63 + 122632.97 = 150274.6 \text{ sq. ft.} = 3 \text{ acres } 1 \text{ rood } 31 \text{ rods } 29 \text{ yards } 3.85 \text{ feet, Ans.}$$

$$2. \frac{2}{3} \times 17.56 = 77.4396 \text{ square chains, area of one triangle;}$$

$$\frac{7}{12} \times 17.56 = 67.8694 \text{ square chains, area of the other triangle;}$$

$$77.4396 + 67.8694 = 145.309 \text{ square chains, the area of the whole field; } 145.309 \div 10 \text{ (10 square chains make 1 acre)} = 14.5309 \text{ acres} = 14 \text{ acres } 2 \text{ rods } 5 \text{ rods, nearly, Ans.}$$

(ART. 626, p. 429.)

$$1. 172.05 \times 250 \times 5 = 107531.25 \text{ square feet, Ans.}$$

$$2. 308.305 \times 356 \times 6 = 329269.74 \text{ yd., Ans.}$$

$$3. 60^2 \times 4.828427 = 17382.3372 \text{ yards} = 3 \text{ acres } 2 \text{ rods } 14 \text{ rods } 19 \text{ yards, nearly, Ans.}$$

$$4. 243^2 \times 7.694209 = 454335.34724 = 10 \text{ acres } 1 \text{ rood } 28 \text{ rods } 24 \text{ yards } 6.34724 \text{ feet, Ans.}$$

(ART. 633, p. 430.)

$$1. 144 \times 3.141592 = 452.389248, \text{ Ans.}$$

$$2. 7964 \times 3.141592 = 25019.638688, \text{ Ans.}$$

$$3. 512 \times 2 \times 3.141592 = 3216.9984 \text{ feet} = 4 \text{ furlongs } 34 \text{ rods } 5 \text{ yards } 1 \text{ foot, Ans.}$$

(ART. 634, p. 430.)

$$1. 1043 \times .318309 = 331.997, \text{ Ans.}$$

2. $25000 \times .318309 = 7957.74$, Ans.

3. $50 \times .318309 = 15.91549$, Ans.

(ART. 635, p. 430.)

1. $761^2 \times .785398 = 454840.475158$, Ans.

2. 1 mile = 320 rods; $320 \times 3 = 960$; $960^2 \times .785398 = 723822.7968$ square rods = $4523.89 +$ acres, Ans.

3. $1284^2 \times .079577 = 131195.098512$ square yards = 27 acres 17 rods 0.848512 square yards, Ans.

4. $1\frac{1}{2}^2 \times 5\frac{1}{2}^2 = 22477$ inches = 17 yards 3 feet 13 inches, Ans.

(ART. 636, p. 430.)

1. $79 \times 4\frac{1}{2} = 1856.5$, Ans.

2. The area of a circle of which the radius is $25 = 50^2 \times .785398$; then, $360^\circ : 26^\circ :: 50^2 \times .785398 : 141.8$ sq. ft., Ans.

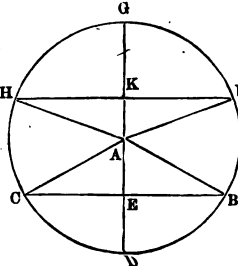
(ART. 637, p. 431.)

1. $360^\circ : 49.25^\circ :: 24^2 \times .785398 : 61.889$, area of sector A B C E; $\sqrt{12^2 - 5^2} = 10.908 =$ perpendicular D E; $10.908 \times 5 = 54.54$, area of triangle A C E; $61.889 - 54.54 = 7.35$, nearly, area of the segment A B C A, Ans.

2. By the second rule, $\frac{15 \times 24 \times 2}{3} + \frac{15^3}{24 \times 2} = 310.3125$ square rods = 1 acre 3 roods 30 rods 9.4 yards, Ans.

(ART. 638, p. 431.)

1. The first step in the working of this problem is to find the height of the segments lying on each side of the zone; the heights are elements necessary in calculating their areas. Let H F B C be the zone; the radii A C, A B, A F, A H, are each 12 feet;



$$A H = 12; 12^2 = 144;$$

$$H K = \frac{20.8}{2} = 10.4; 10.4^2 = 108.16.$$

$$144 - 108.16 = 35.84; \sqrt{35.84} = 5.9 = A K; 12 - 5.9 = 6.1 = G K, \text{ the height of the segment whose cord is } 20.8; \text{ then, } \frac{20.8 \times 6.1 \times 2}{3} + \frac{6.1^3}{20.8 \times 2} = 90.03, \text{ area of the segment } H G F H;$$

$$A C = 12; 12^2 = 144;$$

$$C E = \frac{23.25}{2} = 11.625; 11.625^2 = 135.14; 144 - 135.14 = 8.86;$$

$$\sqrt{8.86} = 2.97 = A E; 12 - 2.97 = 9.03 = D E, \text{ the height of the segment whose cord is } 23.25; \text{ then } \frac{23.25 \times 9.03 \times 2}{3} + \frac{9.03^3}{23.25 \times 2} = 155.79, \text{ area of the segment } C D B C;$$

$$24^2 \times .7854 = 452.39, \text{ area of the whole circle;}$$

$$155.79 + 90.03 = 245.82, \text{ area of the two segments;}$$

$$\frac{245.82}{206.57}, \text{ area of the zone, Ans.}$$

2. Let 10, the radius, be the hypotenuse of a right-angled triangle, and 8, half the cord, be the base; then, $10^2 - 8^2 = 36$; $\sqrt{36} = 6$, the perpendicular from the centre to the cord; $10 - 6 = 4$, height of each segment; then, by the rule (Art. 637), $\frac{16 \times 4 \times 2}{3} \times \frac{4^3}{16 \times 2} = 44.66$, area of each segment; $44.66 \times 2 = 89.32$, area of both segments; $20^2 \times .785 =$

$$314, \text{ area of the circle;}$$

$$\frac{89.32}{224.68}, \text{ area of the segments.}$$

$$224.68, \text{ or } 224.7, \text{ nearly,}$$

area of zone, Ans.

(ART. 639, p. 431.)

$$\begin{aligned}
 1. \text{ By rule, Art. 637, } & \frac{72 \times 30 \times 2}{3} + \frac{30^3}{72 \times 2} = 1627.5 \text{ sq. ft.} \\
 & \frac{72 \times 20 \times 2}{3} + \frac{20^3}{72 \times 2} = 1015.5 \text{ sq. ft.} \\
 & \text{Ans. 612 sq. ft.}
 \end{aligned}$$

(ART. 640, p. 431.)

1. $(20 + 10) \times (20 - 10) \times .7854 = 235.62 \text{ sq. yd., Ans.}$
2. $(157 + 128) \times (157 - 128) \times .7854 = 6491.331 \text{ sq. yd.}$
 $= 1 \text{ acre } 1 \text{ rood } 14 \text{ rods } 17 \text{ yards } 7.4 \text{ feet, Ans.}$

(ART. 641, p. 431.)

1. $50 \times .886227 = 44.31135 \text{ rods, Ans.}$
2. $360 \times .282094 = 101.55 \text{ rods, Ans.}$
3. $10000 \times .282094 = 2820.94 \text{ rods, Ans.}$

(ART. 642, p. 432.)

1. $44.31135 \times 1.12838 = 50.000041118 \text{ rods, Ans.}$

(ART. 643, p. 432.)

1. $30 \times .866025 = 25.98 \text{ inches, Ans.}$
2. $5000 \times .275664 = 1378.320 \text{ feet, Ans.}$
3. $80 \times .275664 = 22.05 \text{ inches, Ans.}$

(ART. 644, p. 432.)

1. $30 \times .707106 = 21.21 \text{ inches square, Ans.}$
2. $80 \times .707106 = 56.56848 \text{ feet, Ans.}$
3. $5000 \times .225079 = 1125.395 \text{ rods, Ans.}$
4. $100 \times .225079 = 22.5 \text{ inches square, Ans.}$
5. $18 \div .225079 = 79.97 \text{ inches, Ans.}$
6. $20 \text{ rods} = 330 \text{ feet; } 330 \div .225079 = 1466.15 \text{ feet, Ans.}$

(ART. 645, p. 432.)

1. $86.2 \div 2.155 = 40$ inches, Ans.

(ART. 647, p. 433.)

1. $24 \times 18 \times .785398 = 339.2919$, Ans.
 2. $33\frac{5}{8} \times 20\frac{1}{4} \times .785398 = 531.469$ sq. feet = 59sq. yd.
 67sq. inches, Ans.

SOLIDS.

(ART. 650, p. 433.)

1. $23 + 34 + 19 = 76$ inches = $6\frac{1}{2}$ feet, the perimeter; $6\frac{1}{2} \times 13 = 82.33$ feet, the area of the sides; by Art. 613, $38 \times 15 \times 4 \times 19 = 43320$; $\sqrt{43320} = 208.1$ inches, the area of one of the ends; $208.1 \times 2 = 416.2$; $416.2 \div 144 = 2.89$, area of both ends in feet;
 82.33, area of sides;
 2.89, area of the ends;
 85.22 feet, surface of the prism, Ans.
2. $33 \times 5 = 165$ inches = $13\frac{3}{4}$ feet, the perimeter; $13\frac{3}{4} \times 14 = 192.5$ feet, area of the sides; and, $33\frac{1}{2} \times 1.720477 \times 2 = 3747.19$ sq. inches = 26.02sq. feet, the area of the two ends; then, $192.5 + 26.02 = 218.52$ sq. feet, surface of the prism, Ans.
3. 57in. = $4\frac{3}{4}$ feet; $4\frac{3}{4} \times 13 = 61.75$ sq. feet, the convex surface of cylinder; $(4\frac{3}{4})^2 \times .079577 \times 2 = 3.59$ sq. ft., area of the two ends; then, $61.75 + 3.59 = 65.34$ sq. ft., surface of cylinder, Ans.
4. 1 acre = 43560sq. feet; $1\frac{1}{2} \times 3.14159 = 5.49778$ feet, the circumference of the cylinder; $5.49778 \times 5\frac{1}{4} = 28.86334$, convex surface of the cylinder; $43560 \div 28.86334 = 1509.18$ times, Ans.
5. $16 \times 10 \times 4 = 640$ sq. feet; $640 \div 9 = 71\frac{1}{3}$ sq. yards, Ans.

(ART. 651, p. 434.)

1. $2.5^2 - 1.25^2 = 4.6875$; $\sqrt{4.6875} = 2.16506 =$ the perpendicular of the triangle marked by the end of the prism; and 2.16506×1.25 (half the base) $= 2.706 =$ area of the end of the prism; $2.706 \times 12 = 32.47$ cubic feet, Ans.
2. (ART. 613.) $6 \times 1 \times 2 \times 3 = 36$; $\sqrt{36} = 6 =$ area of the end; then, $6 \times 10 = 60$ cu. feet, Ans.
3. $3\frac{1}{2} \times 2\frac{2}{3} \times 2\frac{1}{2} = 21\frac{1}{2}$ cu. feet, Ans.
4. $6^2 \times .079577 \times 9 = 25.78$ cu. feet, Ans.

(ART. 656, p. 435.)

1. $2\frac{2}{3} \times 2\frac{2}{3} = 7\frac{1}{9}$ feet, area of the base; $2\frac{2}{3} \times 4 = 10\frac{2}{3}$ feet, the perimeter of the base; $7\frac{1}{9} + 10\frac{2}{3} \times \frac{3\frac{3}{4}}{2} = 27\frac{1}{2}$ sq. feet area, Ans.
2. $9 \times 2\frac{1}{2} = 90$ feet, Ans.

(ART. 657, p. 435.)

1. $(2\frac{1}{2})^2 \times .785398 \times 12\frac{1}{2} = 20.45$, Ans.
2. By Art. 613, $9 \times 4 \times 3 \times 2 = 216$; $\sqrt{216} = 14.69693$, the area of the base; $14.69693 \times 14\frac{1}{2} = 71.035$ cu. feet, Ans.

(ART. 658, p. 435.)

(1.)

- $$\begin{array}{rcl} 3 \times 5 = 15, & \text{perimeter of smaller end;} \\ 5 \times 5 = 25, & \text{perimeter of larger end;} \\ \hline 40 \times 5 = 200 & = \text{surface of the sides;} \\ 3^2 \times 1.720477 & = 15.4842 = \text{surface of smaller end;} \\ 5^2 \times 1.720477 & = 43.0119 = \text{surface of larger end;} \\ \hline & 258.4961 \text{ inches, surface of the frus-} \\ & \text{tum, Ans.} \end{array}$$

(2.)

$$\begin{aligned}
 3\frac{7}{12} \times 3.14159 &= 11.2573, \text{ circumference of larger end;} \\
 1\frac{1}{2} \times 3.14159 &= 6.02138, \text{ circumference of smaller end;} \\
 \frac{17.27868}{4\frac{1}{2}} \times 4\frac{1}{2} &= 77.75406 = \text{convex} \\
 &\quad [\text{surface of the frustum;} \\
 &\quad 77.75406, \text{ convex surface;} \\
 (\frac{4}{3})^2 \times .785398 &= 10.08472, \text{ surface of larger end;} \\
 (\frac{2}{3})^2 \times .785398 &= 2.885246, \text{ surface of smaller end;} \\
 &90.724026\text{sq. feet, Ans.}
 \end{aligned}$$

(ART. 659, p. 435.)

1. $27 \times 27 = 729$, area of larger end; $16 \times 16 = 256$, area of smaller end;
 $729 \times 256 = 186624$; $\sqrt{186624} = 432$; $432 + 256 +$
 $729 = 1417$; $1417 \times \frac{18\frac{2}{3}}{3} = 8816.888$; $8816.888 \div$
 $144 = 61.228$ cubic feet, Ans.
2. $2^2 \times .7854 = 3.1416$, area of larger end;
 $1^2 \times .7854 = .7854$, area of smaller end;
 $3.1416 \times .7854 = 2.46741264$; $\sqrt{2.46741264} = 1.5708$;
 1.5708 ,
 3.1416 , area of larger end;
 $.7854$, area of smaller end;
 $5.4978 \times \frac{4}{3} = 73.304$ cubic feet, Ans.

(ART. 666, p. 436.)

1. $24 \times (24 \times 3.14159) = 1809.55\text{sq. in., Ans.}$
2. $7957\frac{3}{4} \times 25000 = 198943750$, Ans.

(ART. 667, p. 436.)

1. $12^3 \times .523598 = 904.78$, Ans.
2. $25000 \times .31831 = 7957.75$, diameter; $7957.75^3 \times .5236$
 $= 263858149120.06886875$, Ans.

(ART. 668, p. 437.)

1. $12\frac{1}{2} \times 3.141592 \times 2 = 78.54$ sq. feet, Ans.
2. $797.0 \times 3.141592 \times 2143.623553 = 53673093.12$ sq. miles,
Ans.

(ART. 669, p. 437.)

1. $(3^2 + (4\frac{1}{2})^2 \times 3) \times 3 \times .5236 = 109.56$ cu. feet, Ans.
2. $(9^2 + 10^2 \times 3) \times 9 \times .5236 = 1795.42$ cu. feet. Ans.

(ART. 670, p. 437.)

1. $(21 + 4) \times 4 \times 9.8696 = 986.96$ sq. in., Ans.

(ART. 671, p. 437.)

1. $(25 + 5) \times 5^2 \times 2.4674 = 1850.55$ cu. in., Ans.

(ART. 672, p. 437.)

1. $20^2 \times 32 \times .523598 = 6702.05$ cu. in., Ans.
2. $38^2 \times 48 \times .523598 = 36291.62$ cu. feet, Ans.

(ART. 674, p. 438.)

1. $24 \times \frac{2}{3} = 16$ feet, Ans.
2. $30 \times 1\frac{1}{3} = 40$ feet, Ans.
3. $(26 + 14) \div 2 = 20$; $30 \times 20 \div 12 = 50$ feet, Ans.

(ART. 675, p. 438.)

1. $3 \times 5 \times 15 \times 3 = 675$; $675 \div 12 = 56\frac{1}{4}$ feet, Ans.
2. $2 \times 6 \times 10 \times 20 \div 12 = 200$ feet, Ans.
3. $10 \times 17 \times 20 \div 12 = 283\frac{1}{3}$ feet, Ans.

(ART. 676, p. 438.)

1. $30 \times 10^2 \div 144 = 20\frac{5}{8}$ feet, Ans.
2. $50 \times 14^2 \div 144 = 68\frac{1}{8}$ feet, Ans.
3. $90 \times 30^2 \div 144 = 562\frac{1}{2}$ feet, Ans.

(ART. 679, p. 439.)

1. $27 + (8 \times .70) = 32.6$; $32.6^2 \times 45 \times .0034 = 162.602$
wine gallons, Ans.
2. $30 + (8 \times .65) = 35.2$; $35.2^2 \times 42 \times .0034 = 176.9349$
wine gallons, Ans.
3. Consider the tub as the frustum of a cone, and apply the rule in Art. 659; by this rule get the contents of the tub in inches, and change the inches to their equivalent in gallons;
 $30^2 \times .7854 = 706.86$, area of smaller end;
 $40^2 \times .7854 = 1256.64$, area of larger end;
 $1256.64 \times 706.86 = 888268.5504$; $\sqrt{888268.5504} = 942.48$;
 $942.48 + 706.86 + 1256.64 = 2905.98$; $2905.98 \times \frac{1}{2} = 4843$ cubic inches; $4843 \div 231 = 209.66\frac{2}{3}$ liquid gallons, Ans.
4. $\frac{10 \times 5 \times 4 \times 1728}{231} = 1496\frac{8}{77}$ wine gallons, Ans.
5. $\frac{12 \times 6 \times 2 \times 1728}{282} = 882\frac{1}{2}$ beer gallons, Ans.
6. $\frac{15 \times 5 \times 7 \times 1728}{2150.42} = 421.8$ bushels, Ans.

TONNAGE.

(ART. 681, p. 440.)

1. $191\frac{6}{12} = 22\frac{2}{3}$; $36\frac{5}{12} = 4\frac{3}{4}$; $22\frac{2}{3} - \frac{2}{3}$ of $4\frac{3}{4} = 10\frac{1}{6}$;
 $10\frac{1}{6} \times 4\frac{3}{4} \times 4\frac{3}{4} \times \frac{1}{5} = 1184\frac{427}{3200}$ tons, Ans.
2. $184\frac{6}{12} - \frac{2}{3}$ of $38\frac{1}{2} = 26\frac{5}{6}$; $26\frac{5}{6} \times 38\frac{1}{2} \times 19\frac{1}{2} \times \frac{1}{5} = 12842\frac{11947}{1200}$ tons, Ans.
3. $195\frac{2}{12} - \frac{2}{3}$ of $39\frac{1}{12} = 171\frac{1}{6}$; $171\frac{1}{6} \times 39\frac{1}{12} \times 19\frac{1}{12} \times \frac{1}{5} = 1397\frac{182}{1225}$ tons, Ans.
4. $78 - \frac{2}{3}$ of 21 = 65.4; $65.4 \times 21 \times 9 \times \frac{1}{5} = 130\frac{52}{75}$ tons, Ans.
5. $141 \times 30 \times 15 \times \frac{1}{5} = 667\frac{1}{5}$ tons, Ans.
6. $479 - \frac{2}{3}$ of 80 = 431; $431 \times 80 \times 40 \times \frac{1}{5} = 14517\frac{1}{5}$ tons, Ans.

MISCELLANEOUS QUESTIONS.

(PAGE 441.)

1. $\frac{7}{8} \times \frac{5}{8} = \frac{35}{64}$; $1 - \frac{35}{64} = \frac{29}{64}$, Ans.

2. As there is a son and a daughter, the son will have $\frac{4}{7}$ of the estate, the wife $\frac{2}{7}$, and the daughter $\frac{1}{7}$. If there had been only a daughter, her share would have been $\frac{2}{3}$; consequently she loses $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$. Hence

$$\frac{8}{21} : \frac{2}{7} :: \$2400 : \$2100, \text{ Ans.}$$

3. From the conditions of the question, it will readily be seen that it was a little more than half-past 5. At $5\frac{1}{2}$ o'clock the minute-hand was at 6, and the hour-hand half-way between 5 and 6, or $2\frac{1}{2}$ spaces from 6. The minute-hand moves twelve times as fast as the hour-hand; hence, while the minute-hand was moving from 6 to the required position, the hour-hand moved $\frac{1}{12}$ as far, and was then as far from the 6 point as the minute-hand was beyond. Therefore, the sum of the spaces passed over by the hour and minute hands = $2\frac{1}{2}$ minute spaces. Hence $2\frac{1}{2} \text{ m.} = \frac{1}{2}$ of the required time beyond half-past 5, and $\frac{1}{2} = 2 \text{ m. } 18\frac{2}{3} \text{ s.}$
 $5 \text{ h. } 30 \text{ m.} + 2 \text{ m. } 18\frac{2}{3} \text{ sec.} = 5 \text{ h. } 32 \text{ m. } 18\frac{2}{3} \text{ sec., Ans.}$

(4.)

$$6) 27 \text{ deg. } 55 \text{ m. } 7 \text{ fur. } 35 \text{ rd. } 4 \text{ ft. } 6 \text{ in. } (16 \text{ deg.}$$

$$\begin{array}{r} 96 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \hline \end{array}$$

$$\begin{array}{r} 69\frac{1}{8} \\ \hline \end{array}$$

$$6) 124\frac{1}{8} (20 \text{ m.}$$

$$\begin{array}{r} 120 \\ \hline \end{array}$$

$$\begin{array}{r} 4\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$6) 40\frac{1}{3} (6 \text{ fur.}$$

$$\begin{array}{r} 36 \\ \hline \end{array}$$

$$\begin{array}{r} 4\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ \hline \end{array}$$

$$\begin{array}{r} 208\frac{1}{3} \\ \hline \end{array}$$

$$6) 208\frac{1}{3} (34 \text{ rd.}$$

$$\begin{array}{r} 204 \\ \hline \end{array}$$

$$\begin{array}{r} 4\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 16\frac{2}{3} \\ \hline \end{array}$$

$$6) 75\frac{1}{2} (12 \text{ ft.}$$

$$\begin{array}{r} 72 \\ \hline \end{array}$$

$$\begin{array}{r} 3\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$6) 48 (8 \text{ in.}$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

Ans. 16deg. 20m. 6fur. 34rd. 12ft. 8in.

5. $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$; $\frac{1}{2} = \frac{1}{2}$; $\frac{1}{4} = \frac{1}{4}$;
 $\frac{1}{2} : \frac{1}{4} :: \$100,000 : \$57,142\frac{2}{3}$, A's part, } Ans.
 $\frac{1}{2} : \frac{3}{4} :: \$100,000 : \$42,857\frac{1}{4}$, B's part, }
6. If the first man's share be subtracted from the whole, there will remain $\frac{1}{8} - \frac{1}{8} = \frac{1}{8}$; and $\frac{1}{8}$ of $\frac{1}{8} = \frac{1}{64}$ = the second son's share. And $\frac{1}{8} - \frac{1}{64} = \frac{7}{64}$ = difference of their legacies. $\frac{1}{8} = \frac{8}{64}$; $\frac{8}{64} + \frac{7}{64} = \frac{15}{64}$ = legacy of both sons. Hence $\frac{15}{64} - \frac{15}{64} = \frac{1}{64}$ = wife's legacy. Therefore
 $\frac{15}{64} : \frac{1}{64} :: 257\text{£. } 3\text{s. } 4\text{d.} : 635\text{£. } 0\text{s. } 10\frac{3}{4}\text{d.}$, Ans.
7. $63 \times 12 \times 12 \times 1000 \times 8 = 27216000$; $27216000 \div 16 = 1701000\text{lb.}$; $1701000 \div 2000 = 850\frac{1}{2}$ tons, Ans.
8. 4ft. = 48in.; 6in. $\times 2 = 12\text{in.}$; $48 - 12 = 36\text{in.}$; $36 \div 2 = 18\text{in.}$; $18 + 12 = 30\text{in.}$; 48in. : 30in. : : 200lb. : 175lb., Ans.
 $48 - 30 = 18\text{in.}$; 48in. : 18in. : : 200lb. : 75lb., Ans.
9. 25ft. 4in. = 304in.; 4ft. 5in. = 53in.; 3ft. 5in. = 41in.;
 $53 \times 41 = 2173$; $53 - 41 = 12\text{in.}$; $12 \times 12 = 144$;
 $144 \div 3 = 48$; $2173 + 48 = 2221$; $2221 \times .785398 = 1744.368958$;
 $1744.368958 \times 304 \times 8 = 4242305.305856$; $4242305.305856 \div 1728 = 2455.037792$ = cubic feet in the pillars.
 $2455.037792 \times 3000 = 7365113.376$ = weight in ounces; $7365113.376 \div 16 = 460319.586\text{lbs.}$;
 $460319.586 \div 2000 = 230.15\frac{1}{2}$ tons, Ans.
10. If $\frac{3}{4}$ of a certain sum be taken, and \$410 be left, it is evident that \$410 is $\frac{1}{4}$ of that sum, which is $410 \times 7 \div 4 = \$717\frac{1}{2}$. Now, if \$717 $\frac{1}{2}$ remain of a certain quantity after $\frac{1}{4}$ be subtracted, it is certain that the number from which it is taken is $\frac{3}{4}$ of \$717 $\frac{1}{2}$ = \$956.66 $\frac{2}{3}$, Ans.
11. $\frac{1}{4} : \$15.60 :: \$100 : \$6240$ = sum remitted; \$96 : \$100 : : \$6240 : \$6500 = value of goods sold; \$6500 - \$6240 = \$260 = commission, Ans.
12. \$107.50 : \$100 : : \$9675 : \$9000; $\frac{9}{10} \times \$9000 = 2025\text{£. sterling}$ = the bill; \$100 - \$0.25 = \$99.75;

\$100 : \$99.75 :: \$9675 : \$9650.81 $\frac{1}{4}$; \$102 : \$100 ::
\$9650.81 $\frac{1}{4}$: \$9461.58 $\frac{3}{4}$ for investment, Ans.

13. $15 \times 30 = 450$; $15 \times 15 = 225$; $225 \div 3 = 75$; $450 + 75 = 525$; $525 \times 220 = 115500 =$ contents of the whole monument, and from this we deduct the contents of the cylinder. $15 \times 11 = 165$; $4 \times 4 = 16$; $16 \div 3 = 5\frac{1}{3}$; $165 + 5\frac{1}{3} = 170\frac{1}{3}$; $170\frac{1}{3} \times .785398 = 133.779459\frac{1}{3}$; $133.779459\frac{1}{3} \times 220 = 29431.481053\frac{1}{3} =$ contents of the cylinder; $115500 - 29431.481953\frac{1}{3} = 86068.51894 +$ cubic feet of the monument, Ans.

(14.)

$$\begin{aligned} \frac{1}{12} \times 1 + \frac{1}{12} \times \frac{1}{2} + \frac{1}{12} \times \frac{1}{3} &= \frac{22}{36}, \text{ A's product;} \\ \frac{1}{12} \times \frac{1}{2} + \frac{1}{12} \times \frac{1}{3} &= \frac{13}{36}, \text{ M's product;} \\ \frac{1}{12} \times \frac{1}{3} &= \frac{4}{36}, \text{ P's product;} \\ &\frac{39}{36}, \text{ sum of the products.} \end{aligned}$$

$$\begin{aligned} \frac{39}{36} : \frac{22}{36} :: \$300 : \$183.33\frac{1}{3} &= \text{A pays,} \\ \frac{39}{36} : \frac{13}{36} :: \$300 : \$83.33\frac{1}{3} &= \text{M pays,} \\ \frac{39}{36} : \frac{4}{36} :: \$300 : \$33.33\frac{1}{3} &= \text{P pays,} \end{aligned} \left. \vphantom{\begin{aligned} \frac{39}{36} : \frac{22}{36} :: \$300 : \$183.33\frac{1}{3} \\ \frac{39}{36} : \frac{13}{36} :: \$300 : \$83.33\frac{1}{3} \\ \frac{39}{36} : \frac{4}{36} :: \$300 : \$33.33\frac{1}{3} \end{aligned}} \right\} \text{Ans.}$$

(15.)

$$\begin{aligned} \text{A } 20 \times \$132 &= \$2640 \\ \text{B } 25 \times \$120 &= \$3000 & 85 \times \$6 &= \$510. \\ \text{C } 40 \times \$100 &= \$4000 \\ \hline 85 && \$9640 \end{aligned}$$

$$\begin{aligned} \$9640 : \$2640 :: \$510 : \$139\frac{1}{2}\frac{1}{4}, &\text{ A receives,} \\ \$9640 : \$3000 :: \$510 : \$158\frac{1}{4}\frac{1}{2}, &\text{ B receives,} \\ \$9640 : \$4000 :: \$510 : \$211\frac{1}{2}\frac{1}{4}, &\text{ C receives,} \end{aligned} \left. \vphantom{\begin{aligned} \$9640 : \$2640 :: \$510 : \$139\frac{1}{2}\frac{1}{4} \\ \$9640 : \$3000 :: \$510 : \$158\frac{1}{4}\frac{1}{2} \\ \$9640 : \$4000 :: \$510 : \$211\frac{1}{2}\frac{1}{4} \end{aligned}} \right\} \text{Ans.}$$

(16.)

$$\begin{aligned} \$100 \times \$5.00 &= \$500 \text{ given for the flour;} \\ \$500 \times \$0.20 &= \$100 \text{ gained on the flour;} \\ &\$600 \end{aligned}$$

$$\begin{aligned} \$100 \times \$0.03,0\frac{1}{2} &= \$3.05 \text{ bank interest of } \$100 \text{ for 6} \\ &\text{months;} \$100 - \$3.05 = \$96.95 : \$100 :: \$600 : \\ &\$618\frac{1}{3}\frac{2}{3}; \$618\frac{1}{3}\frac{2}{3} \div 100 = \$6.18\frac{1}{3}\frac{2}{3}, \text{ Ans.} \end{aligned}$$

17. We first find the number of square feet in an acre. $160 \times 272\frac{1}{2} = 43560$ feet. If we extract the square root of this number, we obtain the side of a square field that will contain an acre; thus, $\sqrt{43560} = 208.712+$ feet. We now divide this number by 3.5 feet, and obtain $59.632+$, the number of divisions in the first row. We perceive, therefore, that there will be 60 hills, there being one more hill than divisions. Now, if we divide $208.712+$ by 59, the quotient will be 3.537 feet; that is, the hills in the first row may be 3.537 feet apart, instead of 3.5 feet. Thus, our first row will contain 60 hills, which will be 3.537 feet apart. Our next row will contain but 59 hills, the hills being planted in the quincunx order, thus:



To find the distance between the rows, we square 3.5 = 12.25; we then take half of $3.537 = 1.768$, which we square = 3.125824; we subtract this last number from 12.25, and obtain 9.124176. The square root of this number is 3.0206 feet, equal the distance between the rows. Now, if we divide $208.712+$ by 3.0206, we obtain $69+$; therefore, the number of rows will be 70. To obtain the number of hills in the field, we multiply 70 by $60 = 4200$. But, as there are 70 rows, and as half of the rows contain only 59 hills, we subtract 35 from 4200. Thus, $4200 - 35 = 4165$ hills, Ans.

18. $\$300 : \$700 :: 20 \text{ months} : 46\frac{2}{3} \text{ months}$, Ans.
19. $\$1500 \div 150 = 10$; $10 + 2 = 12$ children. $\$1500 \times 2 = \3000 ; $\$3000 \div 3 = \1000 ; $\$3000 + \$1000 = \$4000$; $\$4000 \div 4 = \1000 ; $\$4000 + \$1000 = \$5000$; $\$5000 \times 2 = \$10,000$, Ans.
20. $7 - 5 = 2$ miles which B gains each day, and he will have to gain 80 miles before he overtakes A. $2 \text{ miles} : 80 \text{ miles} :: 1 \text{ day} : 40 \text{ days}$, the time which it takes B to

overtake A. And as B travels 7 miles each day, he will have to go, before he overtakes A, $40 \times 7 = 280$ miles, Ans.

21. 16lb. : 80lb. :: 24.4in. : 122 cubic inches of lead. $\frac{1}{4} \times 2 = \frac{1}{2}$ in.; lin. + $\frac{1}{2}$ in. = 1.5 inches; $1.5 \times 1.5 = 2.25$; $2.25 \times .785398 = 1.7671455$ = area of a section of the pipe. From this we subtract the area of a section of the calibre of the pipe. $1 \times .785398 = .785398$; $1.7671455 - .785398 = .9817475$; $122 \div .9817475 = 124.26$ + inches, = 10.35 + feet, Ans.
22. $.785398 \times 2 \times 2 = 3.141592$; $\frac{3}{4} \times \frac{3}{4} \times .785398 = .441786$; $3.141592 - .441786 = 2.699806$; $2.699806 \times 8 = 21.598448$; $\frac{3}{4} \times 2 = \frac{3}{2} = 1\frac{1}{2}$; $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$; $1.5 \times 1.5 = 2.25$; $.785398 \times 2.25 = 1.7671455$; $1.7671455 - .441786 = 1.3253595$; $21.598448 \div 1.3253595 = 16.29$ + in., Ans.
23. Let \$100 represent the *relative* value of what D pays; then $\$100 \times 1.08 = \108 will represent what C pays; $\$108 \times 1.10 = 118.80$, what B pays; and $\$118 \times 1.08 = \133.056 , what A pays. $\$100 + \$108 + \$118.80 + \$133.056 = \$459.856$.
- $\$459.856 : \$100 :: \$100 : \21.7417944 , D pays.
 $\$459.856 : \$108 :: \$100 : \23.4819122 , C pays.
 $\$459.856 : \$118.80 :: \$100 : \25.8314927 , B pays.
 $\$459.856 : \$133.056 :: \$100 : \28.9314417 , A pays.
- $20 \times 20 \times 20 = 8000$; $\$459.856 : \$133.056 :: 8000 : 2314.742006$; $\sqrt[3]{2314.742006} = 13.22$ + ft., which A takes; $\$459.856 : \$251.856 :: 8000\text{ft.} : 4381.47594\text{ft.}$; $\sqrt[3]{4381.47594} = 16.36$ + ft.; $16.36 - 13.22 = 3.14$ + ft. which B takes. $\$459.856 : \$359.856 :: 8000\text{ft.} : 6260.324971\text{ft.}$ $\sqrt[3]{6260.324971} = 18.42\text{ft.}$; $18.42 - 16.36 = 2.06$ + ft. which C takes. $20 - 18.42 = 1.58$ + ft. which remains for D.
24. As B paid 20 per cent. more than A, and 10 per cent. less than C, we obtain their shares paid as follows:

For each dollar A pays, B pays \$1.20, and C pays \$1.33 $\frac{1}{3}$;
 $\$1 + \$1.20 + \$1.33\frac{1}{3} = \$3.53\frac{1}{3}$. Therefore,

$\$3.53\frac{1}{3} : \$1.00 :: \$10.60 : \3.00 , A paid.

$\$3.53\frac{1}{3} : \$1.20 :: \$10.60 : \3.60 , B paid.

$\$3.53\frac{1}{3} : \$1.33\frac{1}{3} :: \$10.60 : \4.00 , C paid.

As A paid \$3.00, his share of the stone will be

$$\frac{300}{1060} = \frac{15}{53}; \text{ B's, } \frac{360}{1060} = \frac{18}{53}; \text{ C's, } \frac{400}{1060} = \frac{20}{53}.$$

$65^2 = 4225$, square of the diameter of the stone.

$3^2 = 9$ in., square of the place for the axle.

$4225 - 9 = 4216$, to be divided among A, B, and C.

$4216 \times \frac{1}{53} = 1193.20$, A's part.

$4216 \times \frac{18}{53} = 1431.84$, B's part.

$4216 \times \frac{20}{53} = 1591$, C's part.

$4225 - 1193.20 = 3031.80$; $\sqrt{3031.80} = 55$ inches.

$65 - 55 = 10$ in.; $10 \div 2 = 5$ inches, A grinds off.

$3031.80 - 1431.8 = 1600$; $\sqrt{1600} = 40$ inches.

$55 - 40 = 15$; $15 \div 2 = 7\frac{1}{2}$ inches, B grinds off.

$1600 - 1591 = 9$; $\sqrt{9} = 3$ inches.

$40 - 3 = 37$; $37 \div 2 = 18\frac{1}{2}$ inches, C grinds off.

A grinds off 5 inches, B $7\frac{1}{2}$ in., and C $18\frac{1}{2}$ in., Ans.

NOTE. — In the solution of this problem we have omitted small fractions.

25. It is evident that in every case the drawing off of one gallon from the cask full leaves in it $\frac{9}{10}$ of its previous contents. Hence the quantity of wine left the first day is $\frac{9}{10}$ of 10 gallons; the second day, $\frac{9}{10}$ of that; and so on, till at the 20th day it is only 10 gallons multiplied by the twentieth power of $\frac{9}{10}$; and if this quantity be taken from 10 gallons, the remainder will be the quantity of water. By similar reasoning it would be shown that the quantity of water contained in the cask, at the end of the second period of twenty days, would be equal to the quantity last mentioned, multiplied also by the twentieth power of $\frac{9}{10}$. Now, the twentieth power of $\frac{9}{10}$, or .9, is .121576-65459. (.9⁴ = .6561; .9⁸ = .43046721; .9⁸ \times .9⁸ \times

$.9^4 = .9^{20} = .12157665459$.) The product of this by 10 taken from 10, the remainder is 8.7842334541, the quantity of water in the cask at the end of 20 days; and the product of this by $.9^{20}$ is 1.0679577+ gallons, or more than a gallon and half-pint, Ans.

26. $18.5 \times 18.5 \times 18.5 \times 8 = 50653$; $\sqrt[3]{50653} = 37$ in. wide; $8 \times 8 \times 8 \times 8 = 4096$; $\sqrt[3]{4096} = 16$ in. deep, Ans.

27. As the metal is lin. thick, the diameter of the inner sphere is 3in.; $5 \times 5 \times 5 \times .5236 \times \frac{1}{15} \frac{2}{3} = 16.8861$ lb., weight of the shell, if it were solid iron; $3 \times 3 \times 3 \times .5236 \times \frac{1}{15} \frac{2}{3} = 3.6473976$ lb., weight of the inner sphere, if it were iron; $16.8861 - 3.6473976 = 13.2387$ +lb., Ans.

28. As the two hands had precisely changed positions, they together had passed round through all the spaces of the dial-face; but, as the minute-hand always goes through 60 spaces while the hour-hand goes through 5, both going through 65, therefore $65 : 5 :: 60$ (the distance passed over by both hands) : $4\frac{2}{3}$ min. or spaces, passed over by the hour-hand, and which is also the distance the minute-hand was in advance of the hour-hand. But at 2 o'clock the hour-hand was 10 minutes in advance of the minute-hand; consequently the minute-hand had gained 10min. + $4\frac{2}{3} = 14\frac{2}{3}$ min. on the hour-hand; then, since the minute-hand always gains 55min. in 60min., how long was it in gaining $14\frac{2}{3}$ min.?

$55 : 60 :: 14\frac{2}{3} : 15\frac{1}{3}$ min. = 15 min. 56 $\frac{2}{3}$ sec. after 2, Ans.

29. $20 \times 20 = 400$; $400 \div 3 = 133.\bar{3}$; $\sqrt{133.\bar{3}} = 11.5469$; $\frac{11.5469^3}{1} = 1539.58$ +cu. in., Ans.

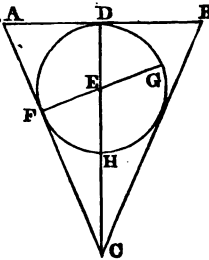
30. $90 \times 40 = 3600$; $\sqrt{3600} = 60$ lb., true weight, Ans. $90 - 60 = 30$ lb.; $60 - 40 = 20$ lb.; 30 lb. : 20 lb. : : 3 ft. : 2 ft.; that is, the arms of the scales are to each other as 2ft. to 3ft., Ans.

31. Both wheels being of the same height, and the outer

wheel making two turns while the inner one makes only one turn, it will follow that the outer ring will be twice the circumference of the inner ring. The distance between the rings being 5 feet, and the circumferences of circles being as their diameters, it will also follow that the diameter of the inner ring will be 10 feet, and the diameter of the outer ring 20 feet. And if the diameter be 20 feet, the circumference will be $62.83+$ feet, Ans.

32. $72 \times 72 \times 3.141598 = 57001d. = 237\text{£}. 10s. 1d., \text{Ans.}$

33. The annexed diagram may represent the conical glass, A B C being the cone, and F D G H a globe or sphere immersed in it. If A B be 5 inches, A D will be 2.5 inches, because A D is half of A B. A D C is a right-angled triangle; therefore the side A C may be found; thus, $\sqrt{A D^2 + D C^2} = A C$; $\sqrt{2.5 \times 2.5 + 6 \times 6} = 6.5 = A C$. Because A D E F is a regular figure, and the angles A D E and A F E being equal, each being a right angle, and the sides D E and F E being also equal, because they are radii of the circle D F H G, the sides A D and A F are also equal. A D is 2.5 inches; A F is also 2.5 inches. If A C be 6.5 inches, and A F 2.5 inches, F C will be 4 inches; $6.5 - 2.5 = 4$ inches. Then, by similarity of triangles (see page 389), C D : D A :: C F : F E; $6in. : 2.5in. :: 4in. : 1\frac{2}{3}in. = F E$. If F E be $1\frac{2}{3}in.$, F G will be $3\frac{1}{3}in. = 1\frac{1}{2}in.$, because F G is the diameter of the sphere, and F E the radius, or semi-diameter. By mensuration of solids (see pages 435 and 436), we find the contents of the cone and sphere in the following manner :



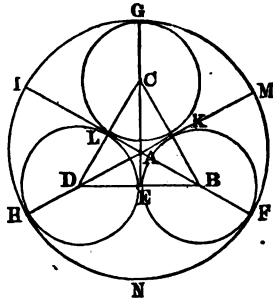
$5 \times 5 \times .785398 \times 2 = 39.2699in. = \text{contents of the cone};$

$1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3} \times .5236 = 19.3925in. = \text{contents of the sphere};$

$19.8774in. = \text{the cubic inches of water}$

that will remain in the cone after the sphere is immersed. Having taken it for "granted" that cones, spheres, and all similar bodies, are to each other as the cubes of their homologous sides, we say, As the quantity of water it requires to immerse the sphere in the *given* cone is to the cube of the diameter of the sphere, so is any other quantity of water in the conical glass to the cube of the diameter of a sphere that may be immersed in it. Now, the quantity of water given to immerse the *required* sphere is $\frac{1}{4}$ of the contents of the conical glass = $\frac{32.2692}{4}$ = 7.85398 cubic inches. The cube of the diameter of the *given* sphere is $13 \times 13 \times 13 = 1297 \text{ in.}$ Therefore $19.8774 \text{ in.} : 1297 \text{ in.} :: 7.85398 \text{ in.} : 14.634114529 \text{ in.};$
 $\sqrt[3]{14.634114529} = 2.445 \text{ in., Ans.}$

34. Let the larger circle of the annexed figure represent the farm of the lady, and the three smaller circles the farms of her daughters. To construct this figure, make the equilateral triangle C B D, each of whose sides is 10 rods, or inches. Bisect each of the sides D C, C B, B D, in the points L, E, K; and draw the lines I F, E G, H M, at pleasure. Upon the points C, D, B, as centres, and with the distance C L as a radius, describe the circles G L K, L H E, K E F. Upon the point A, as a centre, where the lines I F and H M intersect each other, and with the radius A G, describe the circle G I H N F M, and it will touch the peripheries of the smaller circles without cutting them. The sides of the triangle being 10, the diameter of each of the smaller circles will be 10. Because C D E is a right-angled triangle, C E = $\sqrt{C D^2 - D E^2}$; $\sqrt{10 \times 10 - 5 \times 5} = 8.660254 +$;



and as CDE and ADE are similar triangles, CE : CD :: DE : DA; that is, 8.660254 : 10 :: 5 : 5.77-35027 = AD. If we add HD = 5 to DA, we have the semi-diameter of the larger circle, 5.7735027 + 5 = 10.7735027. By multiplying this last number by 2, we have the diameter of the larger circle, 10.7735027 \times 2 = 21.5470054. As the area of a circle may be found by multiplying the square of the diameter by .785398, therefore, by dividing the area by .785398, the quotient will be the square of the diameter. The area of the lady's field is 500 acres = 80000 square rods; 80000 \div .785398 = 101859.18, square of the diameter; $\sqrt{101859.18}$ = 319.154006 + rods = diameter of the lady's farm. To find the diameter of each of the daughters' farms, we say, As the diameter of the larger circle in the diagram is to the diameter of one of the smaller circles in the diagram, so is the diameter of the lady's farm to the diameter of either of her daughters' farms. 21.5470054rd. : 10rd. :: 319.154006rd. : 148.119889 + rd. = diameter of the daughters' farms; and the distance of their houses from each other, 148.119889 \times 148.119889 \times .785398 = 17231.2406 + square rods in each of the daughters' farms; 17231.24 + rd. \div 160 = 107A. 2R. 31.24 + rods = acres, &c., in each of their farms; 107A. 2R. 31.24rd. \times 3 = 323A. 0R. 13.72rd., amount of the 3 daughters' farms; 500A. — 323A. 0R. 13.72rd. = 176A. 3R. 26.28rd. the lady retained. To find the distance of the lady's dwelling-house from those of her daughters, we subtract the semi-diameter of either of the daughters' farms from that of the lady's; thus, 319.154 + rd. \div 2 = 159.577 + rd.; 148.119 + rd. \div 2 = 74.059 + rd.; 159.577 + rd. — 74.059 + rd. = 85.518 + rd.

We therefore find that each daughter's farm contained 107A. 2R. 31.22p. The mother retained 176A. 3R. 26.34p. The distance from one daughter's house to the

other was $148.119817 +$ rods. The mother's dwelling-house was distant from her daughters, $85.51 +$ rods,
Ans.

This question can also be solved by using the principle laid down in Art. 645.

85. The pupil, to understand this problem, will first obtain the number of feet in the diameter of the garden; $10 \times 16.5 = 165$ feet; $165 - 5 = 160$. The trees are, therefore, to be set on a piece of ground 160 feet in diameter. Let the pupil place 1 tree in the centre of the garden; around this let him place 6 other trees, at the distance of 10 feet from each other; he will then perceive that they stand in a hexagonal form. Let him enlarge this hexagon by placing another row of trees around it at the distance of 10 feet each; and this will require 12 additional trees. If we examine this hexagon, we shall find that each side of it contains 3 trees. Let us enlarge this hexagon, by placing another row of trees around it, and we shall find it will require 18 trees, and that each side of the hexagon contains 4 trees. We continue thus to enlarge the hexagon, until we have set 8 rows round the centre tree. Each side of the hexagon will then contain 9 trees. To compute the number of trees in the hexagon, we find the number of trees that compose the periphery of the first hexagon to be 6 trees, and the number that compose the periphery of the larger hexagon to be 48. We therefore add 6 to 48, and multiply the sum by the half of $8 = 4$; thus, $6 + 48 = 54$; $54 \times 4 = 216$. To this we add the tree in the centre, $216 + 1 = 217$. If we now examine our figure, we find we can set 4 more trees at the base of each side of the hexagon, within the limits of the prescribed field. Therefore, 4 times $6 = 24$, to be added to 217; thus $217 + 24 = 241$ trees, Ans.

OPERATION BY ALGEBRA.

90s. = 1080d. ; 3q. 9d. = 45d. Let x = the time in which

B can do the work. Then $9 : x :: 45 : \frac{45x}{9}$ = the sum

which C must receive from A in part payment for his

labor. Then $45 + \frac{45}{x}$ = the money received by C for

his 2 days' labor. Now, it is evident that the sum received by C must bear the same proportion to the sum received for the whole work, as the part of the work which he performs bears to the whole work.

Hence $\frac{45 + \frac{45x}{9}}{1080} = \frac{x + 9}{216}$ denote the part of the work per-

formed by C. Therefore $\frac{x + 9}{216}$ work : 1 work :: 2 days :

$\frac{2}{x + 9} = \frac{432}{x + 9}$ = the time in which C would do the whole work. Now, since from the question and the operation

we see that A performs $\frac{5}{9}$ of the work, B $\frac{x}{9}$ of it, and C

$\frac{x + 9}{216}$ of it, it is evident that $\frac{5}{9} + \frac{x}{9} + \frac{x + 9}{216} = 1$ work.

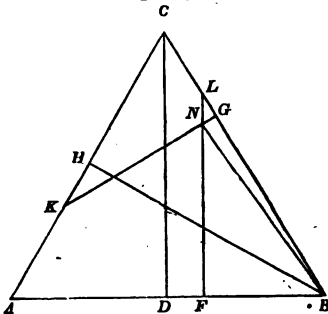
Or, $\frac{x}{9} + \frac{x + 9}{216} = 1 - \frac{5}{9} = \frac{4}{9}$, which, being reduced and transposed, gives $x = 15$ days = the time in which B

would reap the field. $\frac{432}{x + 9} = \frac{432}{24} = 18$ days = the time in which C would reap the field, Ans.

37. $4 - \frac{1}{8} = 3\frac{7}{8}$; $4 : 3\frac{7}{8} :: 40 : 1\frac{1}{4}^{\frac{1}{2}}$; $1\frac{1}{4}^{\frac{1}{2}}\text{yd.} = 24\text{na.}$; $24\text{na.} - 1\frac{1}{4}^{\frac{1}{2}}\text{na.} = 22\frac{1}{2}\text{na.}$; $24\text{na.} : 22\frac{1}{2}\text{na.} :: 2\frac{1}{4}\text{yd.} : 2\frac{3}{8}\text{yd.} = 1\frac{3}{8}\text{yd.}$; $1\frac{1}{4}^{\frac{1}{2}} \times 1\frac{3}{8}^{\frac{1}{2}} = 2\frac{9}{8}\frac{3}{8}^{\frac{1}{2}} = 81\frac{1}{2}\frac{3}{8}\text{yd.}$; $5\text{qr.} = 1\frac{1}{4}\text{yd.}$; $20 - 1\frac{1}{4} = 18\frac{3}{4}\text{yd.}$; $\frac{18\frac{3}{4}}{20} = 1\frac{1}{8}$; $5\text{qr.} = 20\text{na.}$; $20 - \frac{1}{8} = 19\frac{7}{8}$; $19\frac{7}{8} \times 1\frac{1}{8} = 18\frac{9}{32}\text{na.}$; $81\frac{1}{2}\frac{3}{8}\text{yd.} = 1307\frac{1}{8}\text{na.}$; $1307\frac{1}{8} \div 18\frac{9}{32} = 71\frac{7}{8}\text{yd.}$, Ans.

38. Let the tower at A be 30 feet high; that at B, 40; at C, 50.
First. What point F in the side A B is equally distant from the top of the tower at A and the top of that at B?

The square of the distance from F to the top of the tower A is the square of its distance from the foot of A + 900; the square of the distance from F to the top of B is the square of the distance from the foot B + 1600; if the distances of F from the tops of A and B are equal, the square of its distance from the foot A is greater by 700 than the square of its distance from the foot B. $AF^2 - BF^2 = 700$. But $AF + BF = 200$; now, $AF^2 - BF^2 = (AF + BF)(AF - BF)$, $AF - BF = 3\frac{1}{2}$; and therefore $AF = 101\frac{3}{4}$, $BF = 98\frac{1}{4}$. Finding a similar point G on B C, we get $BG = 102\frac{1}{4}$, $CG = 97\frac{3}{4}$.



At what point L does the line FL perpendicular to AB meet B C? Draw C D perpendicular to A B.

Then, as A and B are similarly situated with regard to C, $AD = DB$, and each = 100.

Then $DF = 1\frac{3}{4}$; therefore $CL = 3\frac{1}{2}$, as D F must be $\frac{1}{2}$ of C L, if D B is $\frac{1}{2}$ of C B.

If $CL = 3\frac{1}{2}$, L G must be $CG - CL = 97\frac{3}{4} - 3\frac{1}{2} = 94\frac{1}{4}$.

Draw G K perpendicular to B C.

Now, the foot of the ladder is on the line F L, for every point in F L is equally distant from the top of the two towers A and B. Again, it is on the line G K, for every point in G K is equally distant from the top of the towers B and C; therefore it is on the intersection N of F L and G K.

Now, the triangle L N G is similar to C D B; then L N is 2 times N G, and $LN^2 = 4$ times NG^2 ; then $LG^2 = 3$ times NG^2 ; $LG^2 = 8883.0625$, then $NG^2 = 2961.0208\dot{3}$.

The square of the length of the ladder = height of tower

$$C^2 + C G^2 + N G^2.$$

Square of height of C,	2500
C G ² ,	9555.0625
N G ² ,	2961.02083
Square of length of ladder,	<u>15016.08333</u>
Length of ladder,	122.54+

To find the distance of the foot of the ladder from the foot of each tower, we subtract the square of the height of each tower from the square of the length of the ladder.

This gives

$\overline{N A^2} = 14116.083$	$N A = 118.811$
$\overline{N B^2} = 13416.083$	$N B = 115.827$
$\overline{N C^2} = 12516.083$	$N C = 111.875$

SECOND SOLUTION.

A line drawn from either angle to the middle of the opposite side divides the garden into two equal right-angled triangles; and the length of this line, found in the usual way, is 172.2+ft. Draw, in the same manner, lines from the other two angles, and the three lines will intersect each other at the centre; and the garden will be divided into six equal triangles, *similar* to the first two. Then $172.2+ : 200 :: 100 : 115.6\text{ft.} = \text{distance of the centre from the foot of either tower.}$ A ladder placed on this centre, reaching to the top of one of the towers, will be the hypotenuse of a vertical triangle, of which the tower is the perpendicular, and 115.6ft. the base. Now, since the three vertical triangles have equal bases, and, since the height of the tower B is an arithmetical mean between the heights of the towers A and C, it follows that the square of one base, plus one third of the squares of the three perpendiculars, will equal the squares of an *average* hypotenuse, or the length of a ladder, which, placed at some point, will reach to the top of each of the towers. Finally, find the distance of this point from the foot of each tower, as in the last paragraph of the first solution.

ADDITIONAL SOLUTIONS.

Ex. 16. Arith. p. 295 (Key, p. 156). It is evident that the interest on the note, for the required time, at 6 per cent., is equal to the interest of the *nominal* present worth for the same time, at $6\frac{1}{2}$ per cent. Therefore, the note must exceed the nominal present worth in the ratio of $6\frac{1}{2}$ to 6, or of 13 to 12; hence the *nominal* discount, or interest of the note, must have been $\frac{1}{13}$ of the note. Then, since the interest of any sum for one year at 6 per cent. is $\frac{6}{100}$ of the same sum, we have $\frac{6}{100} : \frac{1}{13} :: 1 \text{ year} : 1 \text{ year}, 3 \text{ months}, 11\frac{1}{3} \text{ days}$, Ans.

Ex. 36. Arith. p. 444 (Key, p. 248). As A could reap the field in 9 days, he would in 5 days reap $\frac{5}{9}$ of the field, and B and C would, in 5 days, reap $1 - \frac{5}{9} = \frac{4}{9}$ of the field. Now, the whole sum received for reaping the field was 90 shillings; consequently A would receive $\frac{5}{9}$ of 90s. = 50s. for his labor, and B and C $\frac{4}{9}$ of 90s. = 40s. for their labor.

But, by the question, B receives 3s. 9d. = 3.75s. less by employing C. Hence, as the sum which B receives is to the sum A receives, so is the sum B receives less by employing C to the sum A receives less by employing C. Of this proportion there are given only the second term, 50s., and the third term, 3.75s.

Since the product of the extremes is equal to the product of the means (Art. 336), it is evident that the product of the first and fourth terms will be $50 \times 3.75 = 187.5$.

Also, since the sum of the first, third, and fourth terms is 40s., and the difference between 40s. and the third term, 3.75s., is 36.25s., it is evident that the sum of the first and fourth terms will be 36.25.

Then, having the sum of the first and fourth terms, 36.25, and their product, 187.5, these terms may be found by Art. 552.

Thus, $36.25 \div 2 = 18.125$, half the sum of the two terms; $(18.125)^2 = 328.515625$, the square of half the sum; $328.515625 - 187.5 = 141.015625$, the square of half the difference; $\sqrt{141.015625} = 11.875$, half the difference of the first and fourth terms; $18.125 + 11.875 = 30s.$, the first term, or the sum B receives; and $18.125 - 11.875 = 6.25s.$, the fourth term, or the sum A receives less by employing C.

As B received 30 shillings, C received $40 - 30 = 10s.$

30s. : 90s. :: 5 days : 15 days, the time B would reap the field.

10s. : 90s. :: 2 days : 18 days, the time C would reap the field.

THE END.



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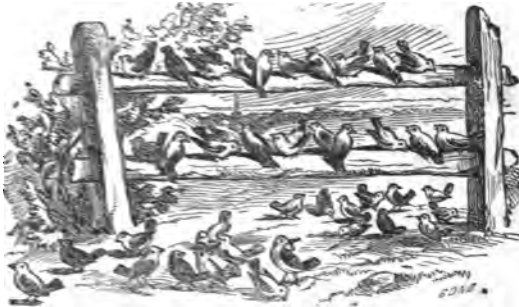
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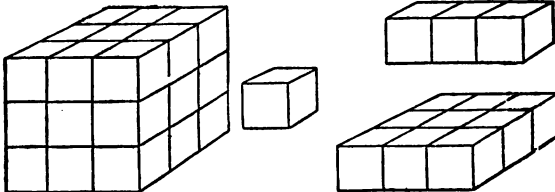
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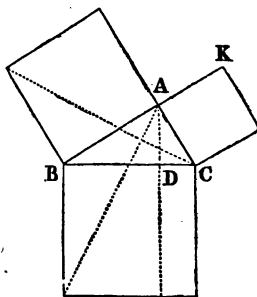
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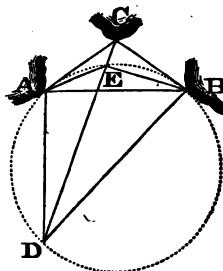
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